A Casimir Effect in Quantum Mesoscopic Physics

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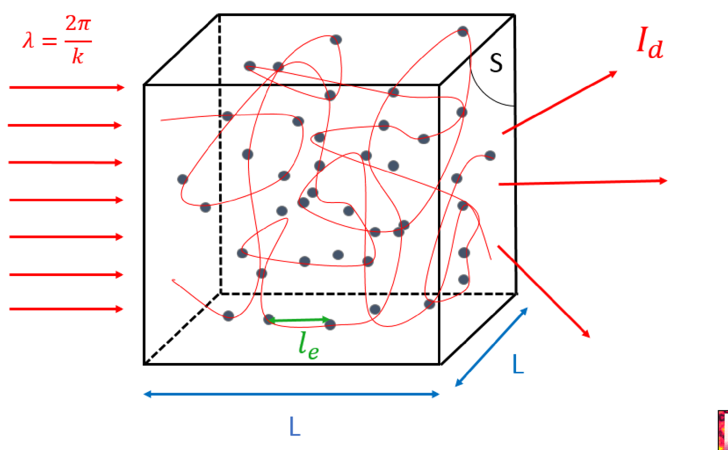
Introduction

Fluctuation induced forces (Casimir forces, so called Casimir effect) are ubiquitous [1], and are caused by the confinement of long-range fluctuations. Fluctuation induced forces have been first predicted and measured using perfectly conducting plates immersed in the (quantum) QED vacuum [2], then in classical systems [3],[4] and more recently in out-of-equilibrium hydrodynamic systems [5],[6].
In the later case of a driven diffusive system, the average particle density fluctuates spatially on a long-range scale around a steady-state profile, which induces a specific behavior of pressure resulting from these fluctuations.
Here, we consider intensity fluctuations of classical light propagating through a scattering medium. In the elastic multiple scattering regime, the average light intensity behaves diffusively, and can be described using an effective Langevin approach which properly incorporates interferences effects [7]. Light intensity fluctuations are spatially long-ranged as a result of underlying mesoscopic coherent effects [8]. Their magnitude depends on the dimensionless conductance g (a parameter that depends on the geometry of the system).
Using the analogy with out-of-equilibrium systems provided by the Langevin equation, we show the emergence of fluctuation induced forces for coherent diffusive light, which constitutes the first example of a Casimir effect for a mesoscopic system. We give analytical and numerical results for two different geometries and light sources.

Mesoscopic Interference for Classical Light - I

The Average Diffusive Light Intensity Satisfies a Diffusion Equation

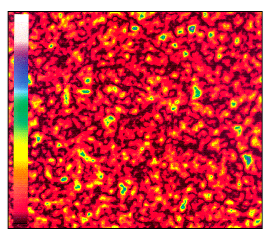


$$g = \frac{k^2 l_e S}{3\pi L}$$

Dimensionless conductance

l_e : elastic mean free path

k : wave number (monochromatic light)



Speckle pattern : "snapshot" of a granular medium ; the color gradient represents the variation of the diffused light intensity resulting of coherent effects (before averaging over disorder)

- Intensity in the medium : $I(\mathbf{r}) = \frac{4\pi}{c}|E(\mathbf{r})|^2$

  where $\Delta E(\mathbf{r}) + k^2(1+\mu(\mathbf{r}))E(\mathbf{r}) = s_0(\mathbf{r})$

  $\mu(\mathbf{r})$: fluctuating dielectric constant (disorder)
  $s_0(\mathbf{r})$: light source distribution.

- Associated light current :

  $\mathbf{j}(\mathbf{r}) = \frac{ic}{2k}\lim_{\mathbf{R}\to\mathbf{r}} \nabla[E(\mathbf{r})E(\mathbf{R})^* - c.c.]$

Average over disorder ($\overline{\cdots}$):

$\overline{I(\mathbf{r})} = I_d(\mathbf{r})$

Weak disorder limit ($kl_e \gg 1$ or $g \gg 1$) :

Diffusion equation : $-D\Delta I_d(\mathbf{r}) = s(\mathbf{r})$

Green's function : $-D\Delta P_d(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$

Fick's law : $\mathbf{j_d}(\mathbf{r}) = -D\nabla I_d(\mathbf{r})$

Intensity Fluctuations around the Average are Spatially Long-Ranged

Fluctuations $\delta I(\mathbf{r}) = I(\mathbf{r}) - I_d(\mathbf{r})$: speckle pattern (illustrated above).

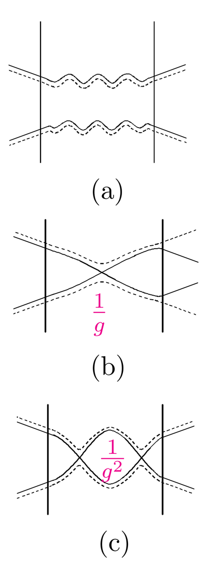
- For weak disorder $g \gg 1$, spatially long-ranged correlations driven by interference processes survive disorder averaging.

$$\boxed{\frac{\overline{\delta I(\mathbf{r})\delta I(\mathbf{r}')}}{I_d(\mathbf{r})I_d(\mathbf{r}')} = C^{(1)}(\mathbf{r},\mathbf{r}')+C^{(2)}(\mathbf{r},\mathbf{r}')+C^{(3)}(\mathbf{r},\mathbf{r}')}$$

(a) $C^{(1)}(\mathbf{r},\mathbf{r}') \propto \delta(\mathbf{r}-\mathbf{r}')$: short-range (incoherent part)

(b) $C^{(2)}(\mathbf{r},\mathbf{r}') = \frac{1}{g}G(\mathbf{r}-\mathbf{r}')$: **long-range** (coherent part); G depends on the geometry and on the boundary conditions.

(c) $C^{(3)}(\mathbf{r},\mathbf{r}') \propto \frac{1}{g^2}$: Constant ; universal conductance fluctuation



Mesoscopic Interference for Classical Light - II

Langevin approach

The above results can be recovered using a Langevin approach:
start with a generalized Fick's law for all length scales and before $\overline{\cdots}$:

$$\mathbf{j}(\mathbf{r}) = -D\nabla I(\mathbf{r}) + \sqrt{\frac{\sigma}{SL}}\boldsymbol{\eta}(\mathbf{r})$$

Continuity equation : $\mathrm{div}\mathbf{j}(\mathbf{r}) = 0$

$D = \frac{cl_e}{d}$: diffusion coefficient ;
$\sigma(I) = \frac{2\pi l_e^2}{3k^2}I_d^2(\mathbf{r})$: noise strength ;

The fluctuating term $\boldsymbol{\eta}(\boldsymbol{r})$ is such that : $\overline{\boldsymbol{\eta}(\boldsymbol{r})} = 0$, and :

$$\boxed{\overline{\eta_\alpha(r)\eta_\beta(r')} = \delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}') + \left(\frac{2\pi c^2}{k^2\sqrt{\sigma}}\right)^2 (K^{(1)}(\mathbf{r},\mathbf{r}') + K^{(2)}(\mathbf{r},\mathbf{r}') + K^{(3)}(\mathbf{r},\mathbf{r}'))}$$

($\alpha,\beta = x,y,z$) are the sources giving rise to the intensity fluctuations $C^{(2)}$ and $C^{(3)}$, and depend explicitly on I_d and P_d :

$$K^{(1)}(\mathbf{r},\mathbf{r}') = \delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\int_{vol} d\mathbf{r_1} P_d^2(\mathbf{r_1},\mathbf{r}')\nabla I_d(\mathbf{r_1})\cdot\nabla I_d(\mathbf{r_1})$$

$$K^{(2)}(\mathbf{r},\mathbf{r}') = K^{(3)}(\mathbf{r},\mathbf{r}') = P_d^2(\mathbf{r},\mathbf{r}')\partial_\alpha I_d(\mathbf{r})\partial_\beta I_d(\mathbf{r}')$$

Fluctuation Induced Forces

- Force on the plate p_1: difference of the (radiative pressure) forces on each side:

$$f_{p_1} = f_1 - f_2 \propto \int_{p_1} dS\,(\mathbf{j_{1,+}} - \mathbf{j_2})\cdot\hat{\mathbf{e_y}}$$

- Force fluctuations on each side j :

$$\delta f_j = f_j - \overline{f_j} = \pm\int_{p_1} dS\left[-D\partial_y\delta I_j(\mathbf{r}) + \sqrt{\frac{\sigma}{SL}}\eta_{y,j}(\mathbf{r})\right]$$

$\overline{\delta f_j} = 0$

$\overline{\delta f_j^2} = \sum_{i=1}^3 \int_{p_1\times p_1} dSdS'D^2\left(\partial_y\partial_{y'}I_{d,j}(\mathbf{r})I_{d,j}(\mathbf{r}')C_j^{(i)}(\mathbf{r},\mathbf{r}') + \left(\frac{2\pi c^2}{k^2}\right)K_j^{(i)}(\mathbf{r},\mathbf{r}')\right)$

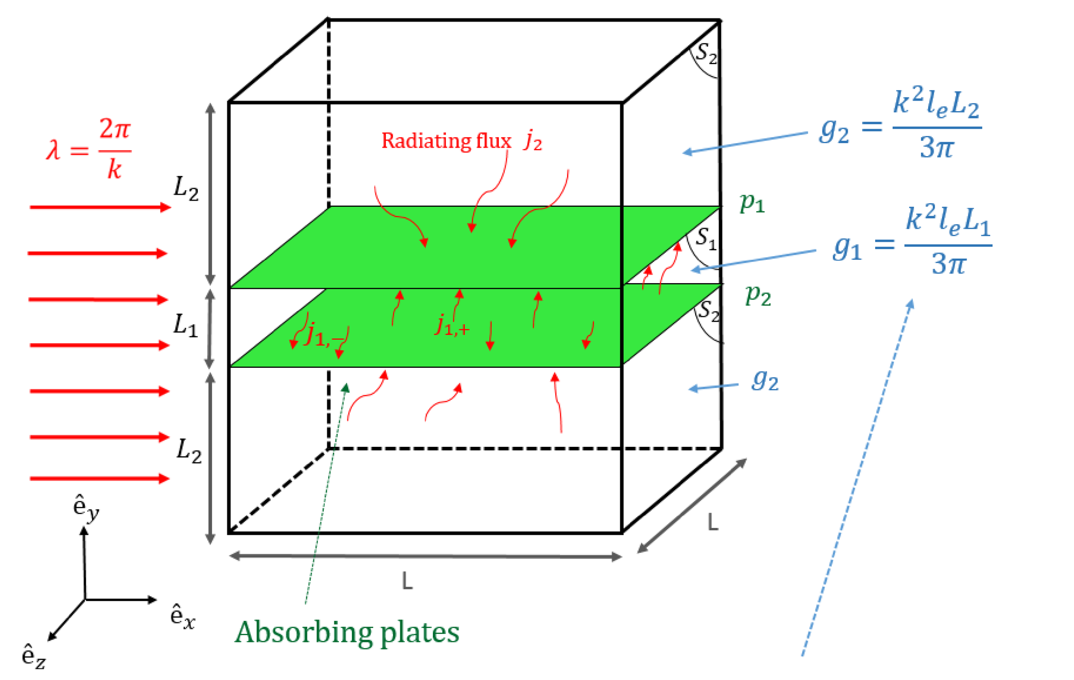
Finally :

$$\boxed{\overline{\delta f_{p_1}^2}/\overline{f_{p_1}}^2 \sim \left(\frac{\alpha_1}{g_1} - \frac{\alpha_2}{g_2}\right)}$$

where the α_j are dimensionless quantities depending on the solutions for I_d and P_d, i.e. on the boundary conditions and geometry, but not on L_1.

The main contribution to the force fluctuations is due to the long-range fluctuations $C^{(2)}(\mathbf{r},\mathbf{r}')$, of strength $\frac{1}{g_j}$.

$$\boxed{g_1 = g_2 \Rightarrow \overline{\delta f_{p_1}^2}/\overline{f_{p_1}}^2 = 0}$$

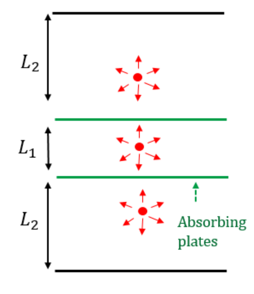


Point Source

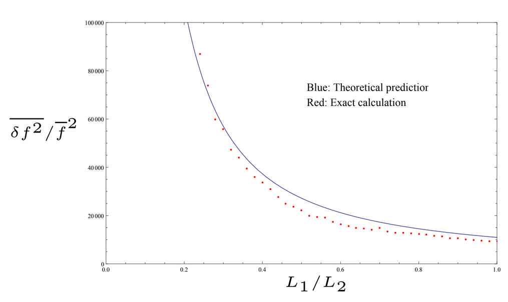
On p_1 : $\overline{\delta f^2}/\overline{f}^2 = \underbrace{\left(\frac{\alpha_1}{g_1} - \frac{\alpha_2}{g_2}\right)}_{\hookrightarrow C^{(2)}} + \underbrace{\left(\frac{\beta_1}{g_1^2} - \frac{\beta_2}{g_2^2}\right)}_{\hookrightarrow K^{(i)}}$

$\frac{L_1}{L_2} \ll 1$: $\boxed{\overline{\delta f^2}/\overline{f}^2 \sim \frac{\alpha_1}{g_1} + \frac{\beta_1}{g_1^2} = \frac{\alpha_1'}{L_1} + \frac{\beta_1'}{L_1^2}}$

α_i, β_i are dimensionless quantities depending on the solution for I_d and P_d, but not on L_1.



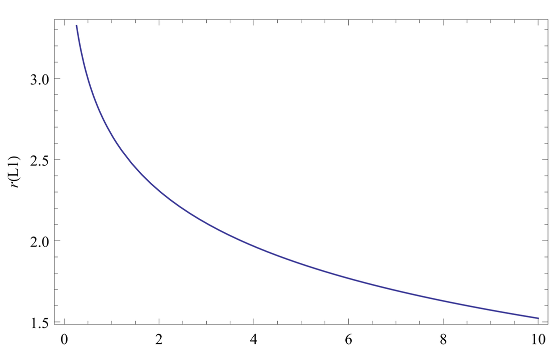
Point source placed in each of the three zones, separated by the plates.



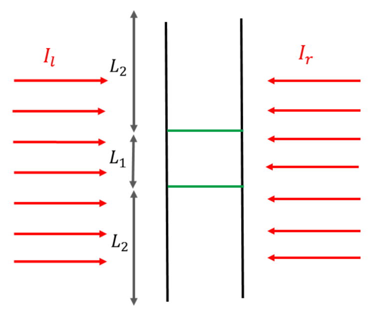
Graph : Theoretical prediction (blue) given by the framed equation and exact calculation (red). The lengths are normalized by $L=1$, and we set $\left(\frac{2\pi c^2}{k^2}\right)^2 = 1$ and $\frac{3\pi}{k^2 l_e} = 1$.

Plane Wave

$$\boxed{\overline{\delta f^2} = \underbrace{\left(\frac{\alpha_1}{g_1} - \frac{\alpha_2}{g_2}\right)}_{\hookrightarrow C^{(2)}} + \underbrace{r(L_1)}_{\hookrightarrow K^{(i)}}}$$



Graph : Contribution of $K^{(i)}$ to the fluctuations.



Medium illuminated by plane waves I_l, I_l.

Fluctuations emerge as long as the system is illuminated, even with $I_l = I_r$ (unlike usual boundary driven out-of-equilibrium systems).

Conclusion and Perspectives

- We demonstrated the existence of fluctuation induced forces due to spatially long-ranged fluctuations of light, resulting from mesoscopic interferences (mesoscopic Casimir effect).
- We highlighted the analogy between the behavior of light in random media and boundary driven out-of-equilibrium systems of particles diffusing between reservoirs of densities ρ_l and ρ_r. However, unlike transport problems where long-range density correlations vanish at equilibrium ($\rho_l = \rho_r$), here the effect is persistent even when illuminating the box from both sides with identical light beams ($I_l = I_r$). The random medium is out-of-equilibrium as long as it is illuminated. The strength and sign of the Casimir forces depend on the long-ranged fluctuations (i.e. on the strength of disorder) and on the geometry, as described by the dimensionless conductances g_j.
- This last point could be used to observe the Anderson localization transition: increasing the disorder, we expect long-range fluctuations to disappear at the transition and thus fluctuation induced forces to vanish.
- Similar behavior is expected for quantum out-of-equilibrium systems (e.g. spin polarized electronic transport in supraconductors).

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