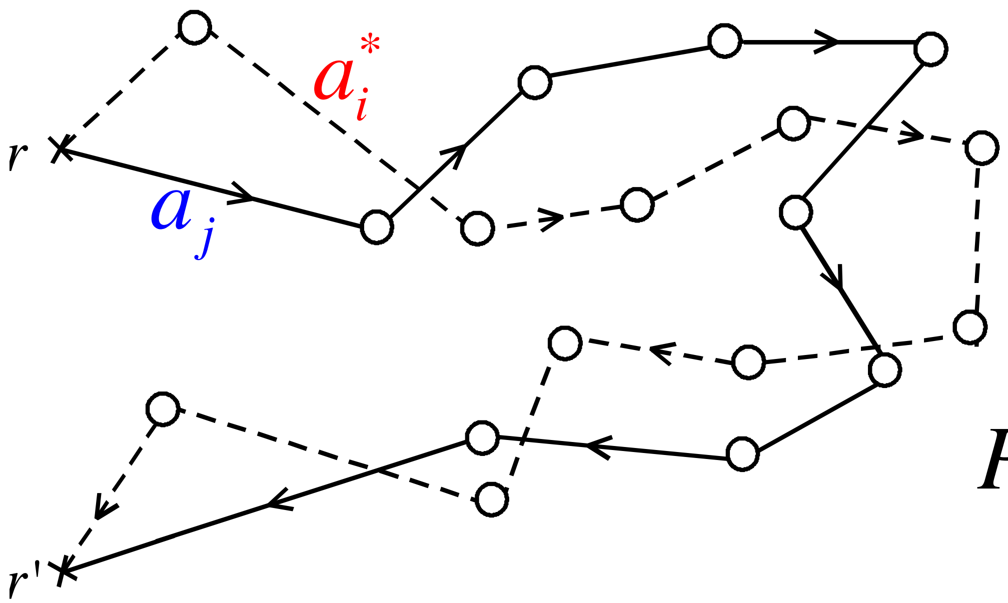
Part 2

Introduction to Mesoscopic Physics

- Transport, transmission and probability of quantum diffusion.
- Mesoscopic limit: characteristic length scales.
- Deviation from classical incoherent transport: quantum crossings.
- Weak localization and Sharvin effect.
- Universal conductance fluctuations.

Do coherent effects survive disorder average?

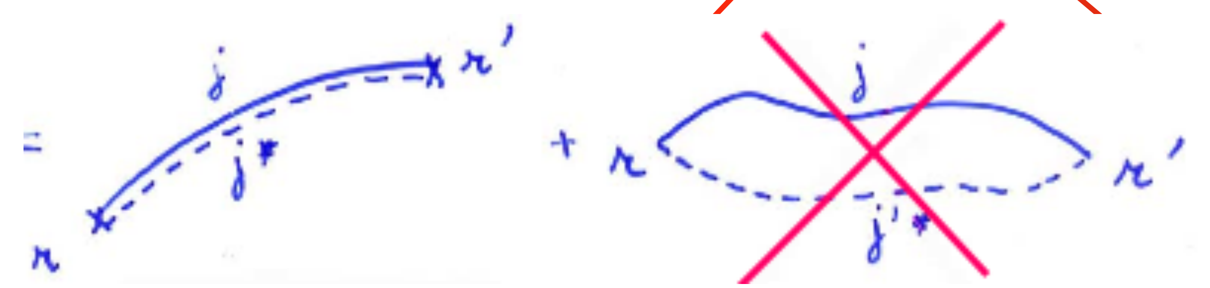
Quantum probability for electron diffusion between two points



$$P(r,r') = \sum_{i,j} \overline{a_i^*(r,r')\, a_j(r,r')}$$

$$P(r,r') = \sum_{j} \overline{\left| a_j(r,r') \right|^2} + \sum_{i \neq j} \overline{a_i^*(r,r') a_j(r,r')}$$

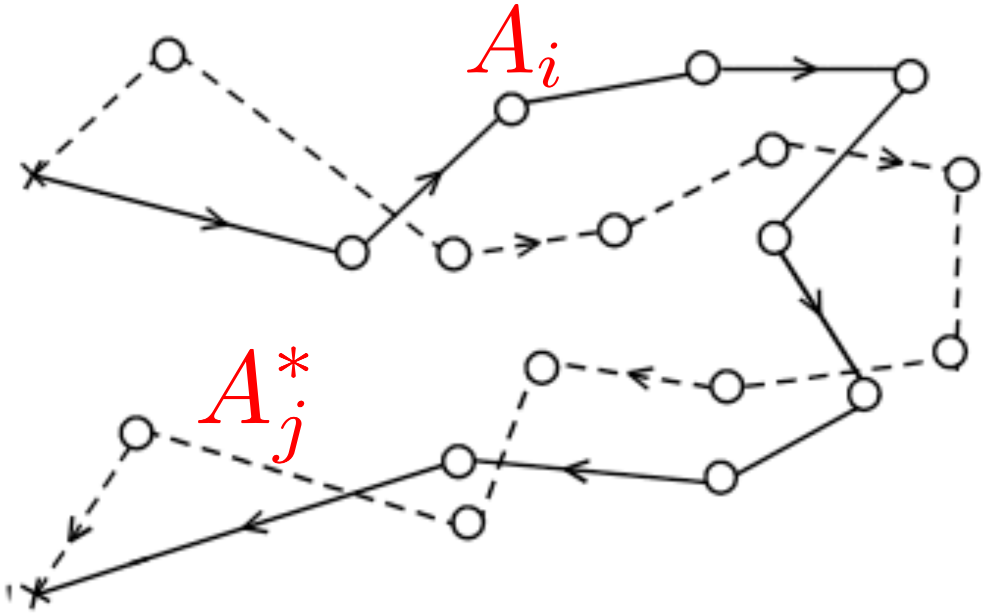
$$a_i^* \, a_j = |a_i| |a_j| e^{i(\delta_i - \delta_j)}$$

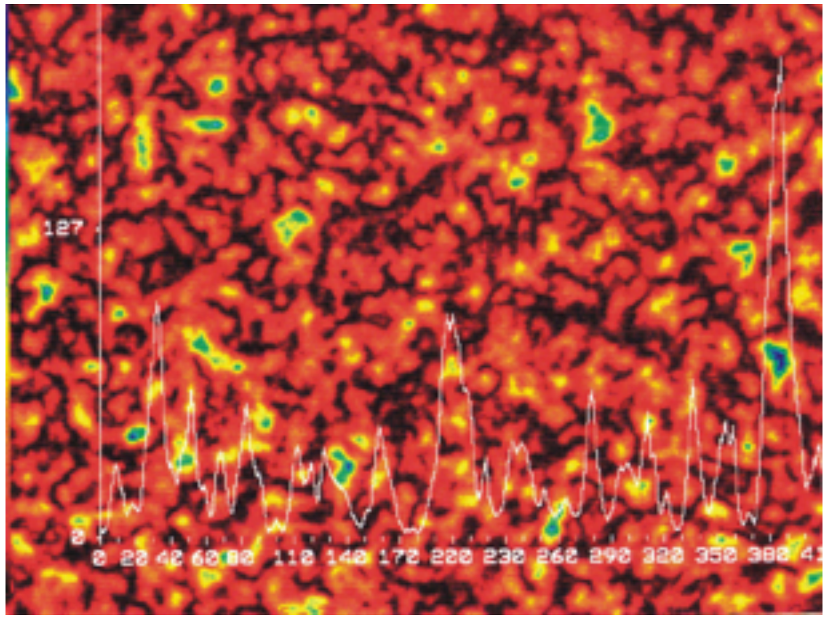


$$\delta_i - \delta_j \gg 1$$

Incoherent propagation !

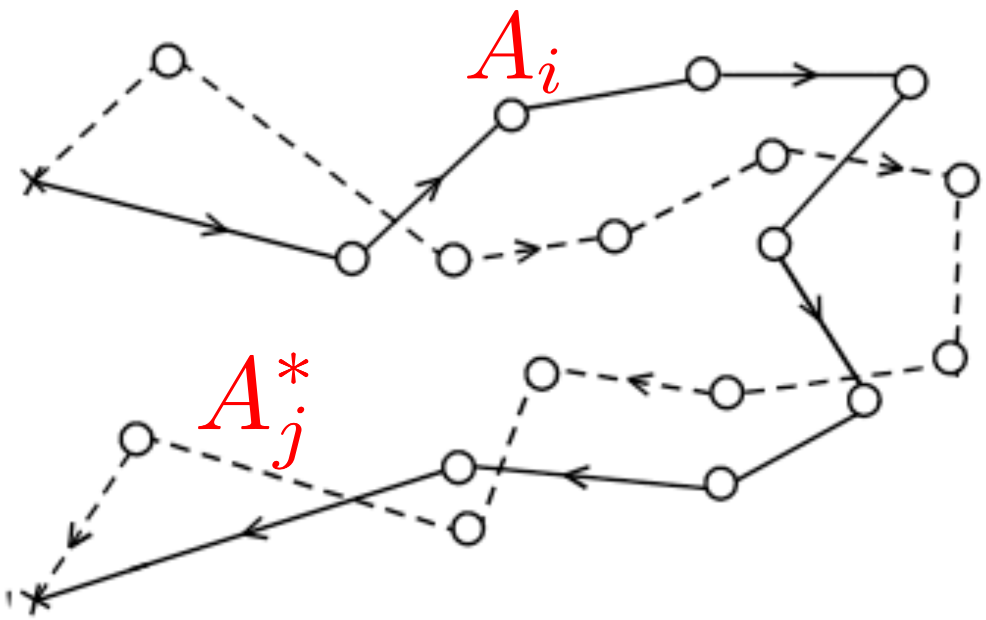
vanishes on average

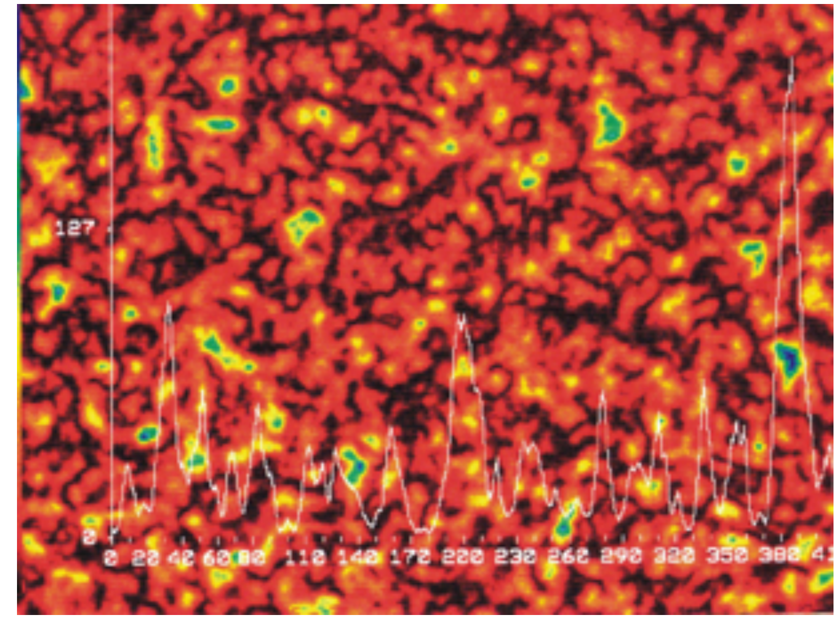




Before averaging : speckle pattern (full coherence)

Configuration average: most of the contributions vanish because of large phase differences.





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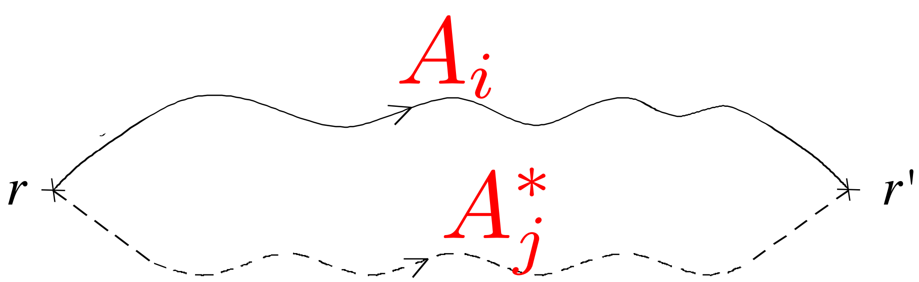
Configuration average: most of the contributions vanish because of large phase differences.

A new design !



Vanishes upon averaging

$$P_{cl}(\mathbf{r},\mathbf{r}') = \overline{\sum_j |A_j(\mathbf{r},\mathbf{r}')|^2}$$

Diffuson

To a good approximation, the incoherent contribution obeys a classical diffusion equation

$$\left(\frac{\partial}{\partial t} - D\Delta\right)P(r,r',t) = \delta(r-r')\delta(t) \quad \Leftrightarrow \quad \left(-i\omega + Dq^2\right)P(q,\omega) = 1$$

Incoherent electrons diffuse in the conductor with a *diffusion coefficient D*

with $D = \dfrac{v_g l}{3}$

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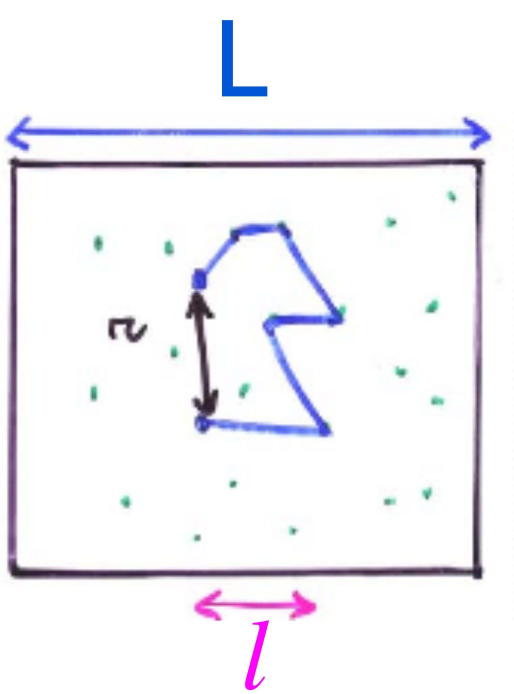


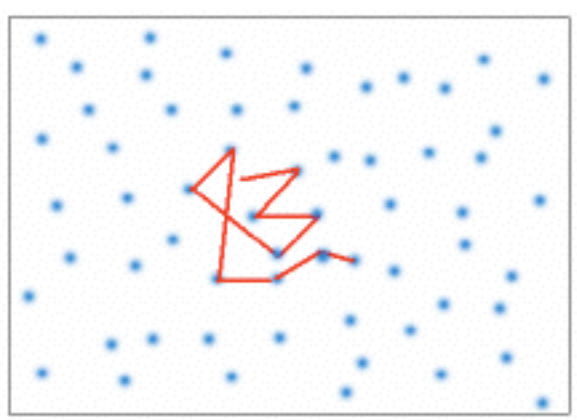
$$\langle r^2 \rangle = 2d\ Dt$$

space dimensionality

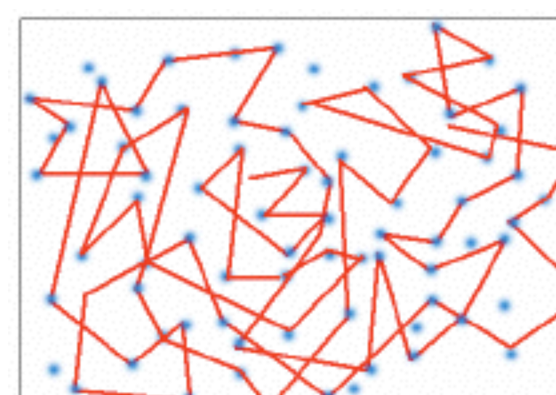
Thouless time

$$L^2 = D\tau_D$$

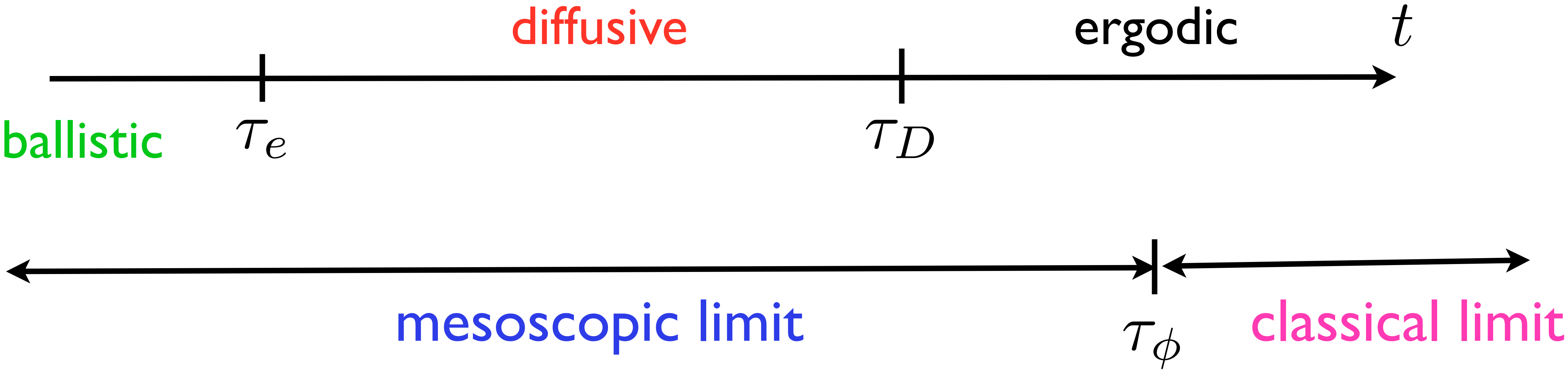
$t \ll \tau_D$

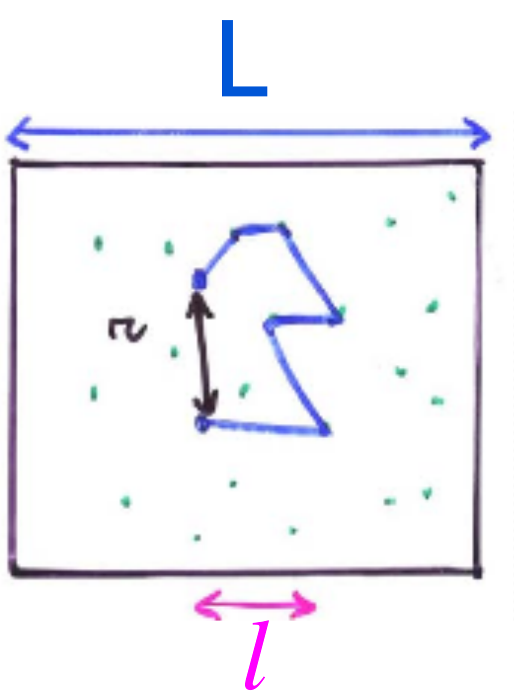


$t \gg \tau_D$



$l \ll L$





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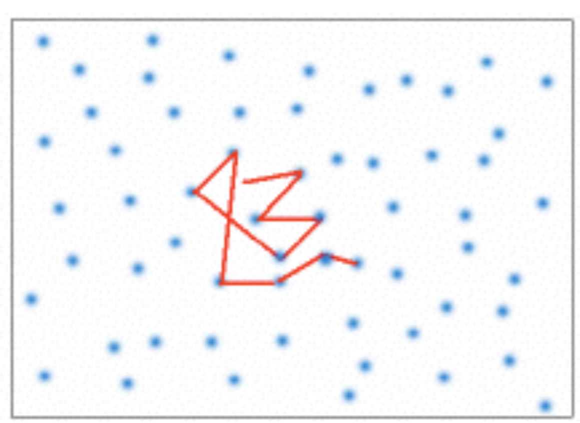

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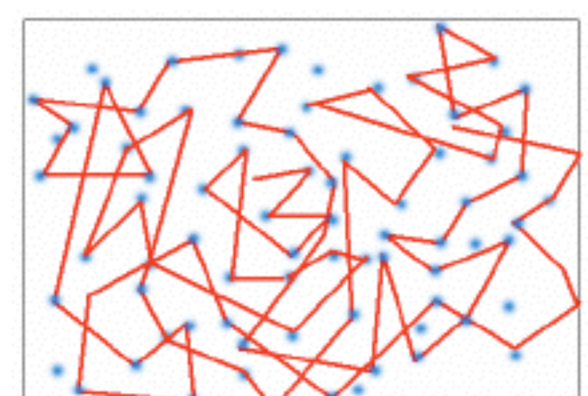
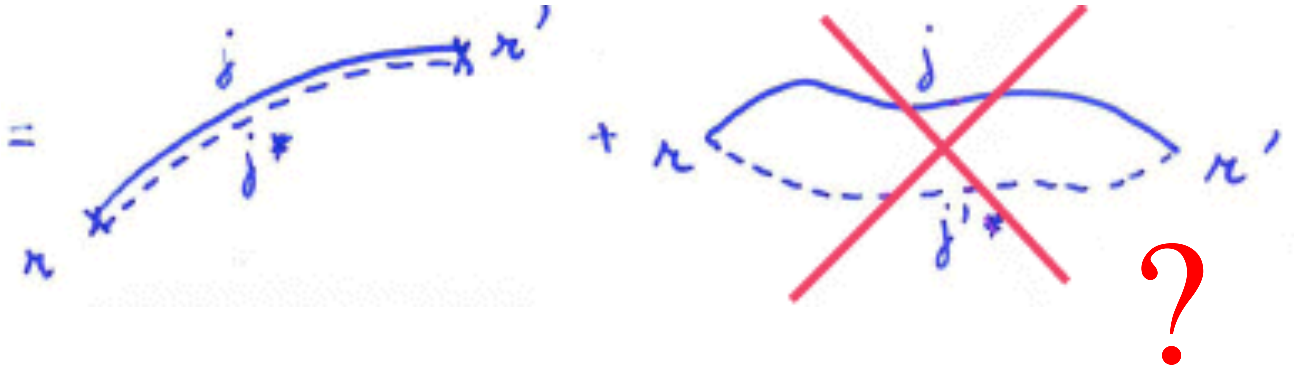


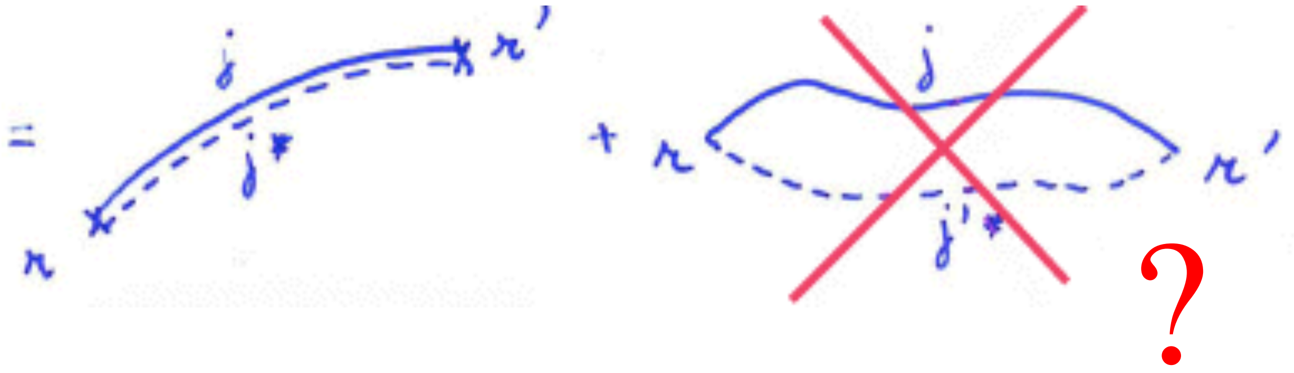
$$l \ll L$$

Coherent effects



What is the first correction *i.e.*, with the *smallest phase shift* ?

Coherent effects



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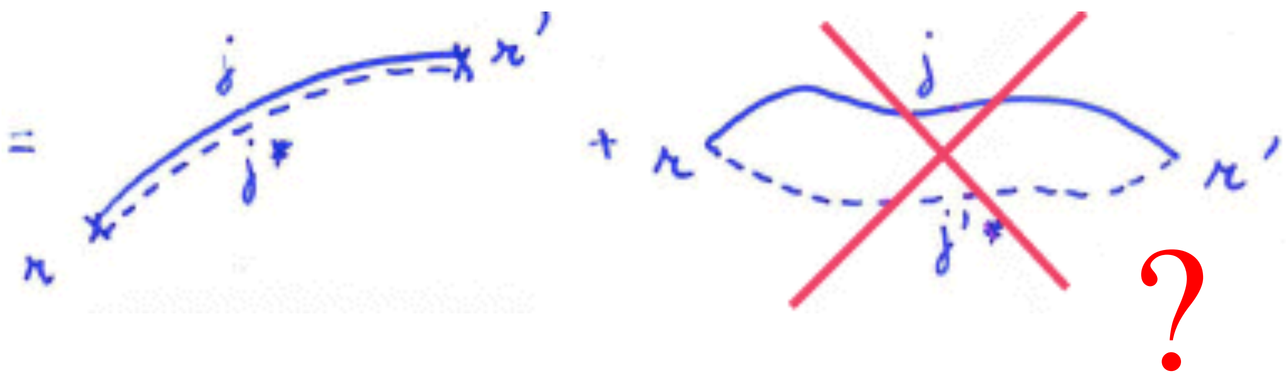
When amplitude paths cross

Coherent effects



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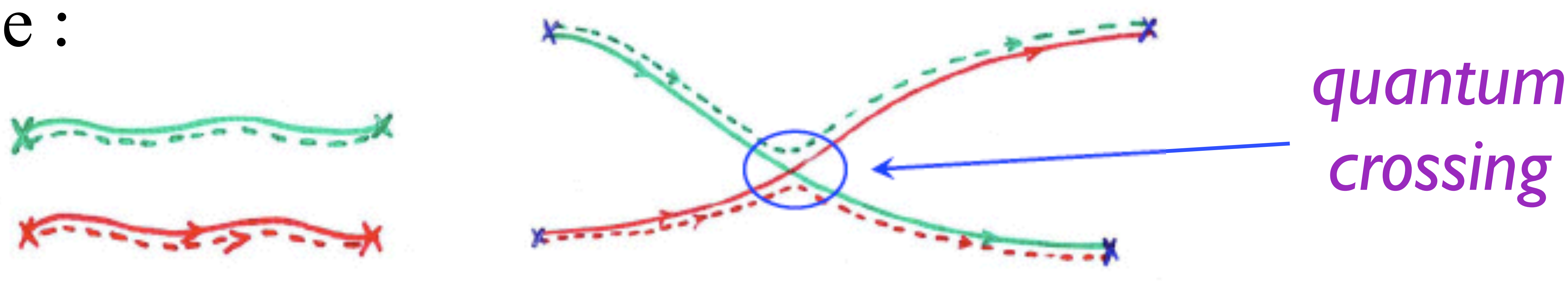
When amplitude paths cross

Example :



Classical diffusion   Exchange of amplitudes

Occurrence of a *quantum crossing* after a time t for a photon diffusing in a volume L^d

$$p_{\times}(t) = \frac{\lambda^{d-1} c t}{L^d}$$

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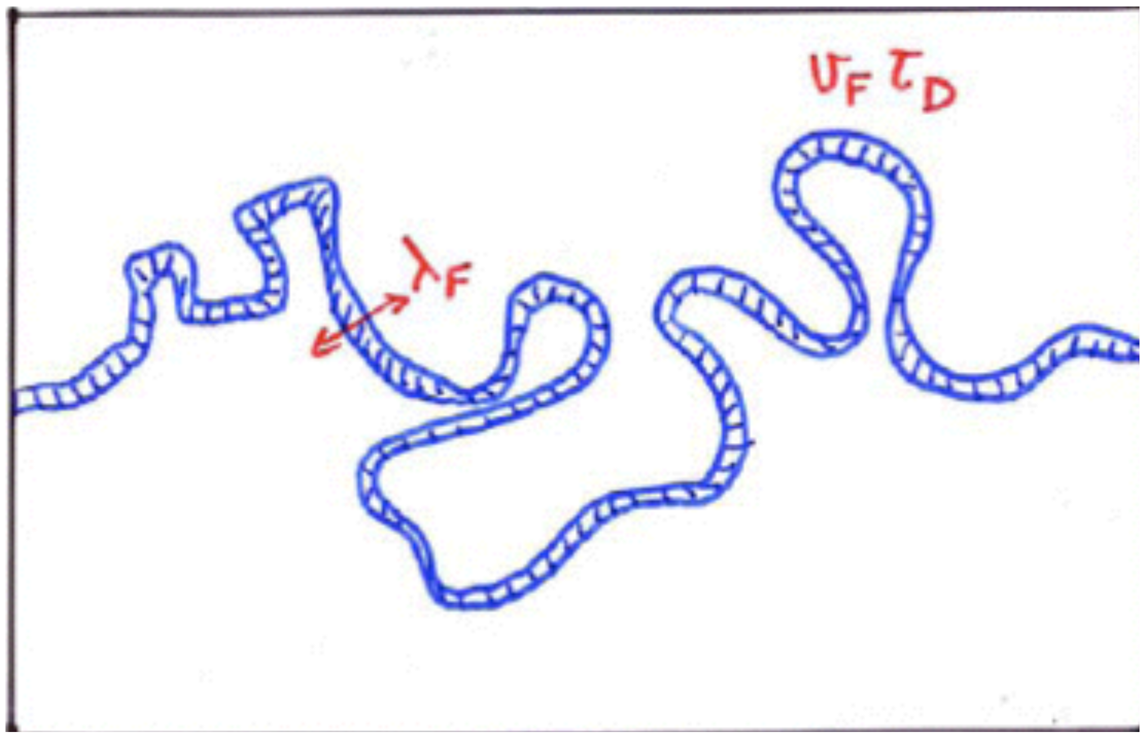

Occurrence of a *quantum crossing* after a time t for a photon diffusing in a volume L^d

$$p_\times(t) = \frac{\lambda^{d-1} c\, t}{L^d}$$

The time spent by a diffusing photon is $\tau_D = L^2/D$ so that

$$p_\times(\tau_D) = \frac{\lambda^{d-1} c\, \tau_D}{L^d} \equiv \frac{1}{g}$$

$$g = \frac{D}{c\lambda^{d-1}} L^{d-2}$$

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Physical meaning of this parameter ?

Electrical conductance of a metal

A metal can be modeled as a quantum gas of electrons scattered by an elastic disorder.

Classically, the conductance of a cubic sample of size L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.

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$$g = \frac{l_e}{3\lambda^{d-1}} L^{d-2} = G_{cl}/(e^2/h)$$

G_{cl} is the classical electrical conductance so that

$$G_{cl}/(e^2/h) \gg 1$$

A direct consequence: quantum corrections to electrical transport

Classical transport : $G_{cl} = g \times \frac{e^2}{h} \, \mathrm{with} \, g \gg 1$

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Classical transport : $G_{cl} = g \times \frac{e^2}{h} \text{ with } g \gg 1$

Quantum corrections: $\Delta G = G_{cl} \times \frac{1}{g}$

so that $\Delta G \simeq \frac{e^2}{h}$

A quantum phase transition: Anderson localization

Expansion in powers of quantum crossings $1/g$ allows to calculate quantum corrections to physical quantities.

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The diffusion coefficient D is reduced (weak localization) and becomes size dependent :

$$D(L) = D\left(1 - \frac{1}{\pi g}\ln\left(L/l\right) + \left(\frac{1}{\pi g}\ln\left(L/l\right)\right)^2 + \ldots\right) \qquad (d=2)$$

This *singular* perturbation expansion is not a simple coincidence but an expression of scaling

A renormalization of *D(L)* changes also *g(L)*:

$$g(L) = \frac{D(L)}{c\,\lambda^{d-1}} L^{d-2}$$

Scaling and its meaning : (P.W. Anderson *et al*.,1979)

If we know $g(L)$, we know it at any scale :

$$g\left(L(1+\varepsilon)\right)=f\left(g(L),\varepsilon\right)$$

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Expanding, we have $g\left(L(1+\epsilon)\right) = g(L)\left(1+\epsilon\beta(g)+O(g^{-5})\right)$

with $\beta(g) = \dfrac{d\ln g}{d\ln L}$ *(Gell-Mann - Low function)*

Scaling behavior :

$$g(L,W)=f\left(L/\xi(W)\right)$$

$\xi(W)$ is the localization length

Numerical calculations on the (universal) Anderson Hamiltonian

$d = 2$ $\quad$ $d = 3$

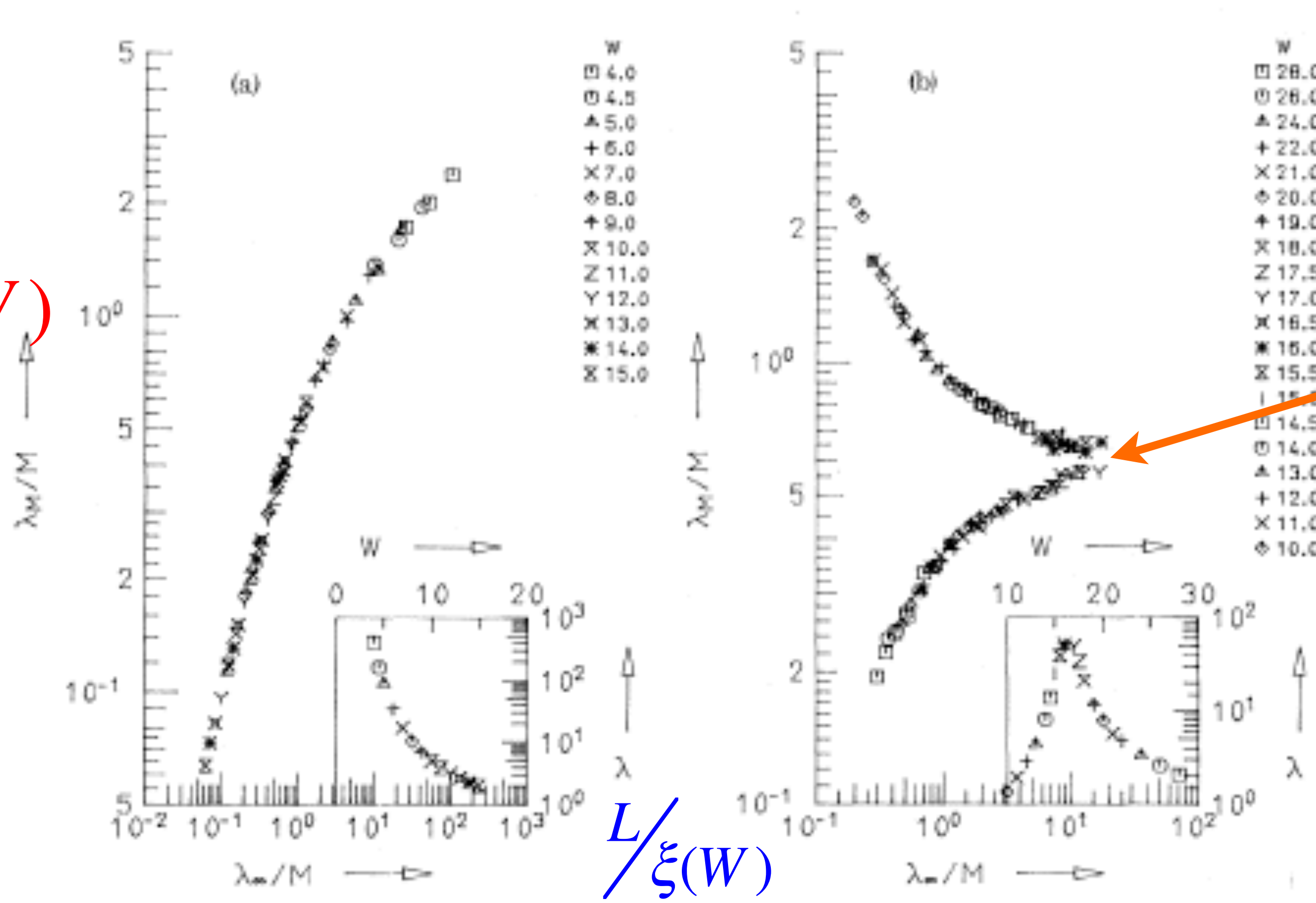

FIG. 1. Scaling function λ_M/M vs λ_∞/M for the localization length λ_M of a system of thickness M for (a) $d=2$ ($M \geq 4$) and (b) $d=3$ ($M \geq 3$). Insets show the scaling parameter λ_∞ as a function of the disorder W.

B.Kramer, A. McKinnon, 1981

Anderson localization phase transition occurs in $d > 2$

Weak disorder physics

Weak disorder limit: $\lambda << l \Rightarrow \mathrm{g} >> 1$

Probability of a crossing $\left(\propto 1/g\right)$ is small: phase coherent corrections to the classical limit are small.

Quantum crossings modify the classical probability (*i.e*. the Diffuson).

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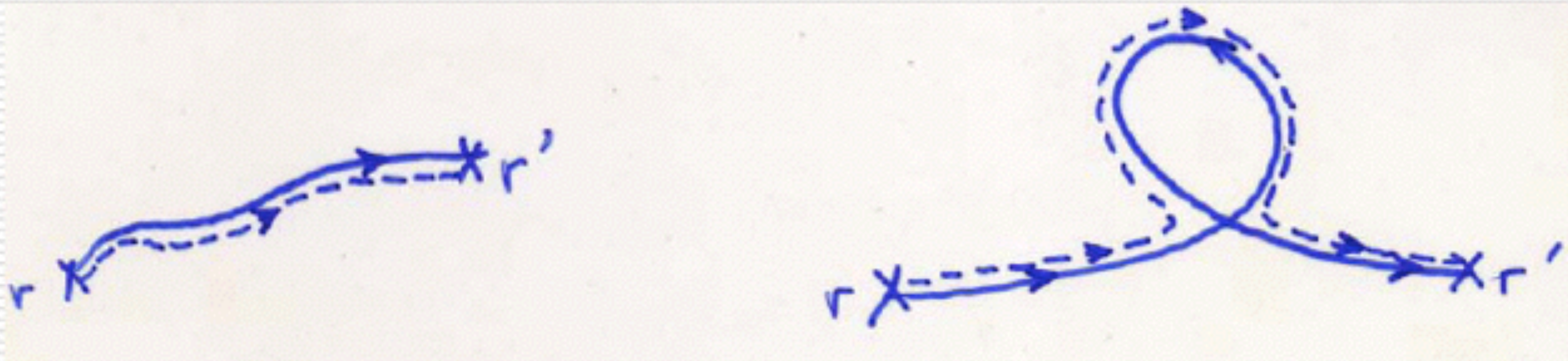
Due to its long range behaviour, the Diffuson propagates (localized) coherent effects over large distances.

Quantum crossings are independently distributed :

We can generate higher order corrections to the Diffuson as an expansion in powers of $1/g$

How to calculate $P_{\text{int}}(t)$?

$P_{\text{int}}(t) = \int P_{\text{int}}(r,r,t)\,d^d r$



$P_{cl}(r,r',t)$

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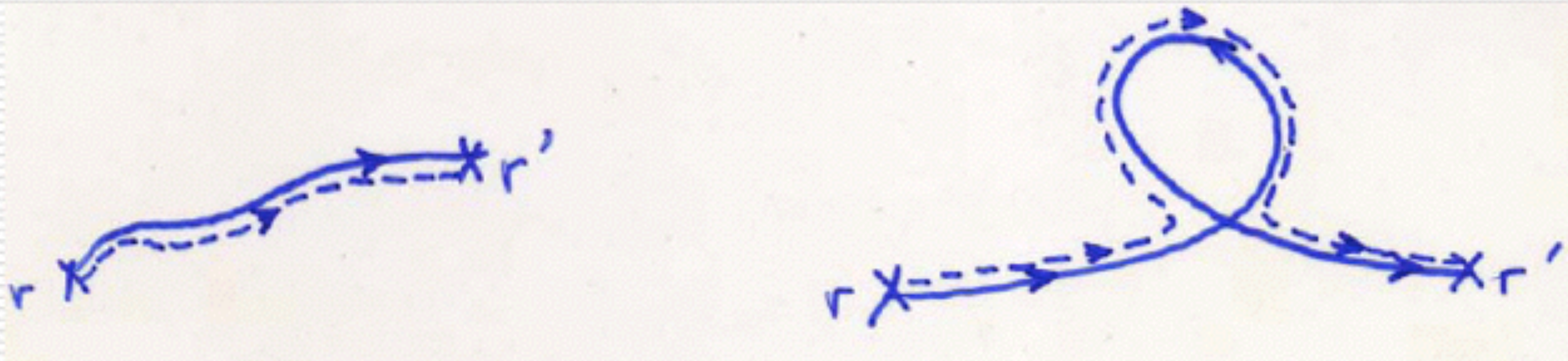

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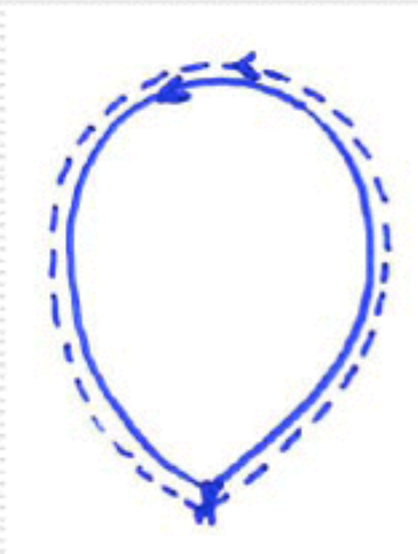
If $r = r'$

Diffuson



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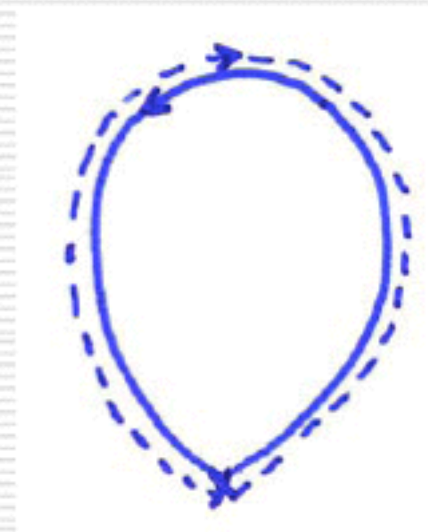


Cooperon

Classical return probability

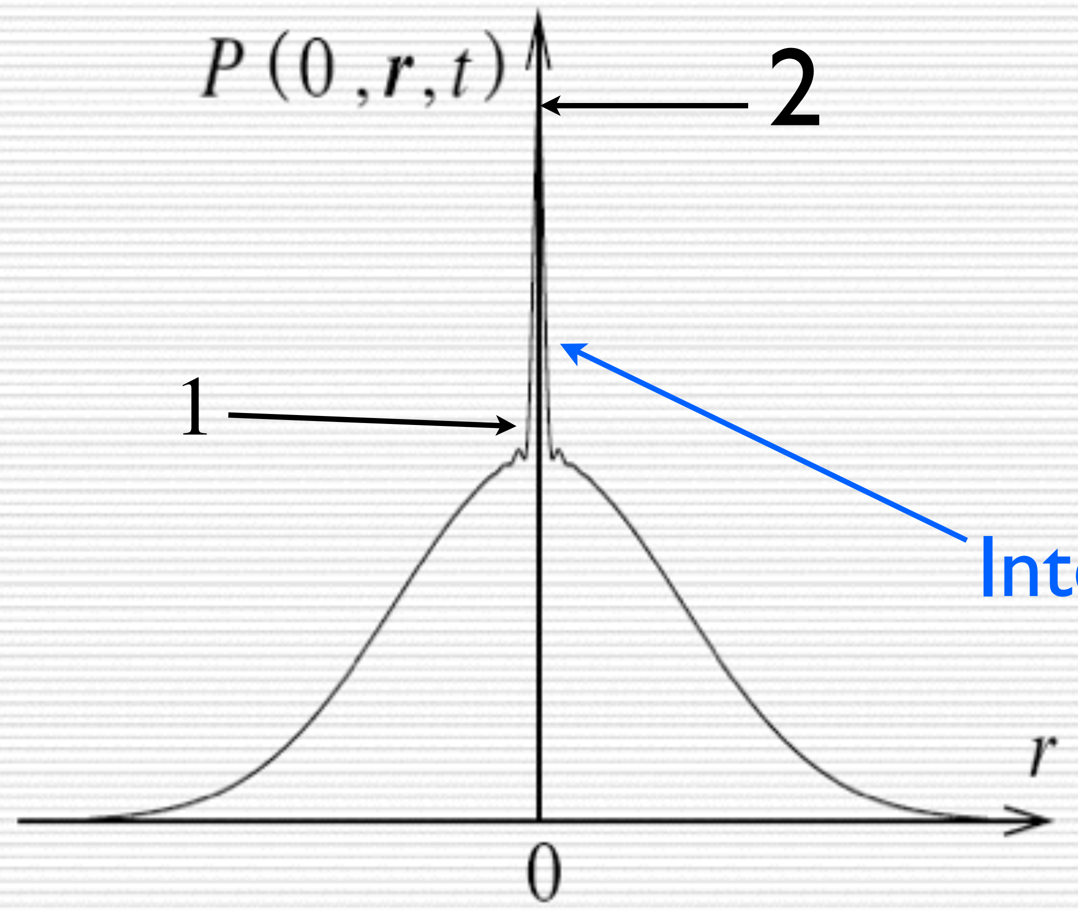
Interference term

$$P_{cl}(r,r,t) = P_{\text{int}}(r,r,t)$$

Return probability is doubled !!

If time reversal invariance

Probability $P_{cl} + P_{int}$



Interference correction

$$P(0,r,t) = \frac{e^{-\frac{r^2}{4Dt}}}{(4\pi Dt)^{d/2}}\left[1+\left(\frac{\sin k_F r}{k_F r}\right)^2\right]$$

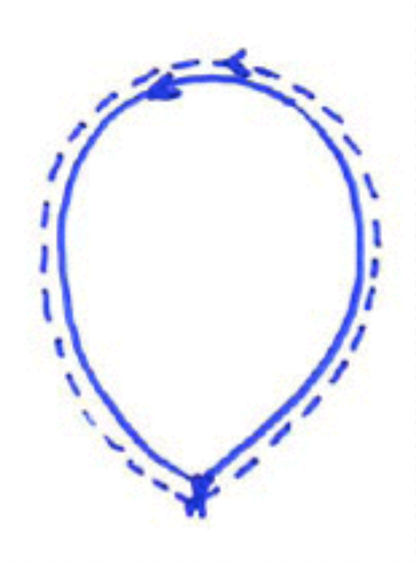
Classical diffusion

Important difference :

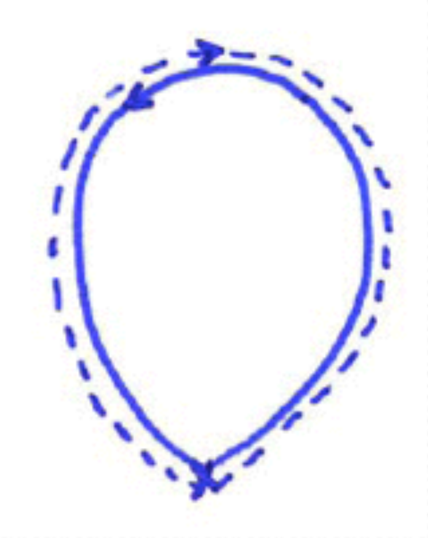
$P_{cl}(r,r',t) \Rightarrow$ paired trajectories follow the same direction

$P_{\text{int}}(r,r',t) \Rightarrow$ paired trajectories follow opposite directions

Diffuson

A_j  A_j^*

Cooperon

A_j  A_j^{*T}

A_j A_j^T have the same phase $\int \vec{p}.\vec{dl}$

$$P_{\text{int}}(r,r,t) = P_{cl}(r,r,t)$$

If time reversal invariance

If phase coherence between the reversed trajectories is preserved

In the presence of a dephasing mechanism that breaks time coherence, only trajectories with $t < \tau_\phi$ contribute.

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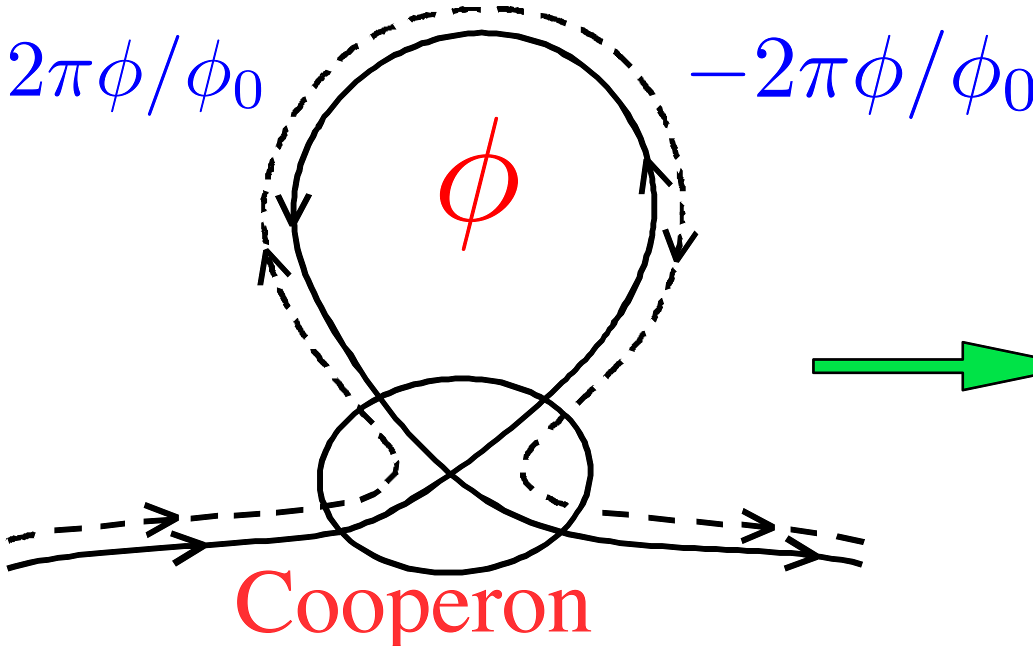
In the presence of an Aharonov-Bohm flux, paired amplitudes in the Cooperon acquire opposite phases:



the phase difference becomes: $4\pi\phi/\phi_0$

→ $\phi_0/2$ periodicity of the Sharvin effect

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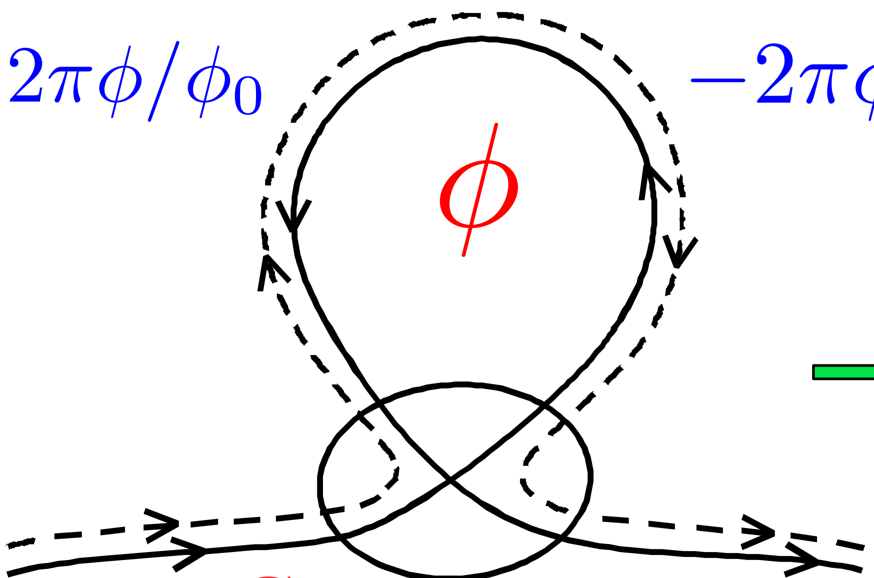


the phase difference becomes: $4\pi\phi/\phi_0$

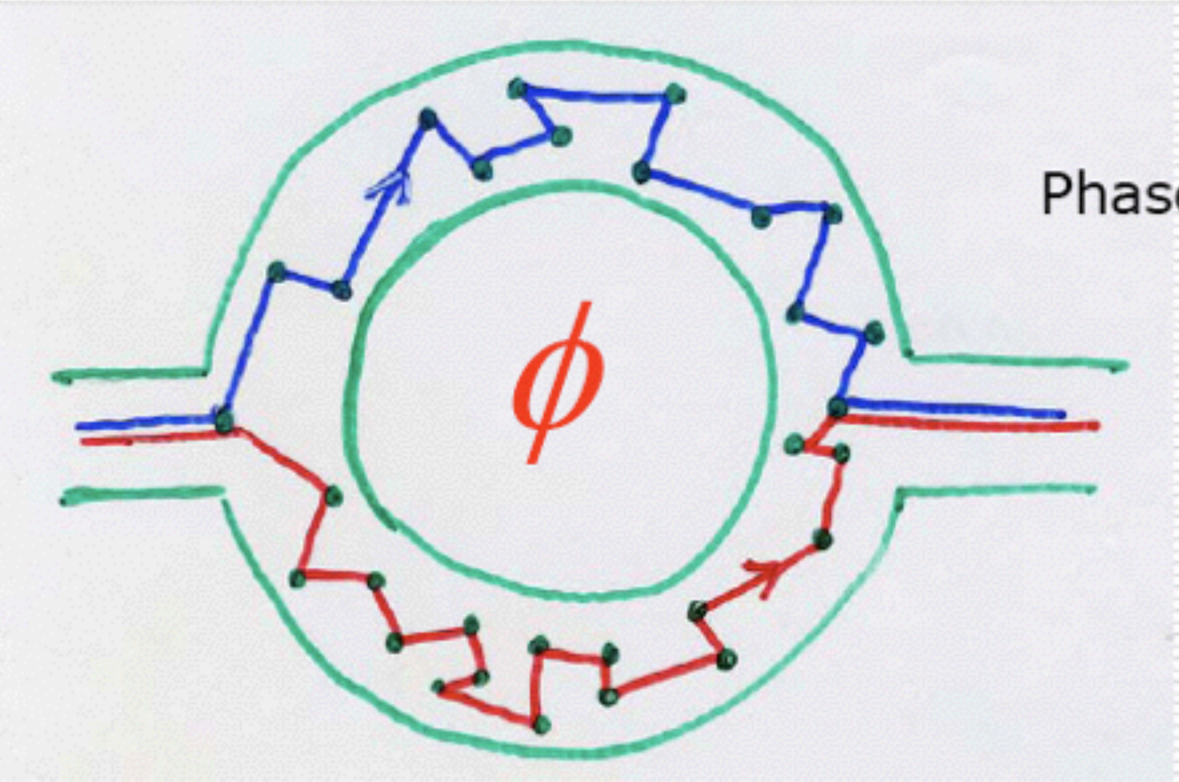
→ $\phi_0/2$ periodicity of the Sharvin effect

$P_{int}(r, r', t)$ is obtained from the *covariant* diffusion equation

effective charge 2e

$$\left(\frac{1}{\tau_\phi} + \frac{\partial}{\partial t} - D\left[\nabla_{r'} + i\frac{2e}{\hbar}\mathbf{A}(r')\right]^2\right) P_{int}(r, r', t) = \delta(r - r')\delta(t)$$

Webb

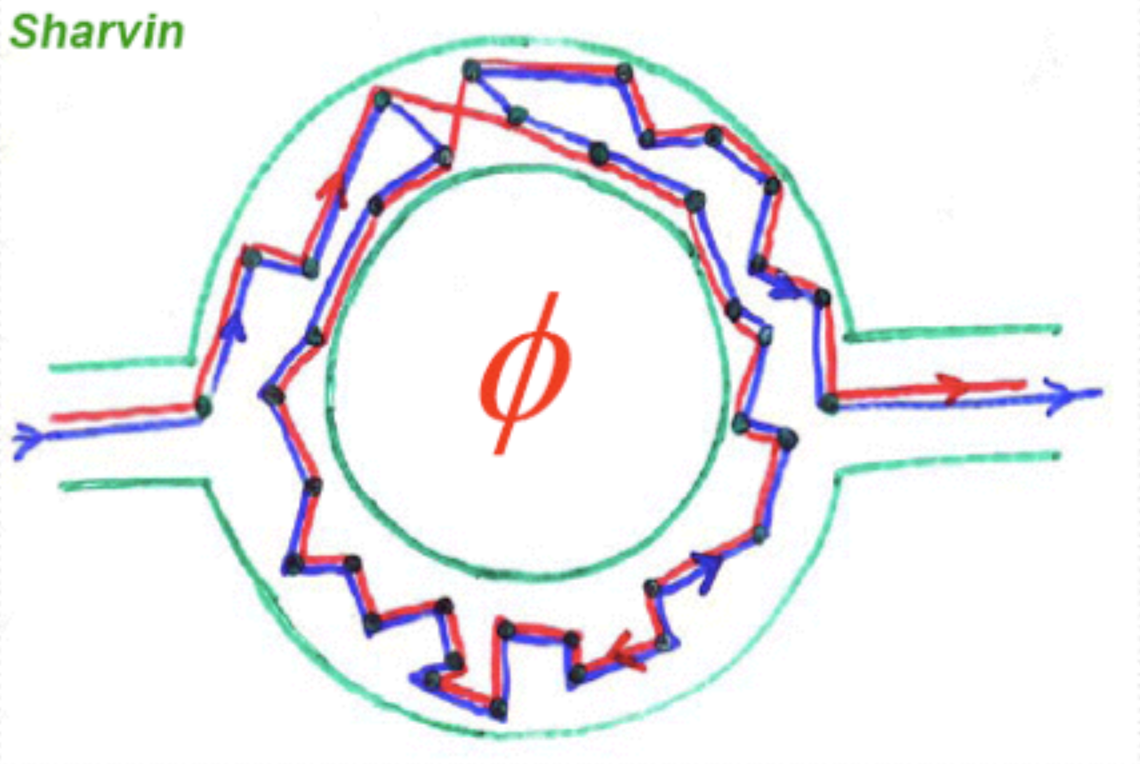


Sample specific interference

Phase difference $2\pi \frac{\phi}{\phi_0}$

Oscillates with period h/e

Sharvin, Sharvin

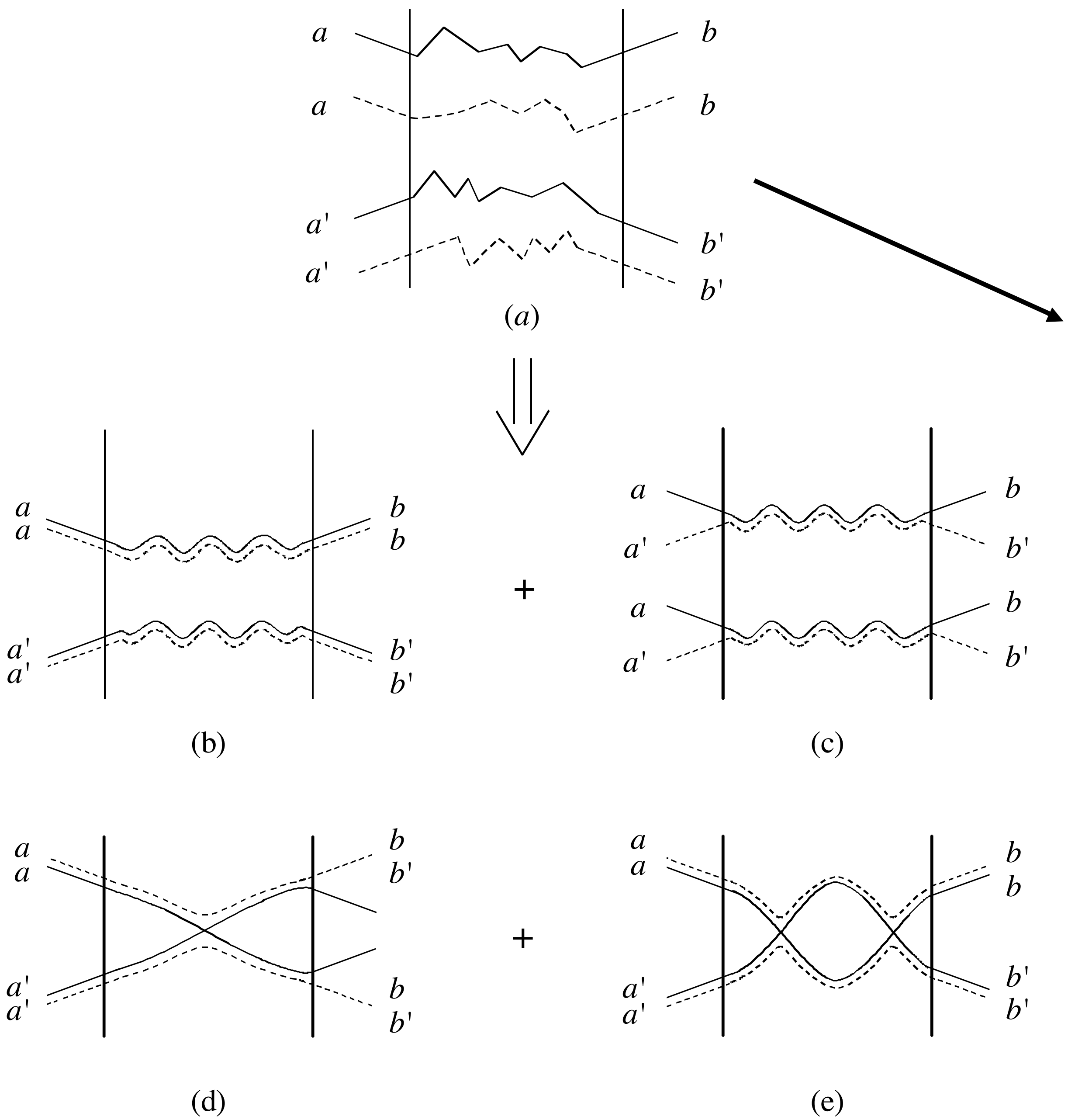


Survives disorder average

Phase difference $4\pi \frac{\phi}{\phi_0}$

Oscillates with period $h/2e$

Fluctuations and correlations



Slab geometry

transmission coefficient

$$T_{ab} = \left| t_{ab} \right|^2$$

correlations involve the product of 4 complex amplitudes with or without quantum crossings

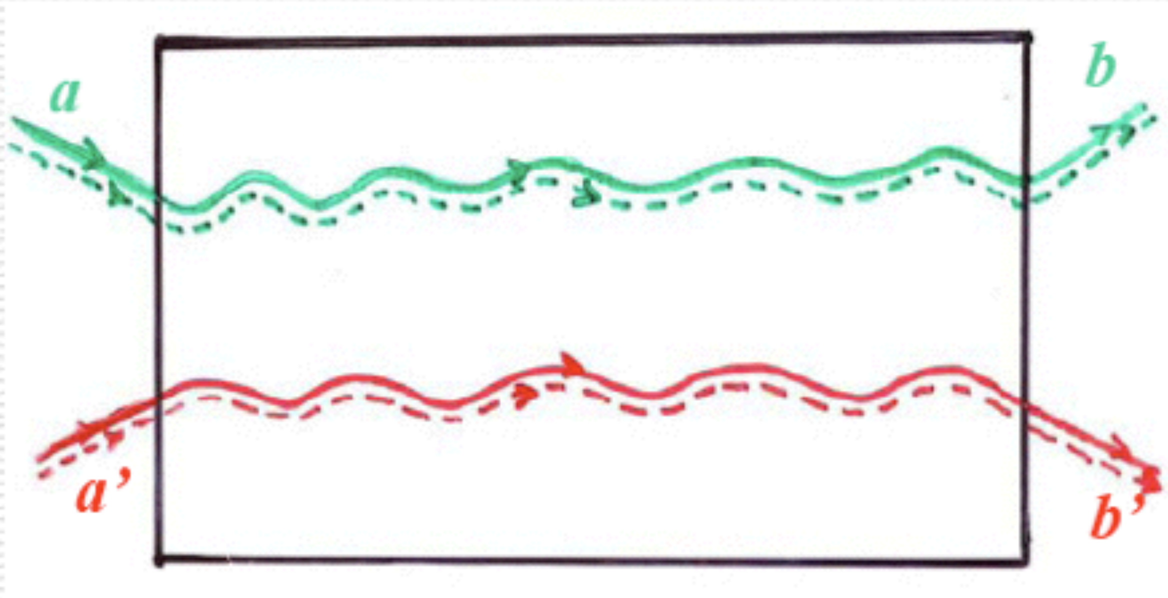
Correlation function of the transmission coefficient :

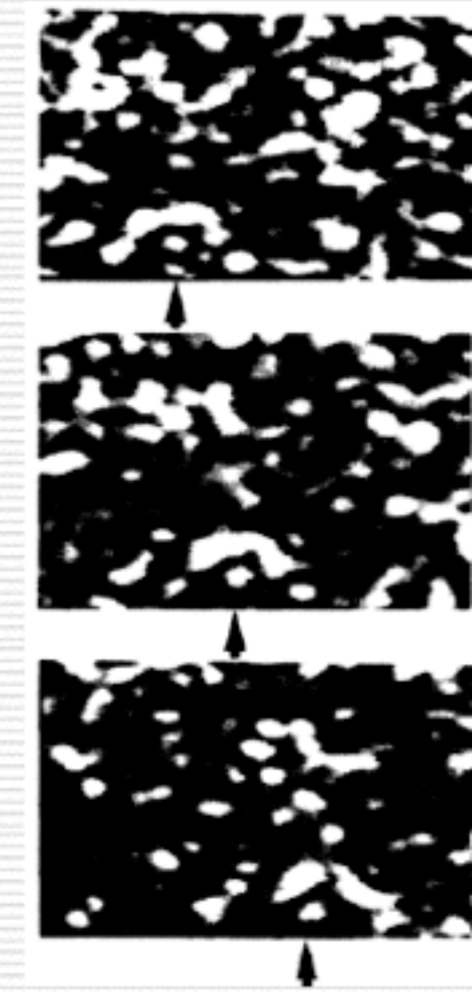
$$C_{aba'b'} = \frac{\overline{\delta T_{ab} \delta T_{a'b'}}}{\overline{T}_{ab} \overline{T}_{a'b'}}$$

Speckle and conductance fluctuations

$$\overline{T_{ab}T_{a'b'}} = \overline{T_{ab}}\;\overline{T_{a'b'}} + \overline{\delta T_{ab}\delta T_{a'b'}}$$

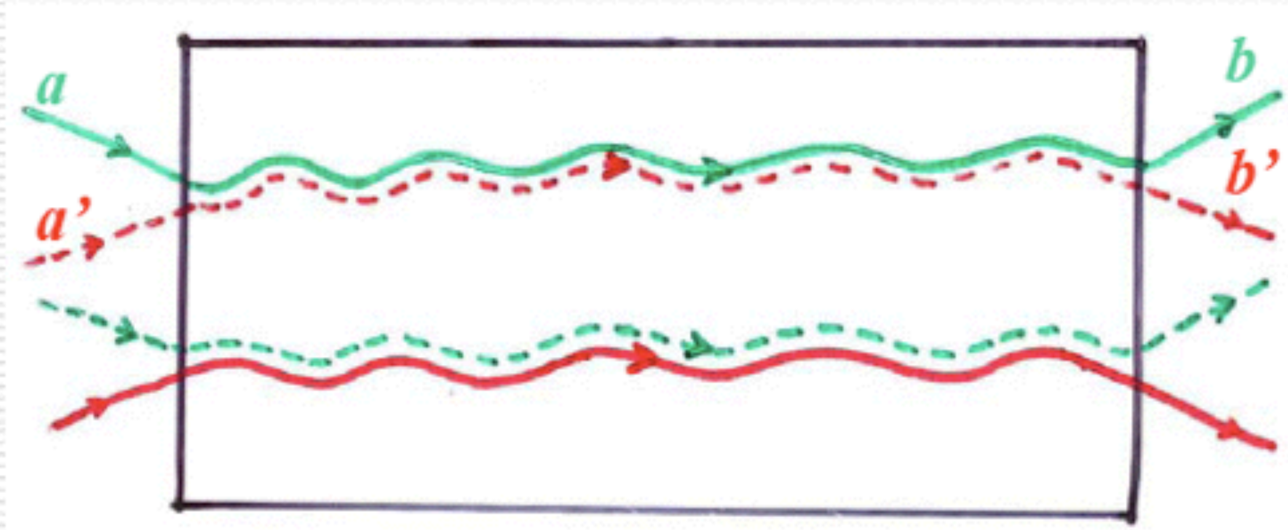
$$\overline{T_{ab}}\;\overline{T_{a'b'}}$$





Memory effect

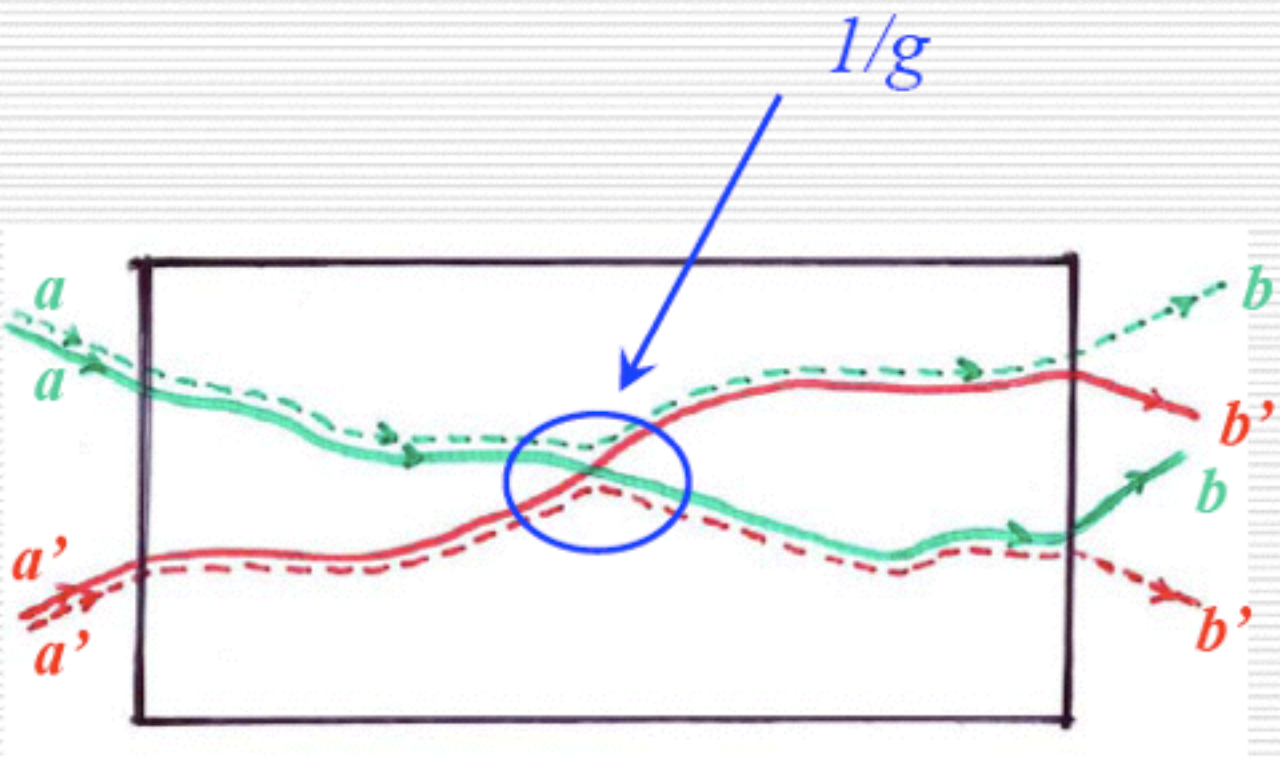
$$\overline{\delta T_{ab}\delta T_{a'b'}}$$



$$= \overline{T_{ab}}\;\overline{T_{a'b'}}\; f(a,a',b,b')$$

$$f(a,a',b,b') = g(\Delta a)\delta(\Delta a - \Delta b)$$

Speckle fluctuations *vs* conductance fluctuations



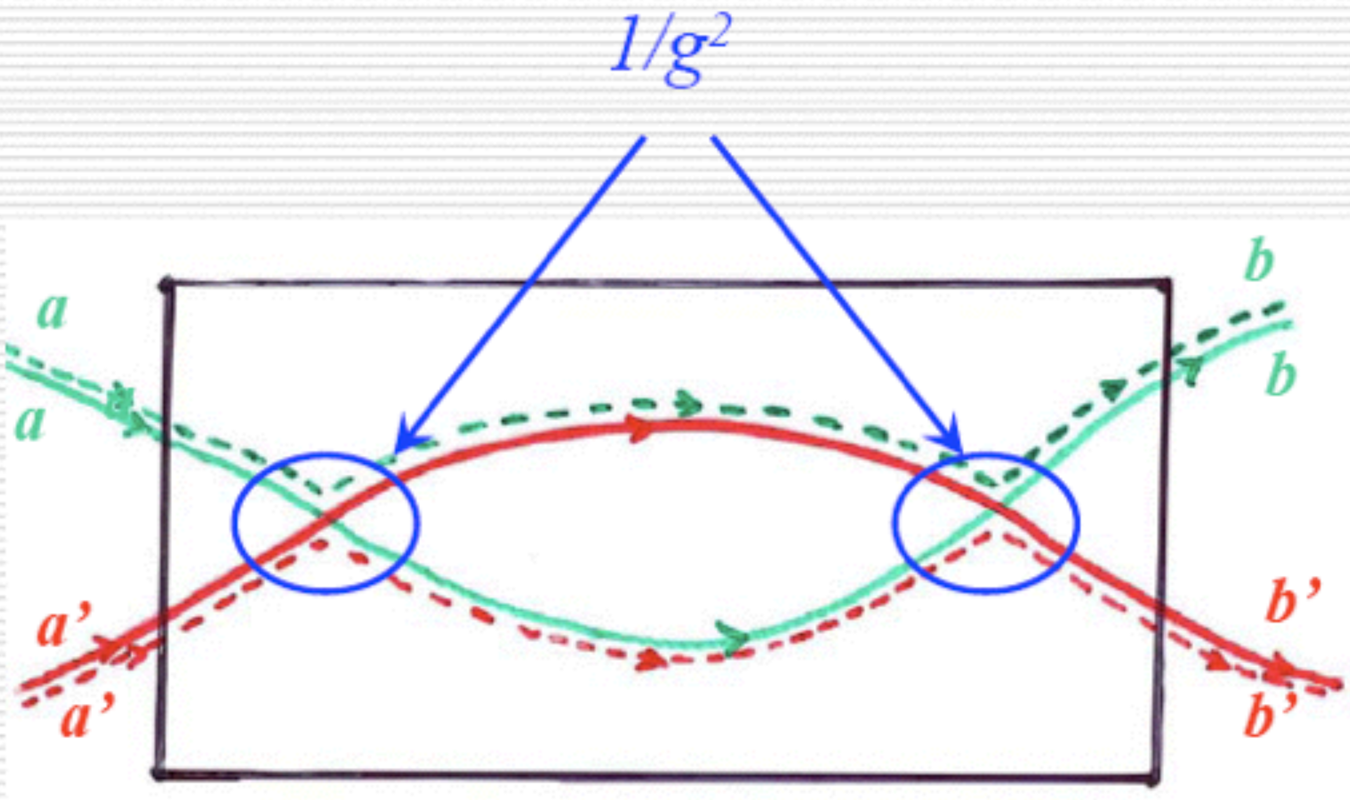
$$\overline{\delta T_{ab}\delta T_{a'b'}} = \frac{2}{3g}\overline{T_{ab}}\ \overline{T_{a'b'}}\ F(b,b')$$

Angular correlations of intermediate range

$$\overline{\delta g^2} = \frac{2}{3g}\sum_{a,a',b,b'}\overline{T_{ab}}\ \overline{T_{a'b'}}\ F(b,b') : 0$$

No conductance correlations !

Speckle fluctuations *vs* conductance fluctuations C_3



$$\overline{\delta T_{ab}\delta T_{a'b'}} = \frac{2}{15g^2}\overline{T_{ab}}\ \overline{T_{a'b'}}$$

Long-range angular correlations, with very weak amplitude

$$\overline{\delta g^2} = \frac{2}{15g^2}\sum_{a,a',b,b'}\overline{T_{ab}}\ \overline{T_{a'b'}} = \frac{2}{15}$$

Universal conductance fluctuations

Universal conductance fluctuations

Landauer description : $G = \frac{e^2}{h} \sum_{ab} T_{ab}$

0 crossing: $\overline{G}^2 = G_{cl}^2 = \left(e^2/h\right)^2 \, g^2$

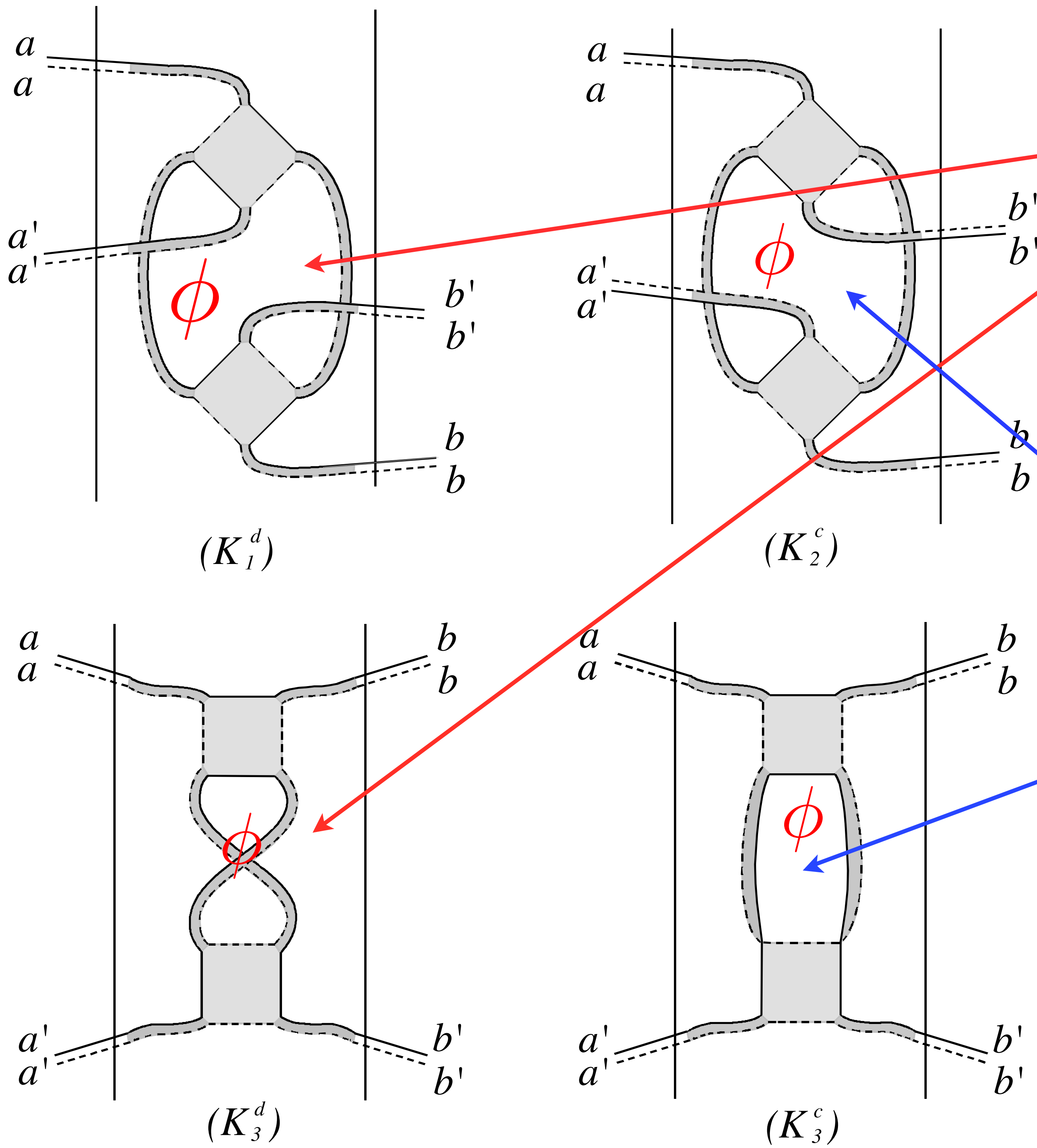
1 crossing: vanishes due to the summation over the channels.

2 crossings: correction $\overline{\delta G^2} \propto \overline{G}^2 / g^2 = (e^2/h)^2$ universal

(very different from the classical self-averaging limit $\overline{\delta G^2} \propto L^{d-4}$)

Dephasing and decoherence

Universal conductance fluctuations



2 Diffusons

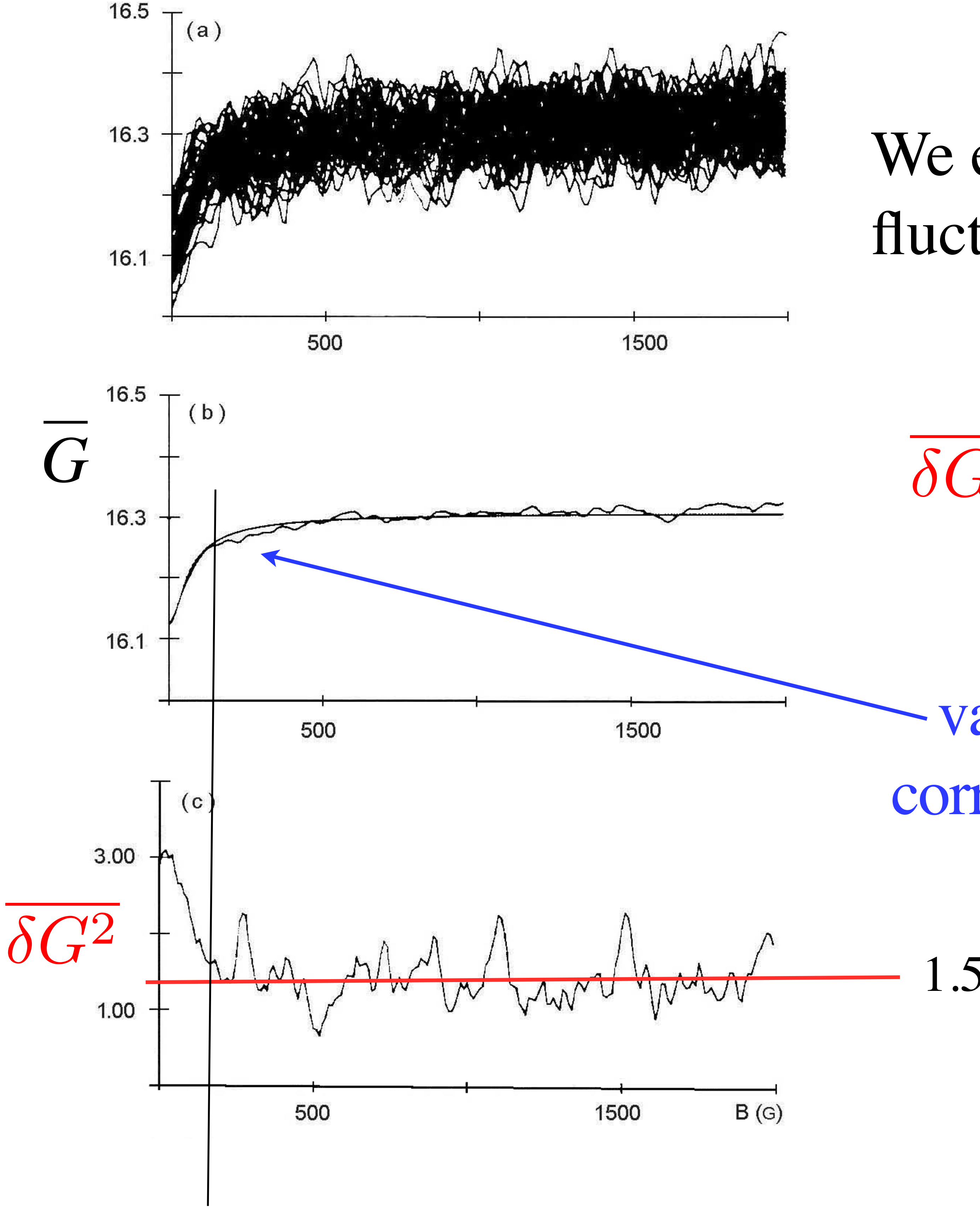
2 Cooperons

sensitive to an applied Aharonov-Bohm magnetic flux

ϕ

46 *Si*-doped *GaAs* samples at 45 mK

(Mailly-Sanquer)



We expect the conductance fluctuations to be reduced by a factor 2

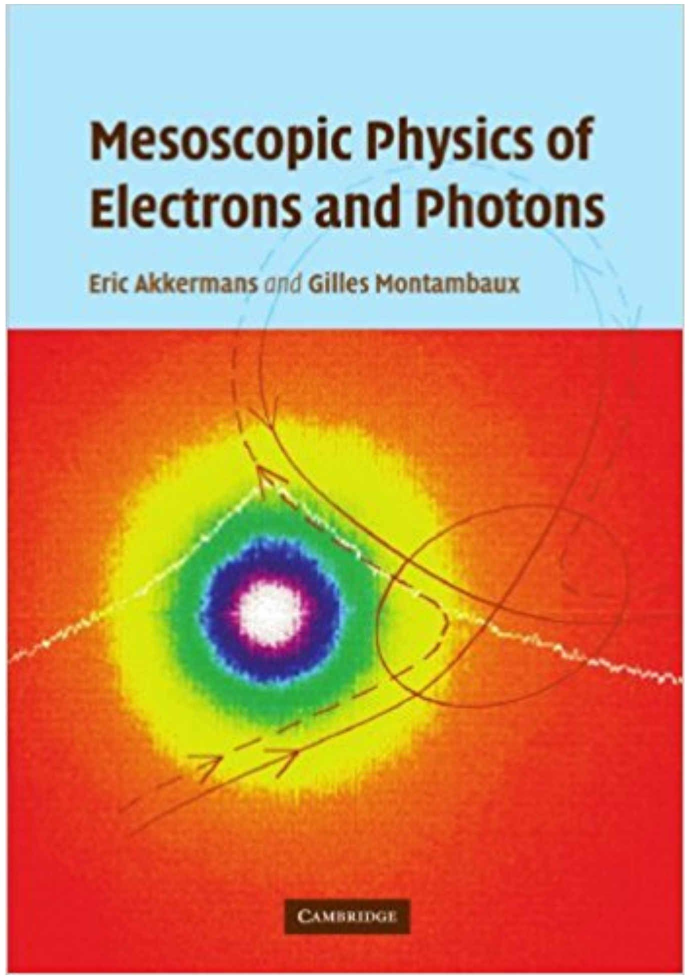
$$\overline{\delta G^2} \xrightarrow{\ \phi\ } \frac{\overline{\delta G^2}}{2}$$

vanishing of the weak localization correction for the same magnetic field

In the presence of incoherent processes $L > L_\phi$:

$$\overline{\delta G^2} \to 0$$

Thank you for your attention.



Based on *Mesoscopic physics of electrons and photons*, by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007