Strong- versus weak-coupling paradigms for cuprate superconductivity

Shahaf Asban,1 Meni Shay,1,2 Muntaser Naamneh,1 Tal Kirzhner,1 and Amit Keren1,*

1Department of Physics, Technion–Israel Institute of Technology, Haifa 32000, Israel
2Department of Physics and Optical Engineering, Or Braude College, P.O. Box 78, 21982 Karmiel, Israel

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Absolutize resistivity measurements as a function of temperature from optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_7 - \delta$, $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$, $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_{2}\text{O}_{8-x}$, and $(\text{Ce}_{0.1}\text{La}_{0.9})(\text{Ba}_{1.65}\text{La}_{0.35})\text{Cu}_3\text{O}_7$ thin films are reported. Special attention is given to the measurement geometrical factors and the resistivity slope between $T_c$ and $T^*$. The results are compared with a strong-coupling theory for the resistivity derivative near $T_c$, which is based on hard core bosons, and with several weak-coupling theories, which are BCS based. Surprisingly, our results agree with both paradigms. The implications of these findings and the missing calculations needed to distinguish between the two paradigms are discussed.

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Two major discoveries were made at a very early stage in the study of cuprate superconductivity. One was the Uemura relation for underdoped samples.1 This relation states in the study of cuprate superconductivity. One was the optimally doped, and overdoped samples: and showed that a broader scaling holds for both underdoped, was based on optical conductivity measurements. In many extrapolates to zero at doping, $T_c$, down to $T^*$, near optimal doping, $T^*$ is similar to $T_c$ and the linear relation extends down to $T_c$. Later on, homes extended the Uemura relation and showed that a broader scaling holds for both underdoped, optimally doped, and overdoped samples: $\rho_s(0) \propto \sigma(T_c) / T_c$, where $\rho_s(0)$ is the superfluid density at zero temperature, and $\sigma(T_c) = 1 / \rho_s(0)$ is the conductivity at $T_c$. This observation was based on optical conductivity measurements. In many low doping models, $\rho_s(0) \propto \lambda^{-2}(0)$. Therefore, the Homes law for optimal doping; it also captures the linear resistivity law in a differential form:

$$
d\rho_s / dT (T > T_c) = 77.378 \left( \frac{\lambda_{ab}(0)}{q} \right)^2 \frac{K_B}{\hbar c^2}. 
$$

(2)

In this Rapid Communication, we check both the WC and SC theories, as accurately as possible, in the small region where both are valid, namely, optimal doping. We use direct current (dc) resistivity versus $T$ measurements in films of $\text{YBa}_2\text{Cu}_3\text{O}_7 - \delta$ (YBCO), $(\text{Ce}_{0.1}\text{La}_{0.9})(\text{Ba}_{1.65}\text{La}_{0.35})\text{Cu}_3\text{O}_7$ (CLBLCO), $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ (LSCO), and $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_{2}\text{O}_{8-x}$ (BSCO). We take the geometrical factors of the film into account and check their influence experimentally. This allows us to determine $\rho_s(T)$ in absolute value, and to demonstrate that our results are indeed film-geometry independent. We then compare $d\rho_s / dT$ to $\lambda_{ab}^2(0)$ and $\lambda_{ab}(0)$, as in the SC and WC theories, respectively. $\lambda_{ab}$ is taken from Refs. 12–15, respectively; the scatter in $\lambda_{ab}$ values as provided by different authors is incorporated in the error bars as described below. Our main results, given in Fig. 1, are represented by the solid symbols. For comparison we also show $d\rho_s / dT$ for single crystals of YBCO, LSCO, BSCO, and $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ (TBCO) taken from Refs. 2, 16, 17, respectively, versus $\lambda_{ab}^2$ for single crystal taken from Refs. 6, 14, 18, respectively (open symbols).

In the case of the LA law, we fit our data to a straight line given by Eq. (2) with $q$ as a fit parameter. We find $q = 1.75(15)$. The fit is shown in Fig. 1. We also depict in
the figure the LA prediction with \( q = 2 \). The experimentally
determined boson charge of 1.75(15)\( e \) is very similar to
theoretical charge of \( 2e \). It means that the HCB model is
self-consistent for the cuprates, and a very good starting
point for understanding the conductivity of optimally doped
samples. We also present our results as a Homes-type plot in the
inset of Fig. 1. Since optical conductivity measures the plasma
frequency which is proportional to \( \lambda^{-2}(0) \), it leaves one free
parameter. To achieve the same scales as Homes we multiply
\( \lambda^{-2}(0) \) by 0.02. This 2\% correction is due to the difference
in the penetration depth and dc conductivity as estimated by
optical conductivity measurements and the techniques used
here.\(^{19} \) With this scale we find that on a log-log plot our data are
not far from Homes’, which are represented by the solid line.
We also show the WC theoretical predictions with \( K = 240 \)
in Eq. (1).

It seems that both WC and SC theories are in agreement
with our experiment. Another important piece of information
is the indication of carriers with charge \( 2e \) around the
superconducting-insulator transition. This indication comes
from doping-temperature scaling relations of the resistivity.\(^{20} \)

The emerging picture is that the superconducting state in the
cuprates is grainy, sometimes called Bose glass.\(^{20} \) The SC
HCB model is a good starting point for describing each grain.
The normal metal between grains, in underdoped and possibly
overdoped samples, plays an important role in determining the
conductivity above the global \( T_c \). This metal is best described
by one of the WC theories. However, at optimal doping one
grain takes over the entire sample. When this happens the
conductivity is related only to the superconducting properties,
as Eq. (2) predicts.

We now describe our experiment in more detail. A cardinal
aspect of our measurement is the determination of the absolute
value of the resistivity and resistivity derivative near \( T_c \). One
strategy is to use single crystals, but in this case one does
not know exactly which route the current takes in the sample
between contacts, and it is difficult to precisely determine the
resistivity. Therefore, such measurements are usually done by
preparing a film and patterning a bridge on it by ion milling. It
is then assumed that the resistance is dominated by the bridge.
However, in high temperature superconductors, close to \( T_c \),
the situation is not that simple. Figure 2 shows resistance
measurements for a set of identical bridges. For this and
other measurements, we used films grown on a 10 \( \times \) 10 mm\(^2 \)
SrTiO\(_3\) (STO) substrate with the \( c \) axis perpendicular to the
film. Due to flux flow resistance, the transition region from normal
to the superconducting state is very rounded and it is
difficult to determine \( \frac{d\rho}{dr}(T \gtrsim T_c) \). There is also variation in the
resistance between different bridges. This variation is due
to the film being less thick near the edges. The inset of Fig. 2
shows the resistance at \( T = 245 \) K as a function of bridge
number. Indeed, the first and last bridges are more resistive,
but the middle ones have very similar resistance. We therefore
abandoned the bridge method, and focused on wide film
measurements which sample the film center and have very
sharp transitions, as shown in Fig. 3. However, in this case
geometrical factors have to be taken into account when measuring resistivity.\(^{21} \)

Our four-point probe measurement setup is shown in the
inset of Fig. 3(b). The two external contacts are used as the
current source and drain and the two internal contacts are
the voltage probes. For a single current source at the origin in
contact with a two-dimensional (2D) infinite conducting plane,
the current density at a distance \( r \) from the source is given by
\( J = I/(2\pi r) \). The electric field on the conducting surface is
set by \( J = \sigma E \). This leads to a logarithmic potential \( V = V_0 =
-\frac{1}{2\pi} r \rho_N \ln r \). In a current source (a) and drain (b), with equal
distance \( s \) between all probes, the potential difference is \( \Delta V =
\frac{1}{2} r \rho_N \). For a finite sheet, the potential difference is found by
introducing an infinite number of images to the original current

FIG. 1. (Color online) Solid symbols: The temperature derivative
of the resistivity of four different optimally doped cuprate films,
at \( T > T_c \), obtained by dc measurements, as a function of their
penetration depth. The solid lines show the best linear fit to the data
(that extrapolates to the origin) and the prediction by the LA model.
The inset shows a Homes-type law on a log-log scale
of the resistivity of four different optimally doped cuprate films,
here.\(^{19} \) With this scale we find that on a log-log plot our data are
consistent with the LA prediction with \( q = 2 \).

FIG. 2. (Color online) Measurements of resistance as a function
of temperature in narrow bridges of optimally doped YBCO. Bridges
1 and 8 are close to the edges of the film. Bridges 3–6 are in the center
of the film.
The resistivity obtained by using Eq. (4). The resistivity is of different dimensions and different distances between contacts. The inset in (b) shows the experimental setup and the set of current images used to generate the correction factor calculated in Eq. (3).

FIG. 3. (Color online) (a) Resistance vs temperature in films of different dimensions and different distances between contacts. (b) The resistivity obtained by using Eq. (4). The resistivity is geometry independent. The inset in (b) shows the experimental setup.

FIG. 4. (Color online) The geometrical factor $C$, calculated in Eq. (3), as a function of the width $w$ for various lengths $l$, and a fixed distance between contacts $s$.

FIG. 5. (Color online) $V$-$I$ measurements of the YBCO film at different temperatures, demonstrating the ohmic behavior of the film. The inset shows an AFM image of the film topography near an etched step.

Current simulations show that only 7% of the total current passes close to the edges of the films where the resistance is high by 7% (see the inset of Fig. 2). This leads to an error of less than 1% on the resistivity due to the thickness measurement.

To check the validity of Eqs. (3) and (4), we produced YBCO films of various geometries and measured their resistance as presented in Fig. 3(a). The figure shows the resistance ($\Delta V/I$) of seven different films with various heights $z$, widths $w$, lengths $l$, and distances between contacts $s$, in units of millimeters. Figure 3(b) depicts the resistivity $\rho_{dc}$ obtained by Eq. (4). The resistivity is indeed geometry independent and linear immediately above $T_c$.

In Fig. 5, we show $V$-$I$ measurements of one of the YBCO films. In the normal state, the films show ohmic behavior up to a current of 140 $\mu$A. Therefore, all our measurements are done in a current of 100 $\mu$A.

Finally, we present resistivity measurements in optimally doped films of YBCO, LSCO, BSCCO, and CLBLCO in Fig. 6. A pure linear behavior is observed only in YBCO, and, as expected, the resistivity extrapolates to zero at zero temperature. In LSCO, the substrate reduces $T_c$ from the bulk value considerably, due to a mismatch in lattice parameters. This lattice mismatch also reduces the $T_c$ of the other compounds, but not as much as in LSCO. To simplify our analysis, we focus on the temperature range 100–200 K, which, for all materials, is higher than the region of fluctuating superconductivity, and lower than $T^*$. In this temperature range, the reduction of $T_c$ in LSCO is not...
relevant. In the inset of Fig. 6, we present the first derivative of the resistivity as a function of temperature. As expected, the derivative is a constant only for YBCO. For the other materials, the derivative varies slowly with temperature. We treat the derivative as a statistical variable and assign to each material an averaged resistivity slope and standard deviation over the entire plotted range. The standard deviation is used to generate the error bars. The summary of our thermal derivative of the resistivity versus magnetic penetration depth results is plotted in Fig. 1. As mentioned before, the penetration depth

\[ \rho_{dc}(T\to 0) = 0. \]

is taken from the literature. For optimally doped YBCO film, \( \lambda_{ab} = 146 \pm 3 \text{ nm} \) was determined in a theory-free method using slow muons.\(^\text{12}\) In this case, the value of \( \lambda_{ab} \) agrees with coated sample resonance (CSR) measurements, which is also a theory-free method,\(^\text{15}\) and the error bar is known. For YBCO crystal, \( \lambda_{ab} = 115 \pm 3 \text{ nm} \) was also measured with slow muons.\(^\text{18}\) For LSCO, there are only crystal measurements and all values reported are scattered around 260 \pm 15 nm.\(^\text{14}\) For BSCCO, the \( \lambda_{ab} = 270 \pm 15 \text{ nm} \) value was taken from CSR with its error bar.\(^\text{15}\) For BSCCO and TBCO crystals the values of \( \lambda_{ab} = 196 \) and \( \lambda_{ab} = 162 \), respectively, are from Ref. 6. They have been measured by a few techniques but no error bar is assigned. Finally, CLBLCO was measured only by standard \( \mu \text{SR} \), where the determination of \( \lambda_{ab} = 250 \text{ nm} \) involves theoretical arguments and the error bar is not known.\(^\text{13}\)

A comparison between our experimental results and both WC and SC theories show that both are valid for optimally doped samples to some extent. To distinguish between the two, the WC theories should be extended to provide \( \sigma(T > T_c) \). Similarly, the SC theory should be broadened to include the doping dependence of \( \rho(T > T_c) \). We believe that there is room for a third theoretical approach that combines the two paradigms into one, in order to account for the full doping and temperature variations. As for optimal doping, the fact that the resistivity above \( T_c \) is determined by a superconducting quantity only is an amazing property of the cuprates.

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