Out-of-Equilibrium Quantum Radiation and Atomic Motion

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Introduction

Black-Body Radiation

- A ***Black-Body*** – is a cavity of volume V, its walls absorb and emit electromagnetic radiation of all frequencies.
- Cavity walls at a constant temperature T, the **radiation inside** is in ***thermal equilibrium*** at the temperature T. Its *spectral energy density* is given by the Planck distribution

$$u_\nu(T) = \overbrace{\frac{8\pi\nu^2}{c^3}}^{\text{density of modes}} \underbrace{h\nu}_{\text{mode energy}} \overbrace{n_\nu(T)}^{\text{mode average occupation}}$$

(isotropic)

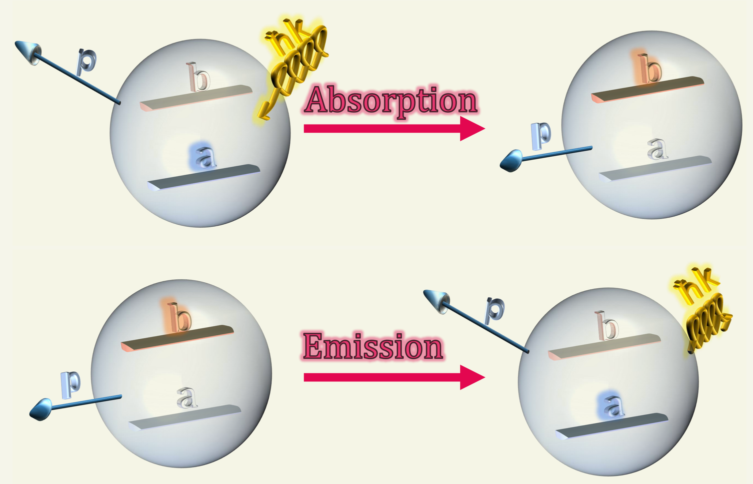
$$n_\nu(T) = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

PLANCK

Atom-Light Interactions: The Einstein model

- Single atom placed inside a black-body cavity at temperature T. The atom has two energy states $\epsilon_a < \epsilon_b$.
- The ground state population $n_a \equiv N_a/N$ evolves according to

$$\dot{n}_a = \overbrace{\left[\underbrace{u_\nu(T) B_{ba}}_{\text{stimulated}} + \underbrace{A}_{\text{spontaneous}}\right]}^{\text{emission rate}} n_b - \overbrace{B_{ab} u_\nu(T)}^{\text{absorption rate}} n_a$$



Steady-state populations are

$$n_a = \frac{e^{\frac{\epsilon}{k_B T}}}{e^{\frac{\epsilon}{k_B T}} + 1}; \quad n_b = \frac{1}{e^{\frac{\epsilon}{k_B T}} + 1}, \quad \epsilon = \epsilon_b - \epsilon_a$$

- Momentum exchanges with the radiation are described by a *Langevin equation*

$$\dot{p}_j = -\frac{\gamma}{m} p_j + \frac{1}{m}\eta_j(t)$$

$$\left\langle \frac{\mathrm{d}p}{\mathrm{d}t} \right\rangle = -\underset{\text{dissipation}}{\gamma} \frac{p}{m}, \quad \left\langle \frac{\mathrm{d}p^2}{\mathrm{d}t} \right\rangle = \underset{\text{fluctuation}}{D_p}$$

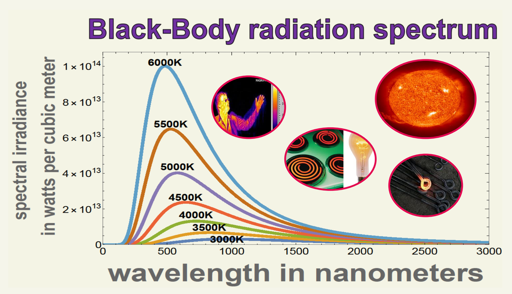
γ – friction coefficient the (inverse) time-scale of relaxation to equilibrium.
D_p – momentum diffusion coefficient magnitude of equilibrium fluctuations.

- ***Einstein relation*** (fluctuation-dissipation)

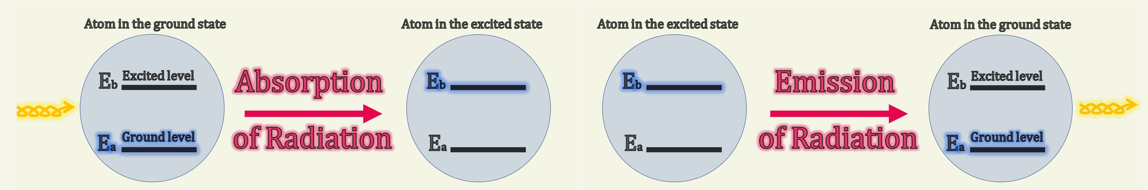
$$\frac{D_p}{\gamma} = k_B T$$

Scope

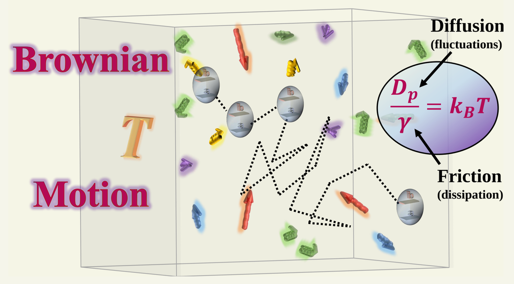
Black-Body radiation is an ubiquitous phenomena in thermal equilibrium physics. Characterized by ***Planck spectral energy density***, examples range from the sun to the human body.



Two-level atom is a simple model for matter, describing interaction with radiation.

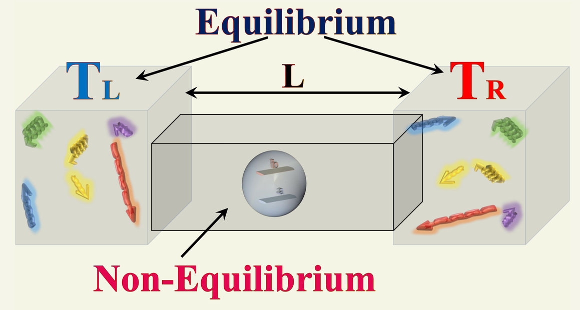


Einstein (1917): two-level atom interacting with black-body radiation performs ***Brownian motion***

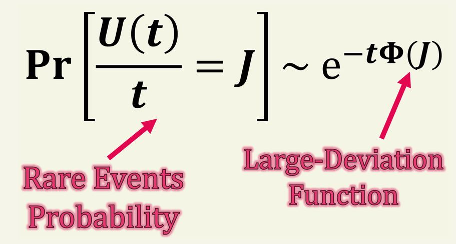


This is at the basis of many physical phenomena. However, does thermal equilibrium physics captures everything?

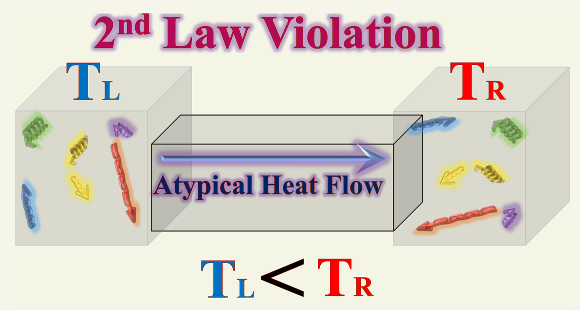
In this work, we address the out-equilibrium extension of Einsteins problem. We consider a system connected to two black-body radiation reservoirs at different temperatures.



We characterize the non-equilibrium steady-state of the radiation, the energy flow $U(t)$ throughout the system and its fluctuations, after a very long time t, by means of the ***large-deviation function*** $\Phi(J)$ – free-energy analog out of equilibrium



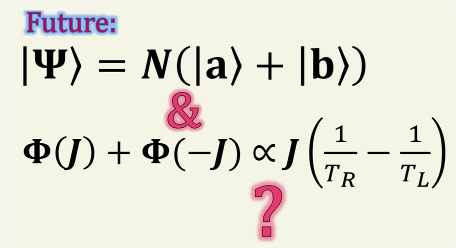
Second law is not obeyed anymore



Gallavotti-Cohen symmetry – microscopic time reversal, is present

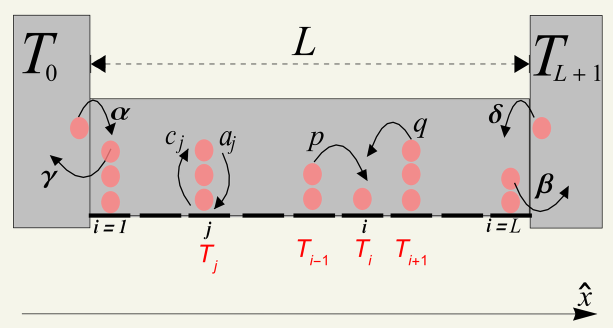
$$\Phi(J) - \Phi(-J) \propto J\left[\frac{1}{T_\mathcal{R}} - \frac{1}{T_\mathcal{L}}\right]$$

Future: is the Gallavotti-Cohen symmetry valid in the presence of quantum entanglement?



The Model

Non-Equilibrium Radiation: Photon lattice gas



Configuration $\eta = \{n_1, \ldots, n_L\}$ n_i – photon occupation number.

Bulk transitions: $\{n_i + 1, n_{i+1}\} \underset{q}{\overset{p}{\rightleftharpoons}} \{n_i, n_{i+1} + 1\}$

Creation/Annihilation: $\{n_i\} \underset{a_i}{\overset{c_i}{\rightleftharpoons}} \{n_i + 1\}$

Left boundary: $\{n_1, \ldots\} \underset{\gamma}{\overset{\alpha}{\rightleftharpoons}} \{n_1 + 1, \ldots\}$

Right boundary: $\{\ldots, n_L\} \underset{\beta}{\overset{\delta}{\rightleftharpoons}} \{\ldots, n_L + 1\}$

Assumption: local equilibrium

$$\frac{c_i}{a_i} = e^{-\frac{h\nu}{k_B T_i}} = \frac{\langle n_i \rangle}{1 + \langle n_i \rangle} \equiv z_i$$

$\langle n_i \rangle$ – average occupation.

Results 1: Non-Equilibrium Radiation

Steady-State

Steady-state probability for photonic configuration

$$P_s(\{n_1, \ldots, n_L\}) = \prod_{k=1}^{L} \pi_k(n_k)$$

$\pi_k(n_k)$ – single site probability: Bose-Einstein distribution

$$\pi_k(n_k) = (1 - z_k) z_k^{n_k}$$

In the continuum limit

$$z_k \to z_\nu(x) = \frac{x}{L}(z_\mathcal{R} - z_\mathcal{L}) + z_\mathcal{L}$$

From $\langle n_k \rangle = z_k/(1 - z_k)$ the ***spectral energy density*** is

$$n_\nu(x) = \frac{z_\mathcal{L} + \frac{x}{L}(z_\mathcal{R} - z_\mathcal{L})}{1 - z_\mathcal{L} - \frac{x}{L}(z_\mathcal{R} - z_\mathcal{L})}$$

$$u_\nu(x) = g_\nu h\nu \, n_\nu(x)$$

NOT PLANCK

Results 2: Non-Equilibrium Radiation

Energy Flow

Fluctuating energy current density (Langevin)

$$j(x,t) = -\overbrace{D(u_\nu)}^{\text{diffusion}} \partial_x u_\nu(x,t) + \overbrace{\underbrace{\sqrt{\sigma(u_\nu)}}_{\text{dissipation}}\,\eta(x,t)}^{\text{noise}}$$

$$\langle \eta(x,t)\,\eta(x',t') \rangle = \frac{1}{L}\delta(x - x')\,\delta(t - t')$$

$$D(u_\nu) = \frac{1}{(1 + u_\nu)^2}; \quad \sigma(u_\nu) = 2\frac{u_\nu}{1 + u_\nu}$$

Remark: The above results correspond to the ***zero-range process*** [2].The total energy transferred throughout the system up to time t

$$U(t) = L^2 \int_0^{t/L^2} \mathrm{d}\tau \int_0^1 \mathrm{d}x\, j(x,\tau)$$

Assume a large-deviation form: $P\left[\frac{U(t)}{t} = J\right] \sim \exp\{-t\Phi(J)\}$ (for $t \to \infty$); $\Phi(J)$ – large-deviation function. Its Legendre transform $\mu(\lambda) = \sup_J [J\lambda - \Phi(J)]$

$$\mu(\lambda) = \frac{1}{L}\left(1 - e^{-\lambda}\right)\left[e^{\lambda} e^{-\frac{h\nu}{k_B T_\mathcal{L}}} - e^{-\frac{h\nu}{k_B T_\mathcal{R}}}\right]$$

Gallavotti-Cohen symmetry

$$\mu(\lambda) = \mu\left(-\lambda + \frac{1}{2}\frac{h\nu}{k_B}\left[\frac{1}{T_\mathcal{R}} - \frac{1}{T_\mathcal{L}}\right]\right)$$

Results 3: Two-Level Atom

Interacting with Non-Equilibrium Radiation

Ground-state population steady-state (for small $\Delta T = T_\mathcal{R} - T_\mathcal{L}$),

$$n_a(x) = n_a(0,T) - h(T)\frac{\Delta T}{T}\frac{x}{L}$$

$h(T)$ and $n_a(0,T)$ are known functions of T. Average force acting on the atom

$$f(x) = \left(\frac{h\nu}{c}\right) b(T) \frac{h\nu}{k_B T}\frac{x}{L}\frac{\Delta T}{T}$$

$b(T)$ known function of T. The diffusion tensor

$$D_{ij} = \delta_{ij}\left(\frac{h\nu}{c}\right)^2 \left[g_1(T) - g_2(T)\frac{\Delta T}{T}\right]$$

$g_1(T)$, $g_2(T)$ – known functions of T.

NOT A BROWNIAN MOTION

References

[1] A. Einstein. On the Quantum Theory of Radiation. *Verh. d. Deutschen Physikal. Gesellschaft*, 13(14):318, 1917.

[2] Ori Hirschberg, David Mukamel, and Gunter M. Schütz. Density profiles, dynamics, and condensation in the ZRP conditioned on an atypical current. *J. Stat. Mech: Theory Exp.*, 2015(11), 2015.