Stiffness measurements of thin and narrow superconducting rings

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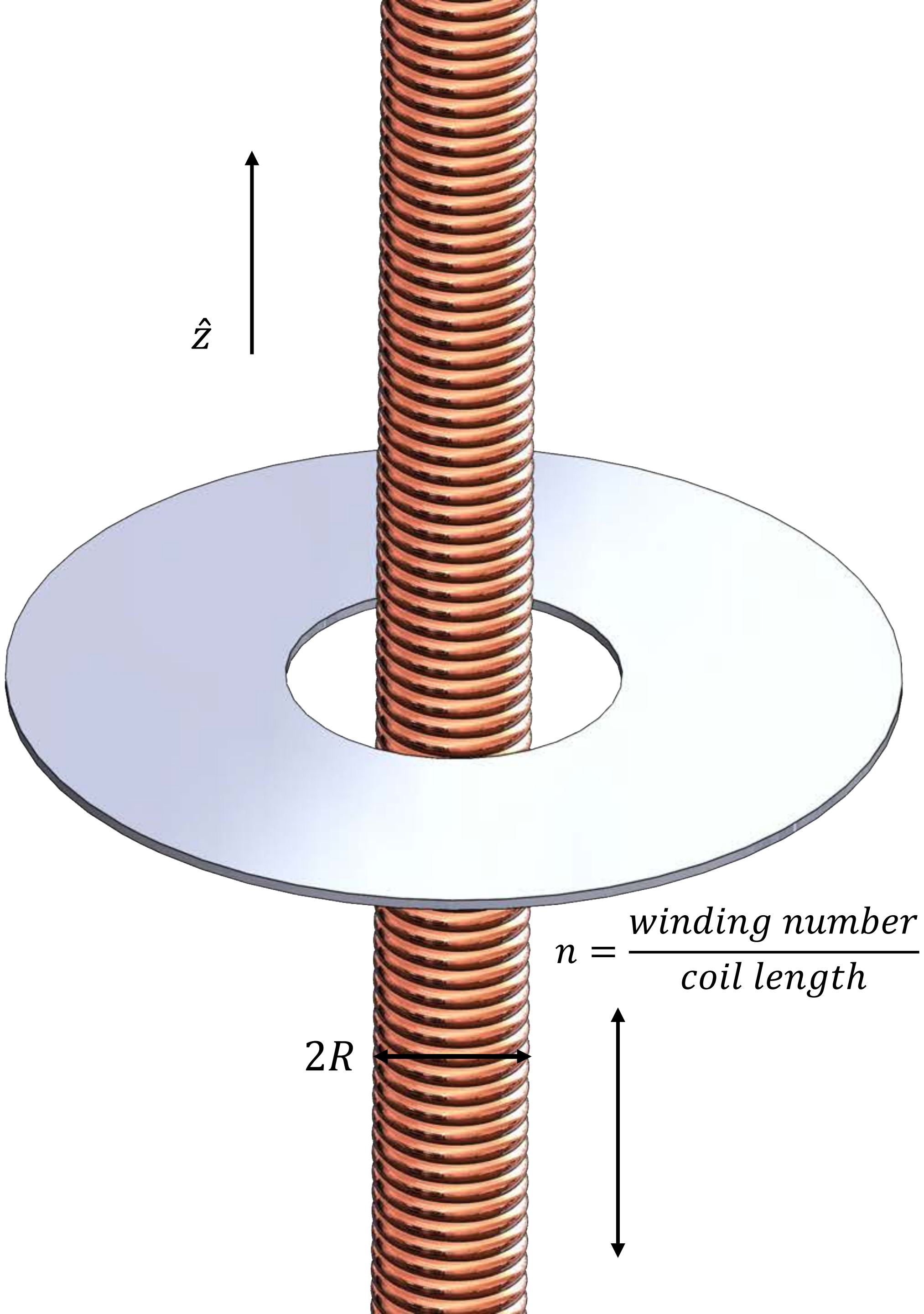
Outline

Stiffnessometer

- Superconducting ring threaded by a long excitation coil with DC current
$$\vec{A} = \frac{\mu_0 n I R^2}{2r}\hat{\varphi}.$$
- The magnetic field H is confined inside the coil, and the ring feels pure vector potential.
- Due to London equation vector potential generates magnetic moment which is measured by a SQUID magnetometer.
- From $m(I)$ we extract the Λ and ξ.



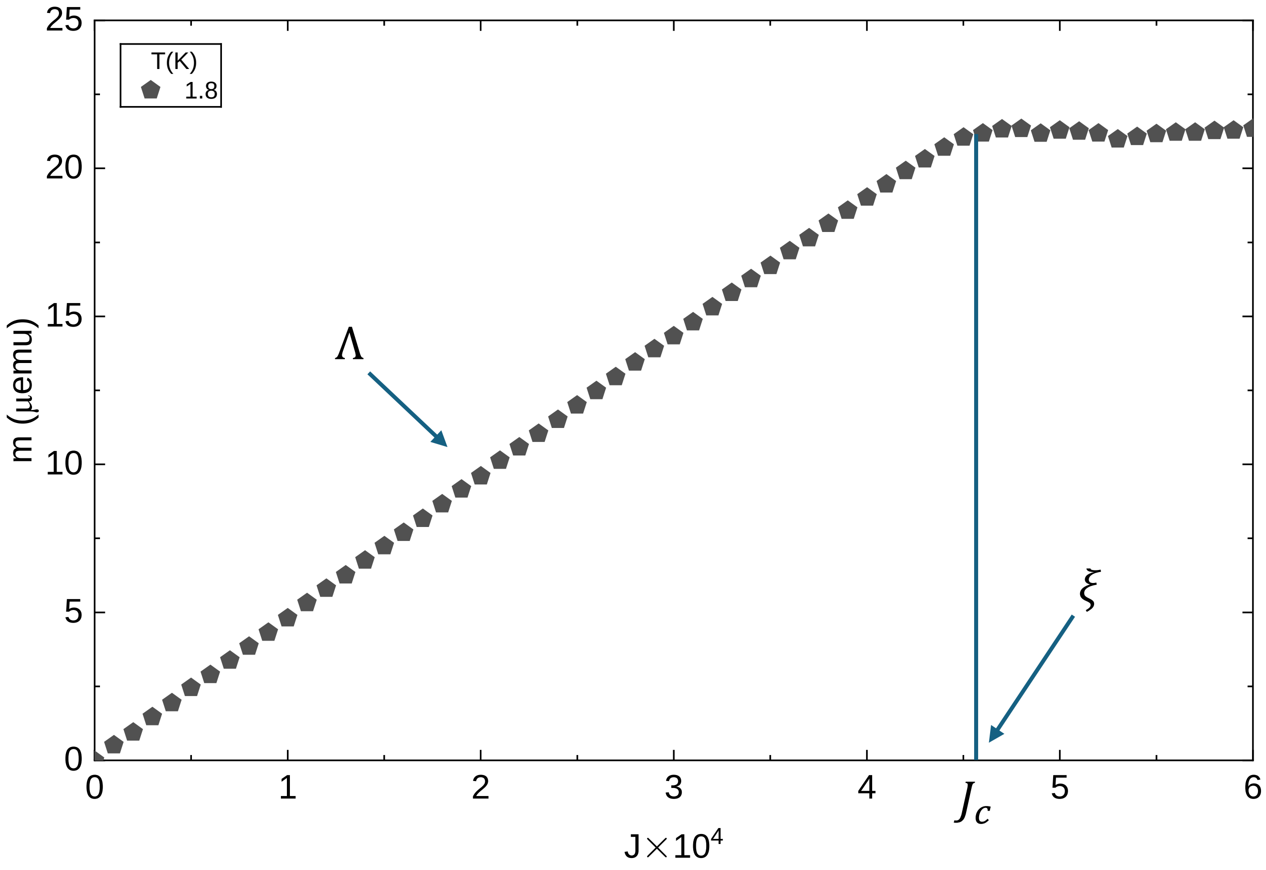
Typical measurement

J is applied flux divided by flux quanta.

$$J = \frac{\Phi}{\Phi_0}$$

Slope is related to Λ

Platue point related to ξ



Superconducting length scales

- London penetration depth

$$\lambda$$

Characteristic decay length of the magnetic field into the superconductor

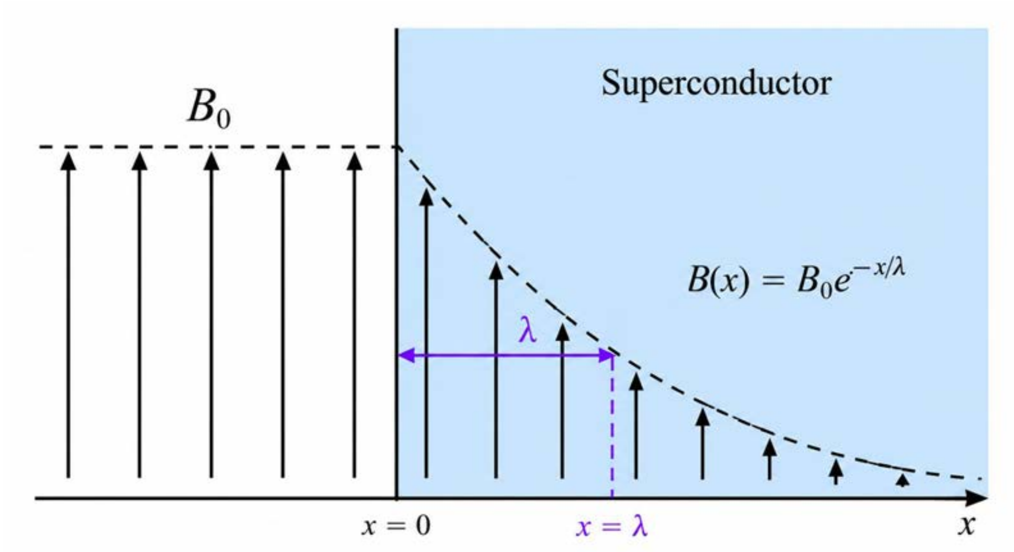
- Coherence length

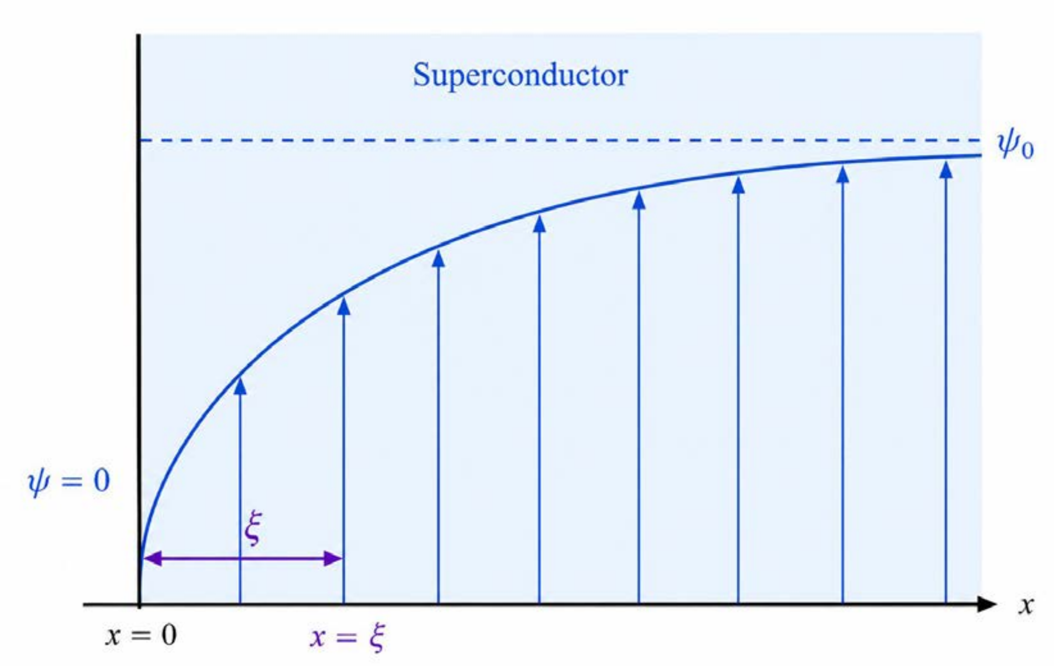
$$\xi$$

Length on which the order parameter varies significantly.

- We define the Pearl length

$$\Lambda = \frac{\lambda^2}{d}$$

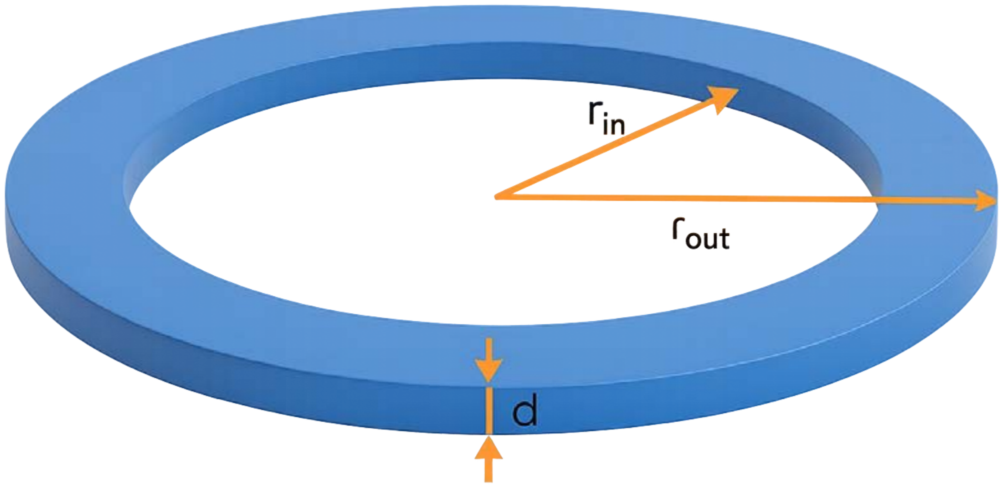




Thin, narrow and very narrow ring

- Thin film: $d \ll \lambda$
- Narrow ring: $r_{out} - r_{in} \ll r_{in}$
- Very narrow ring:

$$\Lambda \gg \frac{1}{3\pi}(r_{out} - r_{in}) \ln \frac{r_{in}}{r_{out} - r_{in}}$$



Order parameter and London equation

- Ginzburg-Landau order parameter

$$\psi(r,\theta) = |\psi_0(r)|e^{i\phi(r,\theta)}$$

- London equation

$$\vec{J}_s = -\rho_s\left(\vec{A}_{tot} - \frac{\Phi_0}{2\pi}\nabla\phi\right)$$

- As $\rho_s = \mu_0 \frac{|\psi|^2 e^{*2}}{m^*} = \frac{1}{\lambda^2}$

Motivation

- AK et al.; based on mean field

$$J_c = \frac{r_{out}}{\xi}$$

- Roy Rabaglia, AK, Ari Turner suggest based on fluctuations

$$J_c = P(r_{in}, r_{out}, \Lambda)\frac{r_{in}}{\xi}$$

- In thin and very narrow ring $P = \frac{1}{\sqrt{3}}$.

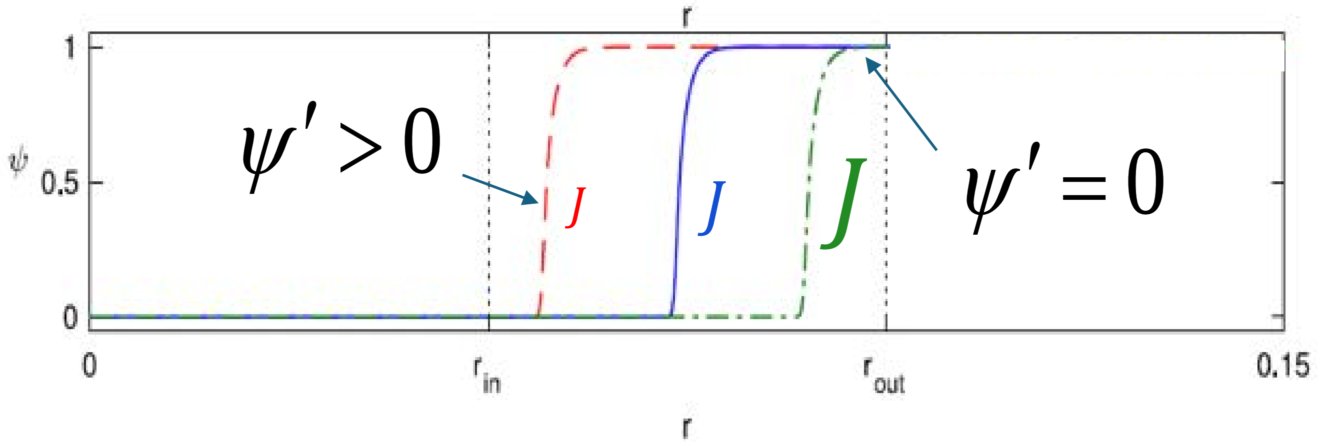
We want to check this experimentally.

Mean field

- Using 2^{nd} GL equation

$$\xi^2\left(\psi_{rr}+\frac{\psi_r}{r}\right)=\left(\xi^2\left(\mathrm{A}_{sc}+\frac{J}{r}\right)^2-1\right)\psi+\psi^3$$

- $J=\frac{\Phi}{\Phi_0}$, Φ_0 is flux quanta.
- $A_{coil}=\frac{J}{r}$.
- At high flux regime $\psi\to 0,\ \psi^3$ is ignored.
- For thin film $A_{sc}\ll\frac{J}{r},\ A_{sc}$ is ignored.
- We are left with $\psi_{rr}+\frac{\psi_r}{r}=\left(\left(\frac{J}{r}\right)^2-\frac{1}{\xi^2}\right)\psi$



At r_{out}: $\psi_{rr}<0,\ \psi_r=0$

$$J_c\equiv\frac{\Phi_c}{\Phi_0}=\frac{r_{out}}{\xi}$$

Stability against fluctuations

- For a thin superconductor $d \to 0$

$$F = \int d^2x \left[\left| \left(\frac{1}{J} \nabla - i\tilde{A} \right) \psi \right|^2 - |\psi|^2 + \frac{1}{2} |\psi|^4 + \frac{\lambda^2}{\xi^2 J^2} \left| \nabla \times \tilde{A} \right|^2 \right]$$

$$\tilde{A} \to \frac{2\pi\xi}{\Phi_0} A \,, \qquad \tilde{A} = \frac{1}{x} \hat{\theta} + \tilde{A}_{sc}, \qquad x = \frac{r}{J\xi}$$

- The GL equations are the extremum of this F, but not necessary the minimum.

- The stability approach askes when does the minimum turns into a saddle point.

Finding instability point $\rightarrow J_c$

- For $J \gg 1$ the derivative are ignored and we are left with

$$F = \xi^{-2} \int d^2 r \left[-\left(1 - \left|\tilde{A}\right|^2\right) |\psi|^2 + \frac{1}{2} |\psi|^4 \right]$$

- Variation by $|\psi|^2$ gives

$$\psi_0(r) = \sqrt{1 - \left|\tilde{A}\right|^2}$$

Fluctuations in $\boldsymbol{\psi(r)}$

- To find the instability we look into the variation in ψ

$$\psi(r,\theta) = \psi_0(r) + \delta\psi(r,\theta)$$

and ask under what conditions $\delta\psi(r,\theta)$ decrease the energy.

- Fluctuation in the energy δF to second order can be written as

$$\delta F = \xi^{-2} \int d^2r \left[\left| \left(\frac{\xi}{r} \frac{\partial}{\partial\theta} - i|A| \right) \delta\psi \right|^2 + Re[\psi_0^2 \delta\psi^2] + (2\psi_0^2 - 1)|\delta\psi|^2 \right]$$

Fluctuations in $\psi(r)$

- We can write $\delta\psi$ in Fourier manner that describe the number of vortices that enter our ring

$$\delta\psi(r,\theta) = \sum_k \delta\sigma_k(r) e^{ik\theta}$$

- The solution is unstable $\delta F \leq 0$.

- $|A_c(r_{in})| = \frac{1}{\sqrt{3}}$ is the smallest value at which instability occur.

Computing $\mathrm{A}(\boldsymbol{r})$ for $|\boldsymbol{A_c(r_{in})}| = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$

- London Eq. $j \propto -A|\psi|^2$

- The current in an infinitesimal ring of radius l (in dimensionless units)

$$I_l = -\frac{\Phi_0}{2\pi\mu_0}\frac{1}{\Lambda\xi}A(l)(1-|A(l)|^2)dl$$

- For a current ring the vector potential[1] is :

$$A_l(r, z=0) = \frac{1}{2\pi}I\left(1+\frac{l}{r}\right)\left[\frac{l^2+r^2}{(l+r)^2}K\left(\frac{2\sqrt{lr}}{l+r}\right) - E\left(\frac{2\sqrt{lr}}{l+r}\right)\right]\hat{\theta}$$

1) Jackson, J. D. (1999). Classical electrodynamics, page 142

Computing $A(r)$

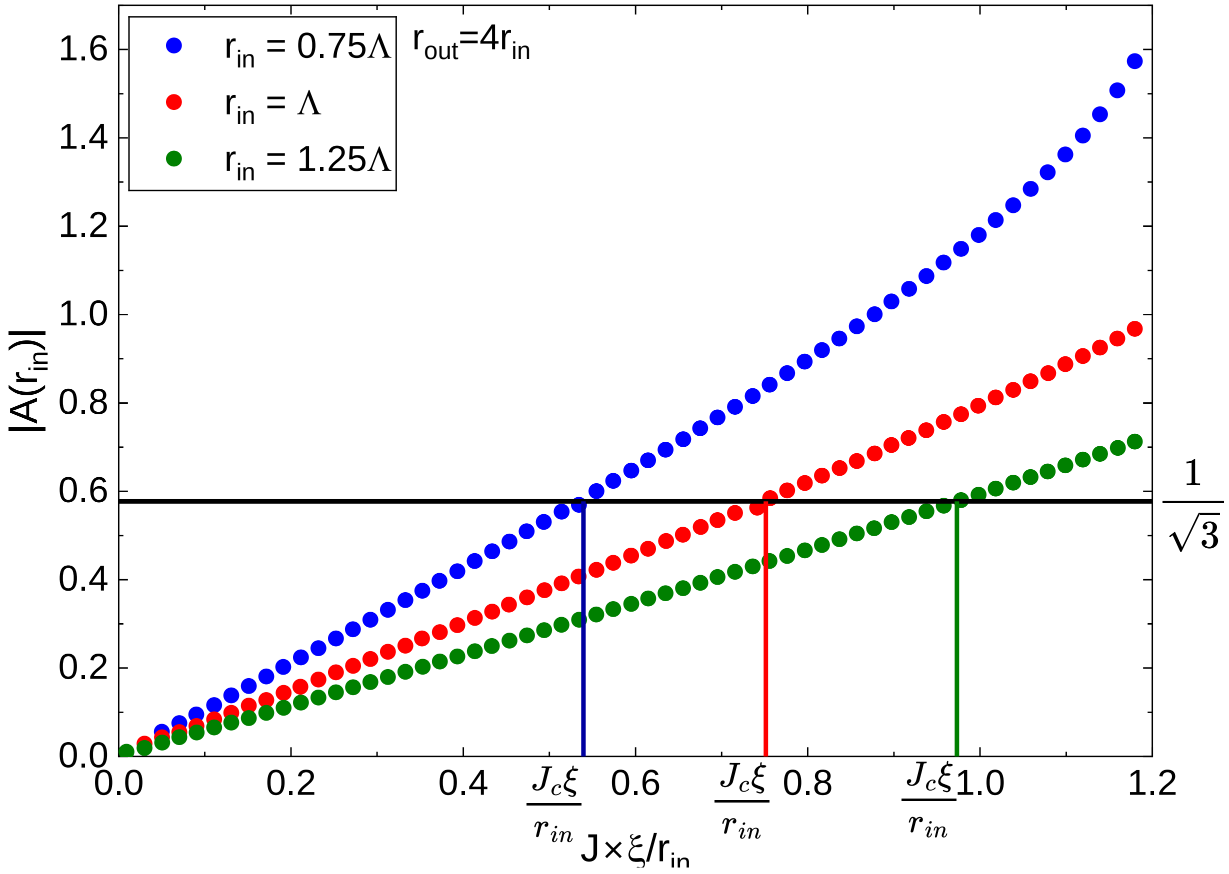
- Summing over infinitesimal rings for the full vector potential

$$|A\ (r)| = \frac{J\xi}{r} - \frac{1}{2\pi\Lambda}\int_{r_{in}}^{r_{out}} |A(l)|(1 - |A(l)|^2)\ \left(1 + \frac{l}{r}\right)\left[\frac{l^2 + r^2}{(l+r)^2} K\left(\frac{2\sqrt{lr}}{l+r}\right) - E\left(\frac{2\sqrt{lr}}{l+r}\right)\right] dl$$

- This integral equation should be solved numerically.

Finding $P(r_{in}, r_{out}, \Lambda)$

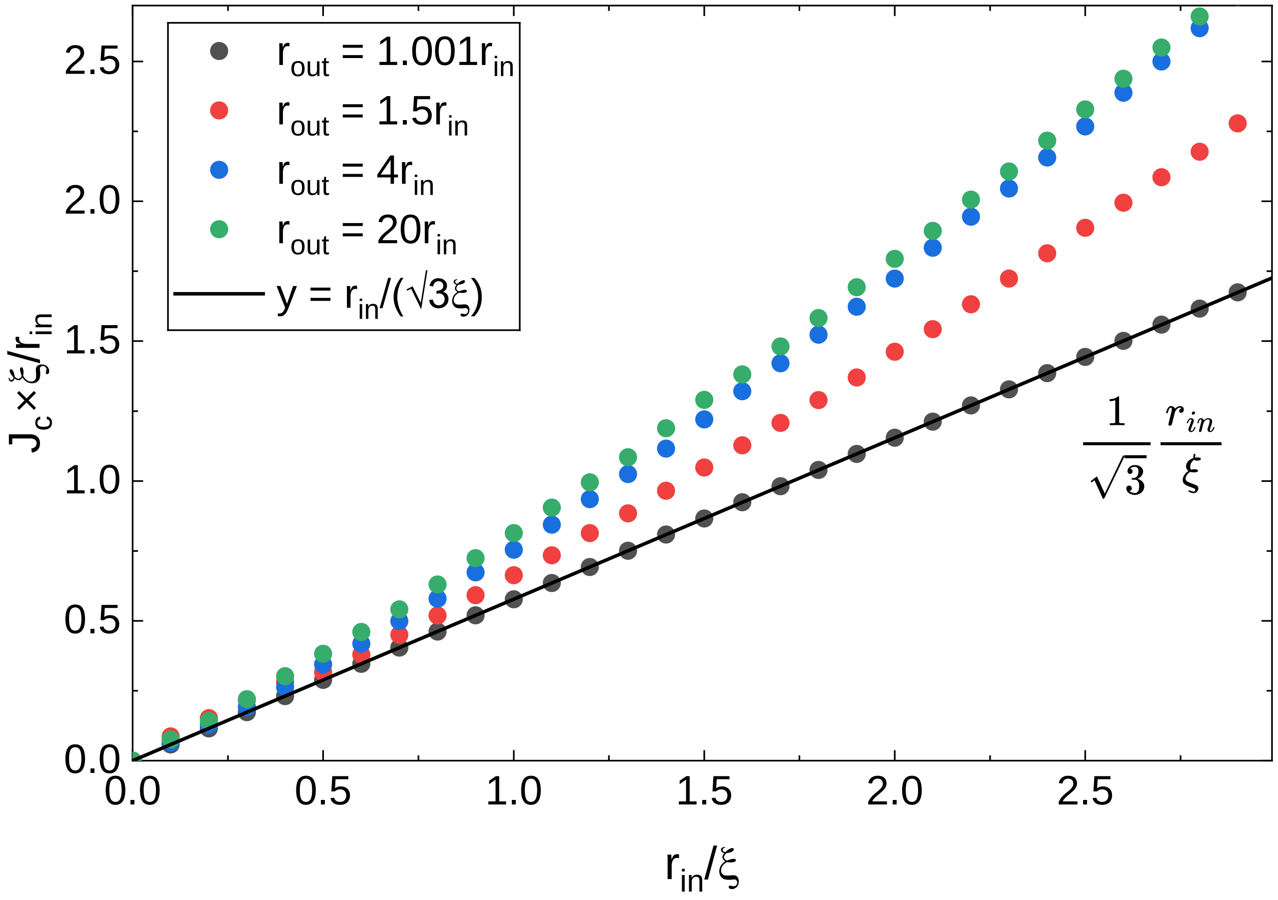
- Looking for $|A(r_{in})| = \frac{1}{\sqrt{3}}$ gives $\frac{J_c \xi}{r_{in}} = P(r_{in}, r_{out}, \Lambda)$.



Finding $P(r_{in}, r_{out}, \Lambda)$

- For thin and very narrow ring

$$P(r_{in}, r_{out}, \Lambda) = \frac{1}{\sqrt{3}} = 0.577$$



Extracting Λ ($J \to 0, \psi = 1$)

- Using London equation in 2D and Amper law

$$\nabla^2 \vec{A}_{SC} = \rho_s\left(\vec{A}_{SC} + \vec{A}_{EC}\right)$$

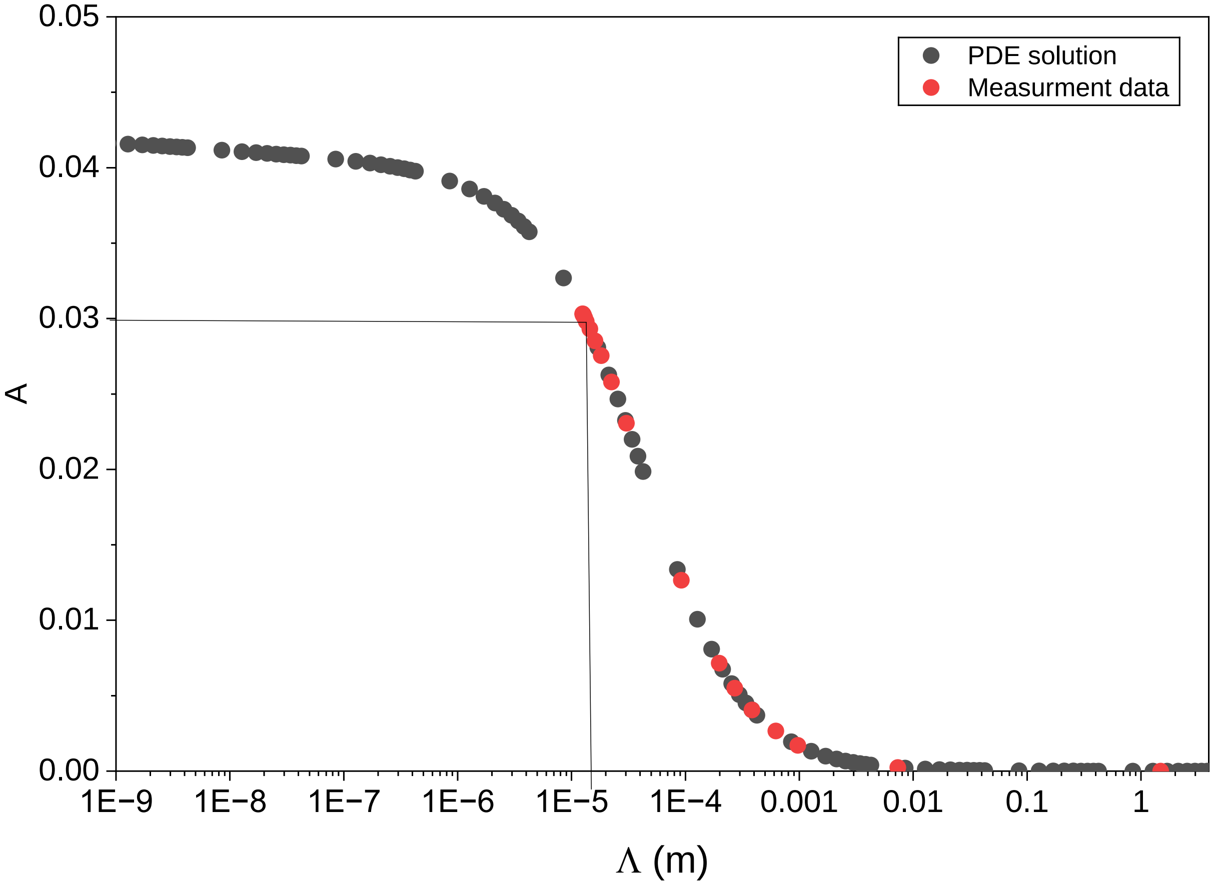
$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - \frac{A}{r^2} = \frac{1}{\Lambda}\left(A + \frac{1}{r}\right)\delta(z)$$

- Solving the PDE using FreeFEM++ (with no mean field approximation) gives the relation $A(\mathrm{r}, \Lambda)$.

- At pickup loop radius R_{pl} our measured m converts to

$$A(R_{pl}, \Lambda) = \frac{A_{sc}(R_{pl}, \Lambda)}{A_{ec}(R_{pl})} = \frac{\frac{\mu_0 m(\Lambda)}{4\pi R_{pl}}}{A_{ec}(R_{pl})} = C_{ec}\frac{m(\Lambda)}{I}$$

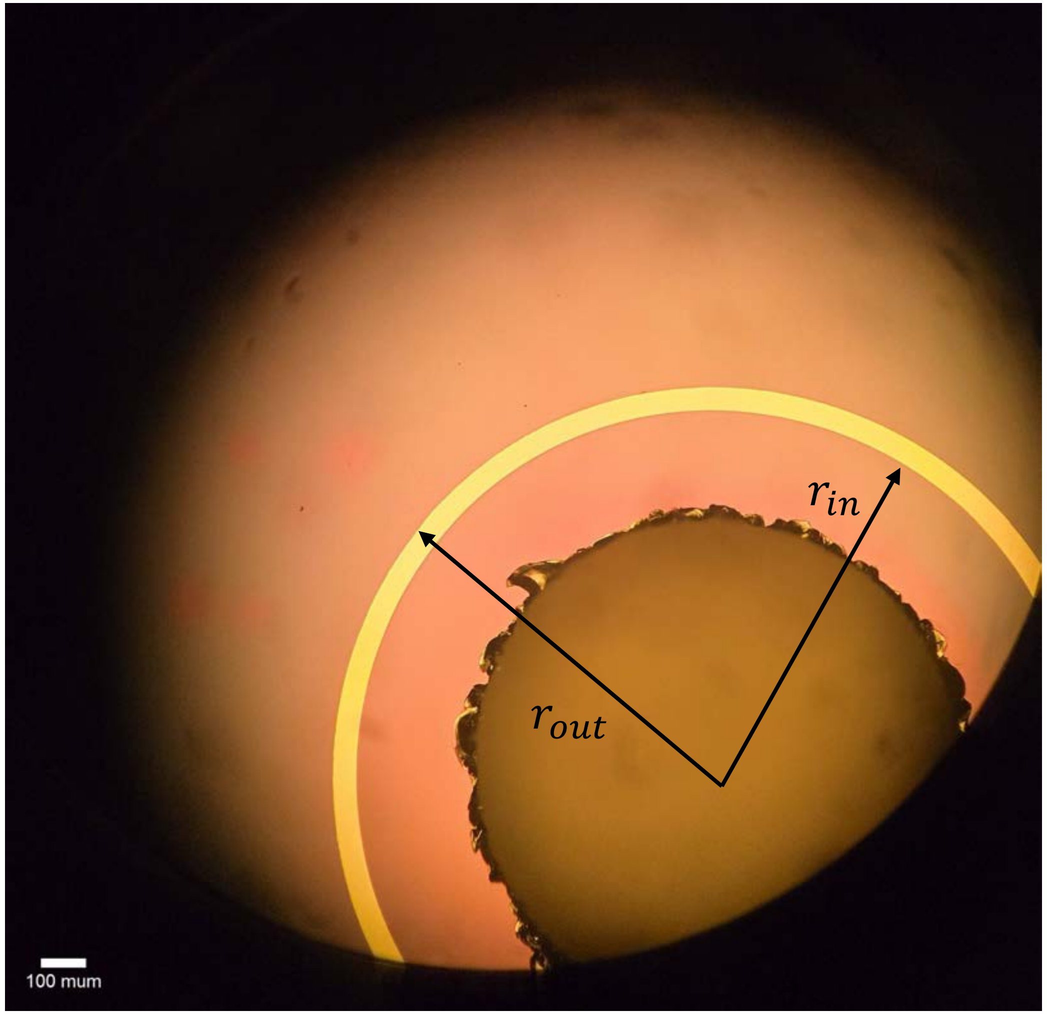
A_{ec} is determined experimentally.



$r_{in} = 750\ \mu m$ and $r_{out} = 825\ \mu m$, Temperatures (1.8-6.8) K

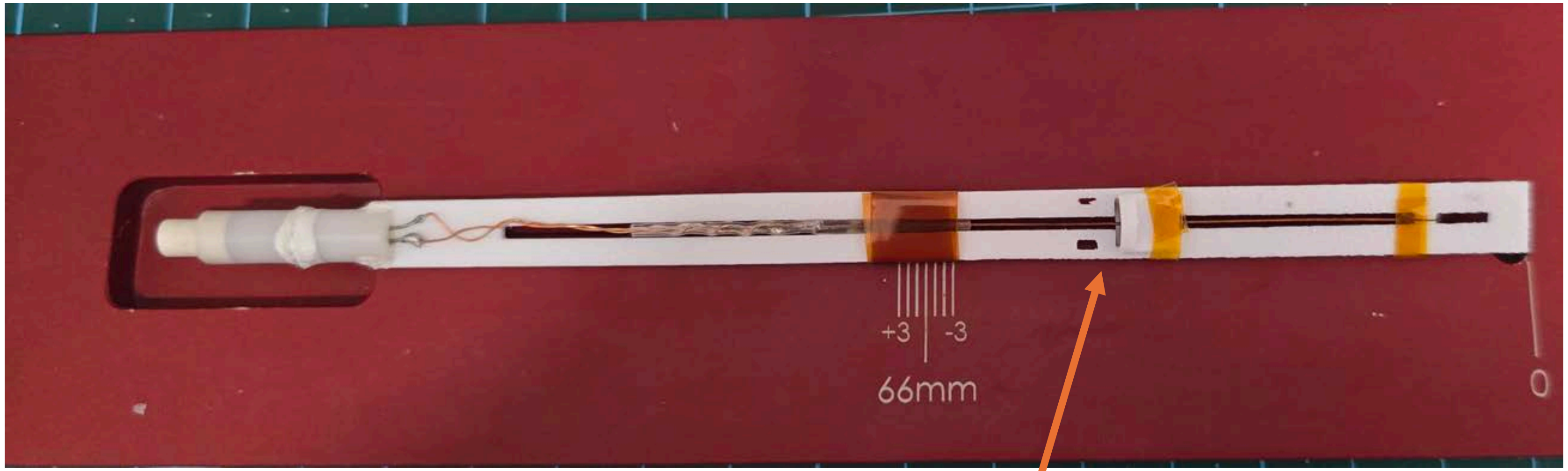
Thin and narrow MoSi sample

- $r_{in} = 750\ \mu m, r_{out} = 800\ \mu m$
- $d = 50\ nm$
- SiO_2 ring substrate



System setup: MPMS3

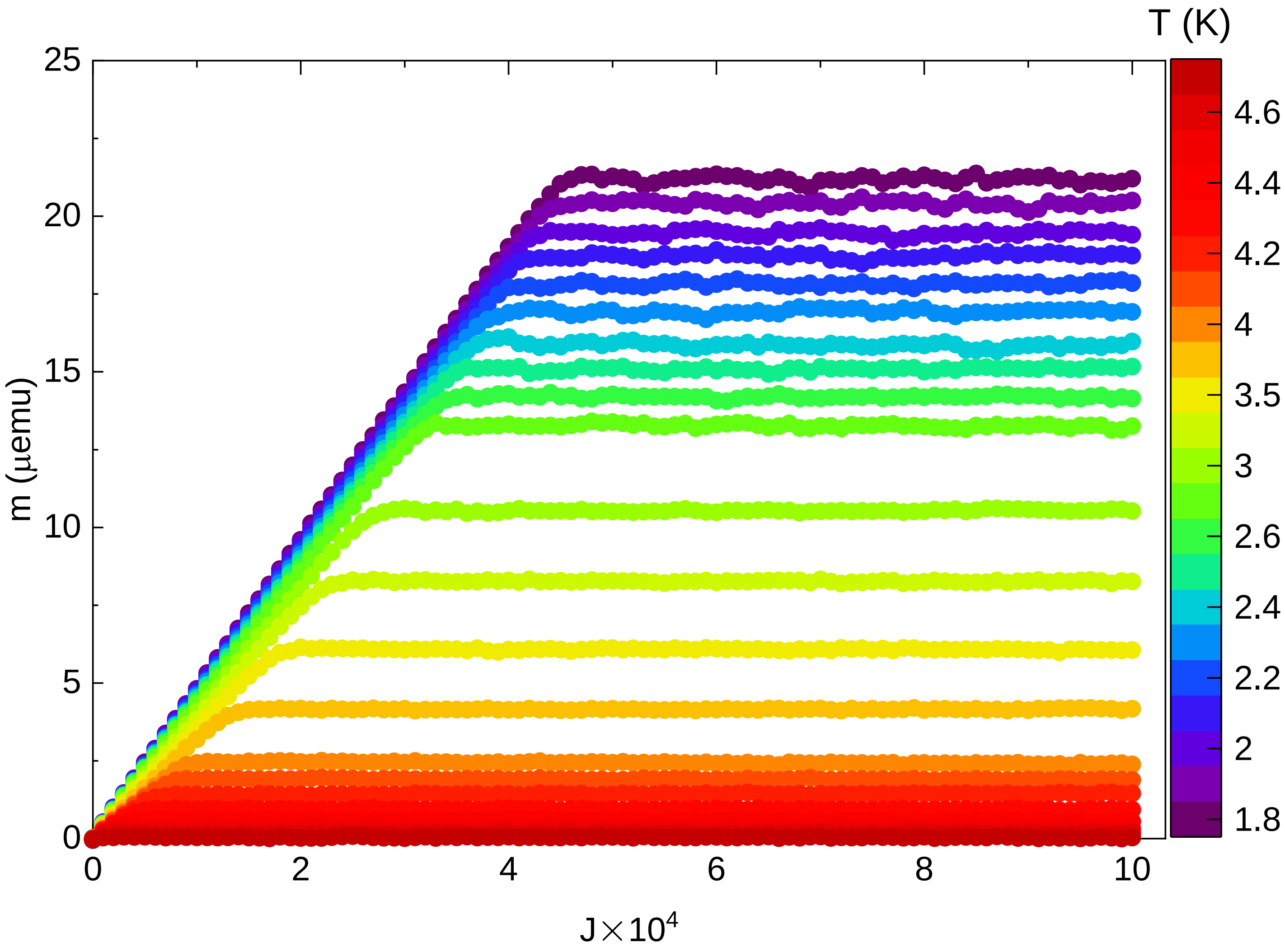
System setup: Sample holder



- Sample glued to the holder threaded with the superconducting (NbTi) excitation coil

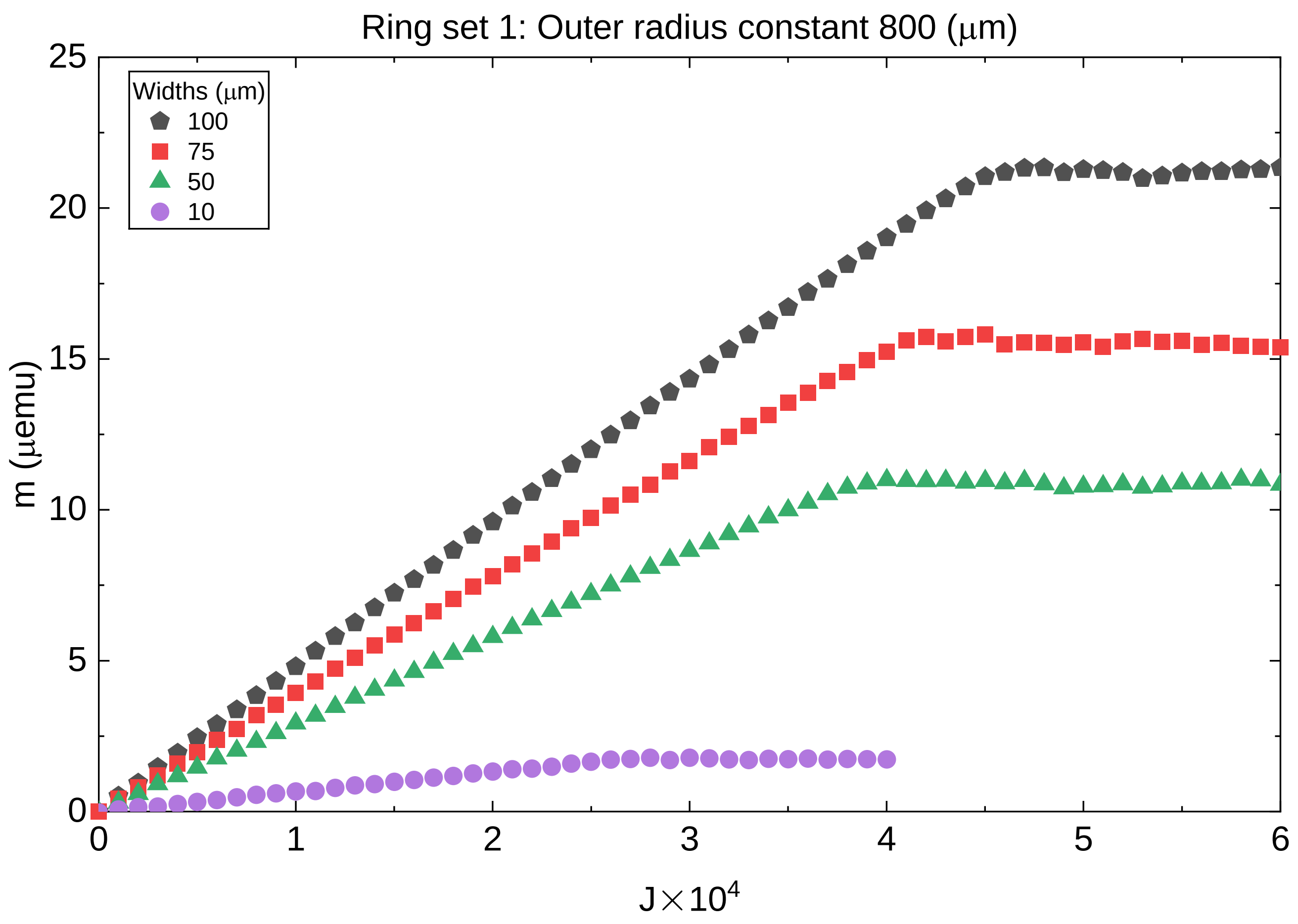
Magnetic response vs Excitation coil flux number

- Cool the sample from above T_c to the desired temperature with no current applied to the coil.
- Gradually increase the DC current in the coil.
- Measure the resulting magnetic moment using the SQUID.



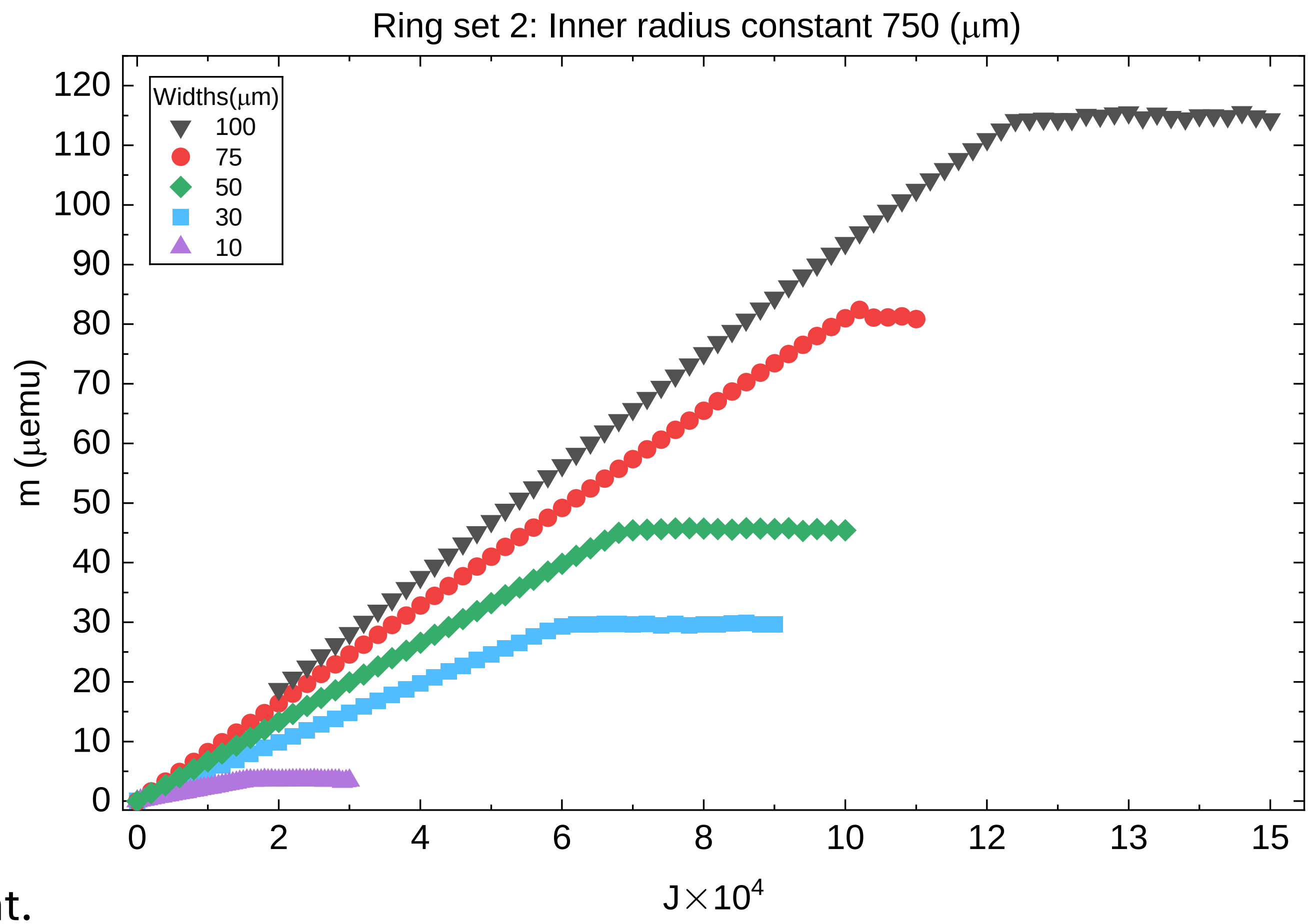
MoSi ring $d = 50$ nm $r_{in} = 700\ \mu m$ and $r_{out} = 800 \mu m$

Magnetic response at $1.8K$



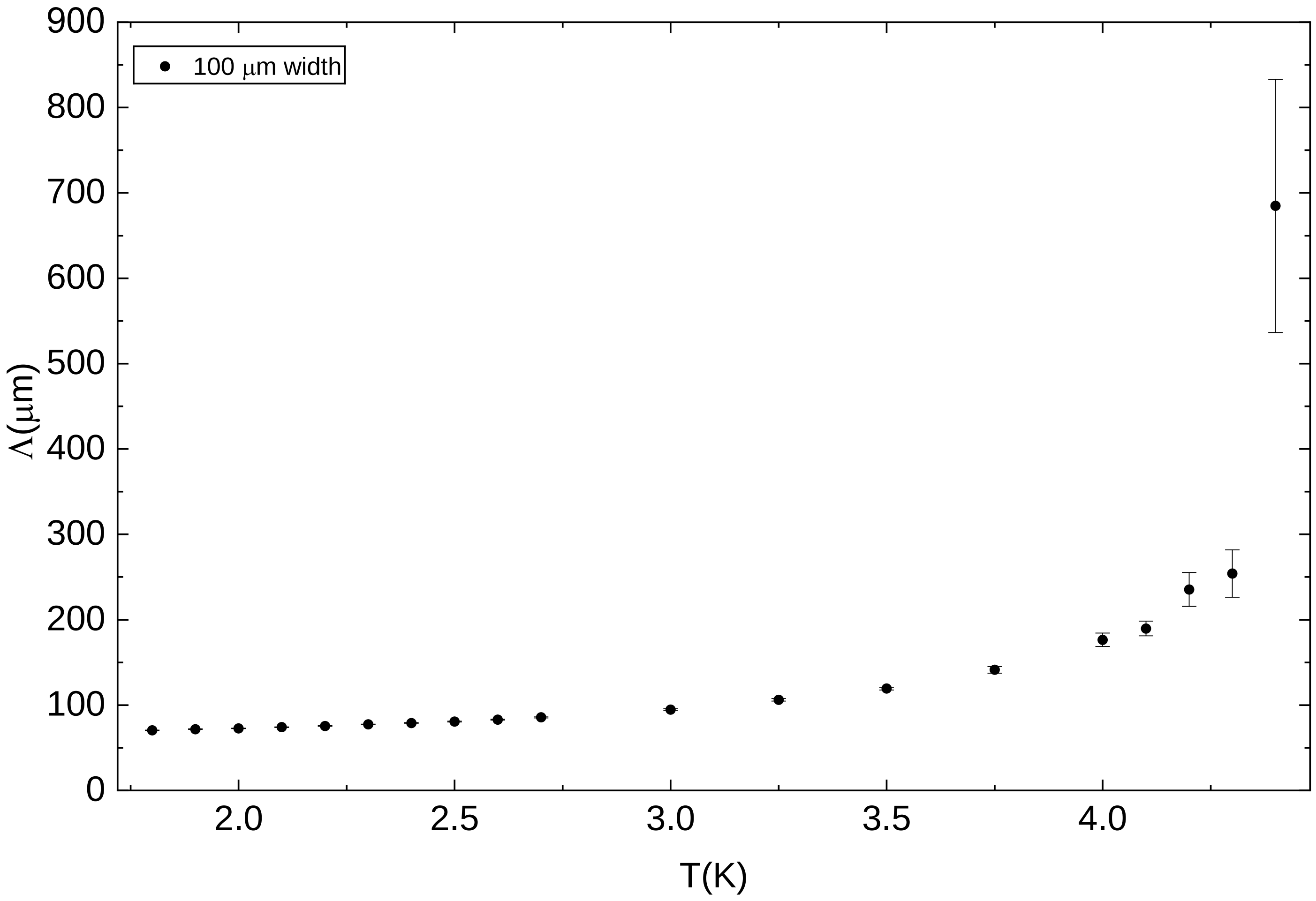
- Old theory ($J_c = \frac{r_{out}}{\xi}$) doesn't work.

Magnetic response at $1.8K$



- The rings are different.
- Neither r_{in} nor r_{out} determine J_c alone.

Pearl Λ vs Temperature



Λ at T=1.8K is 70 μm.

Limits discussion

- We are in the thin limit

From $\lambda = \sqrt{\Lambda d}$,$\lambda_{smallest} = 781\, nm, d = 50\, nm.$

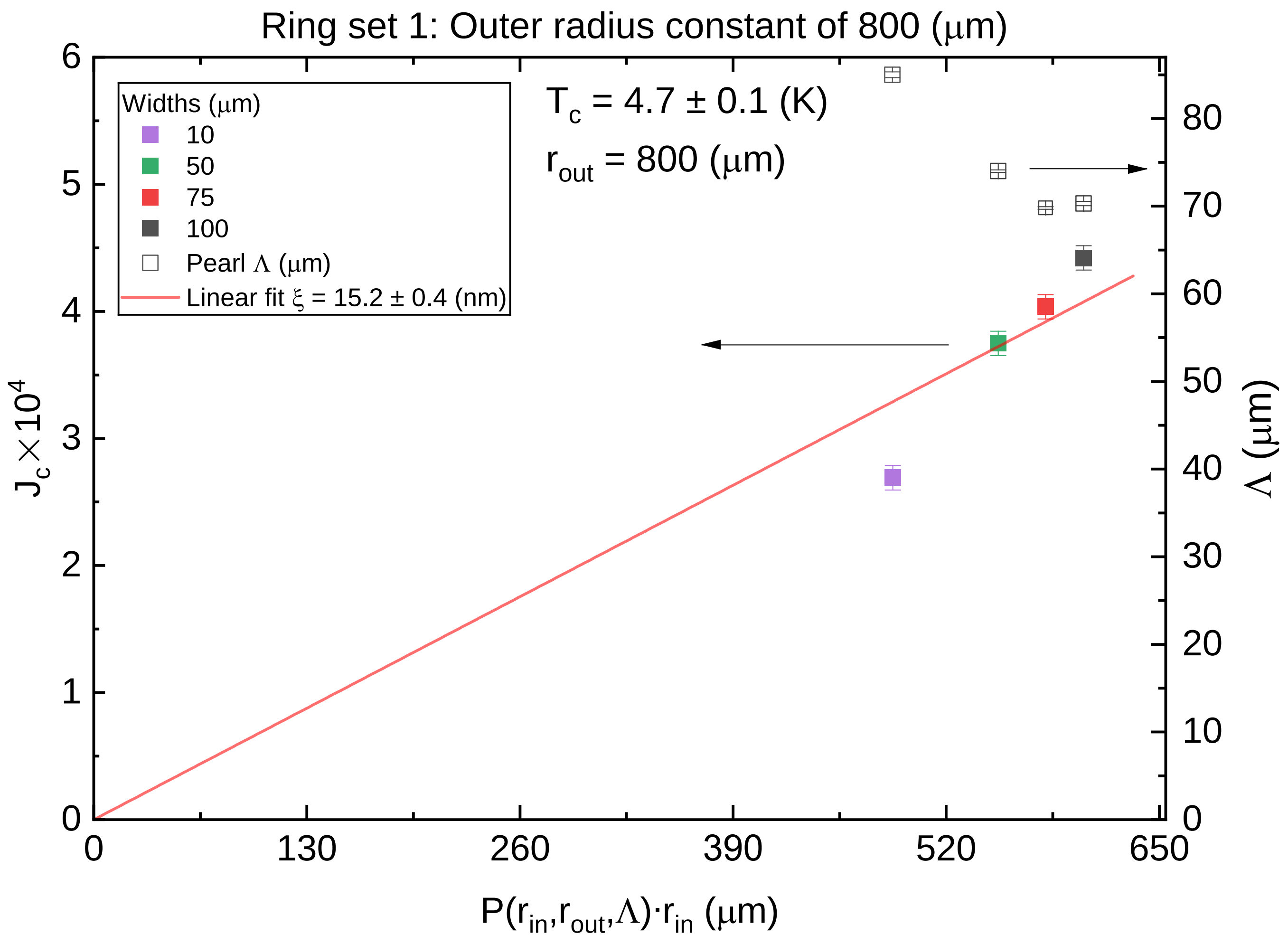
- We are approaching the thin and very narrow ring limit for the 10μm ring.

$\Lambda = 85\, \mu m, r_{in} = 790\, \mu m, r_{out} = 800\, \mu m \to 85 > \frac{1}{3\pi}(r_{out} - r_{in}) \ln \frac{r_{in}}{r_{out}-r_{in}} \approx 4.3$

Indeed, $P(r_{in}, r_{out}, \Lambda) = 0.61$ for the 10μm width ring.
Where for the very narrow limit of $P(r_{in}, r_{out}, \Lambda) = \frac{1}{\sqrt{3}} \approx 0.577.$

Ring set 1: Outer radius constant

$$J_c = P(r_{in}, r_{out}, \Lambda)\frac{r_{in}}{\xi}$$



Ring set 1: Outer radius constant

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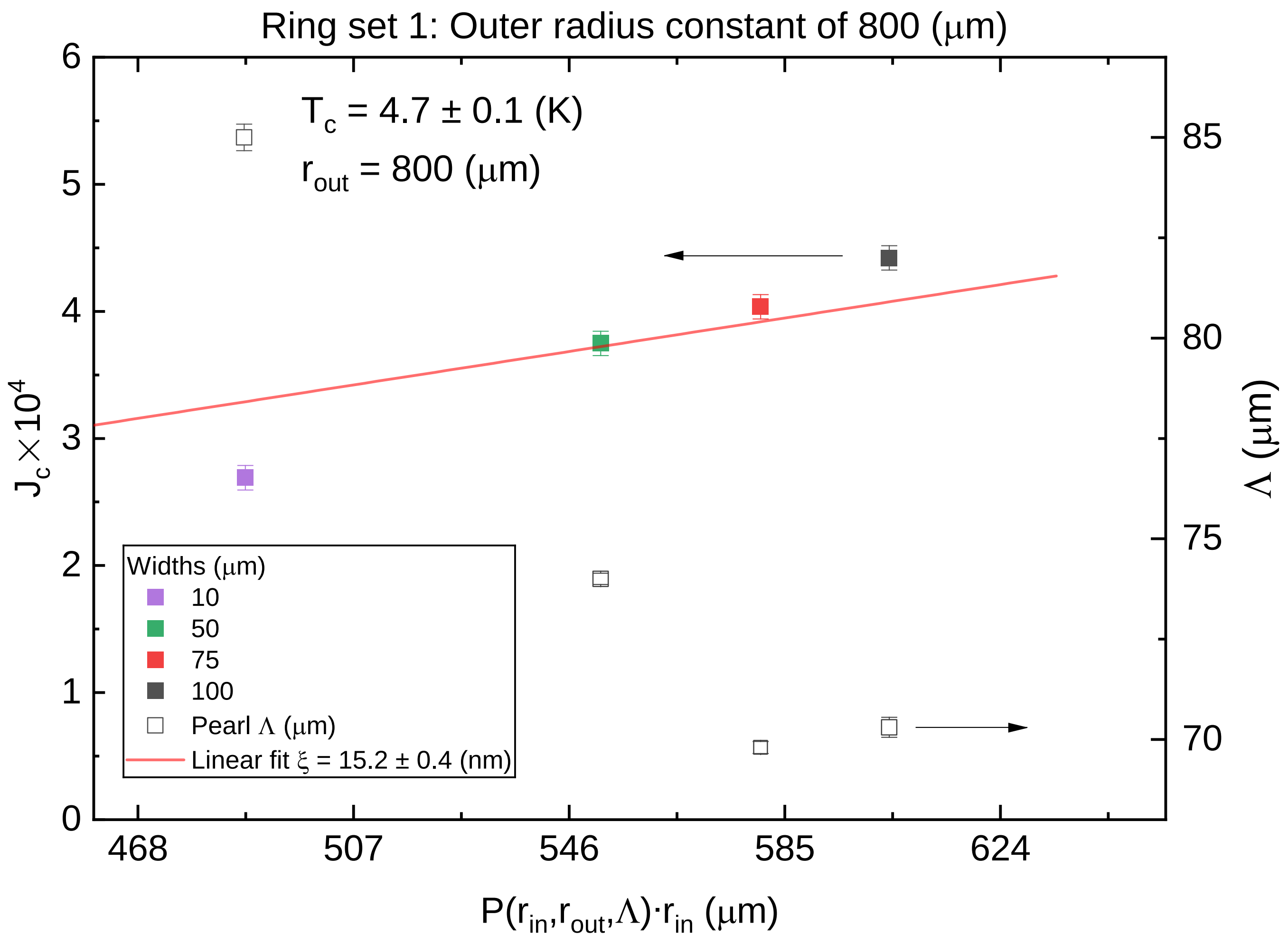
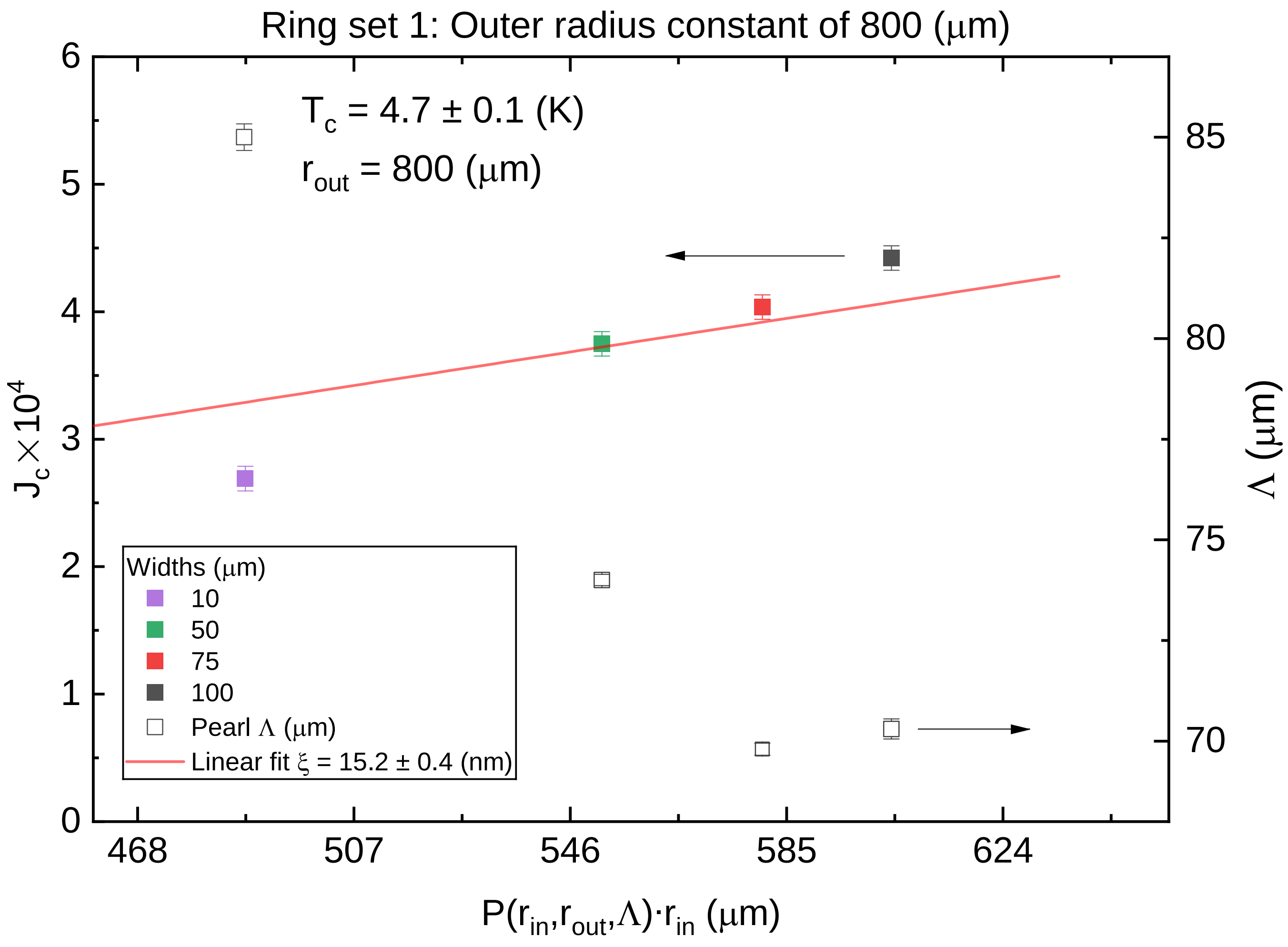


Ring set 1: Outer radius constant

$$J_c = P(r_{in}, r_{out}, \Lambda)\frac{r_{in}}{\xi}$$

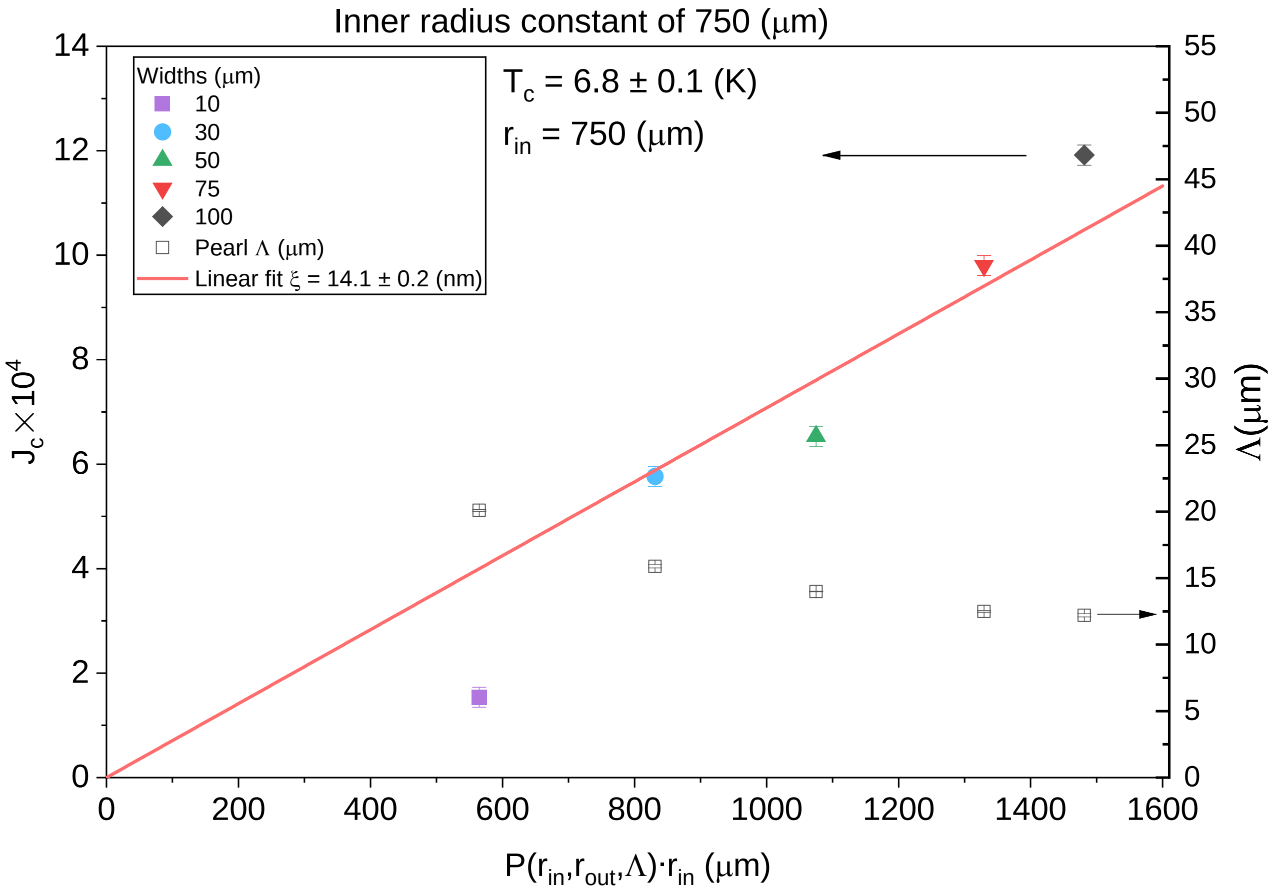
- Despite the big difference in the raw data between samples, Λ turns out to be ~ the same for all rings.
- We observed ~ agreement with the theory apart from w=10 μm.
- The linear fit through the origin gives ξ=15.2(4) nm.



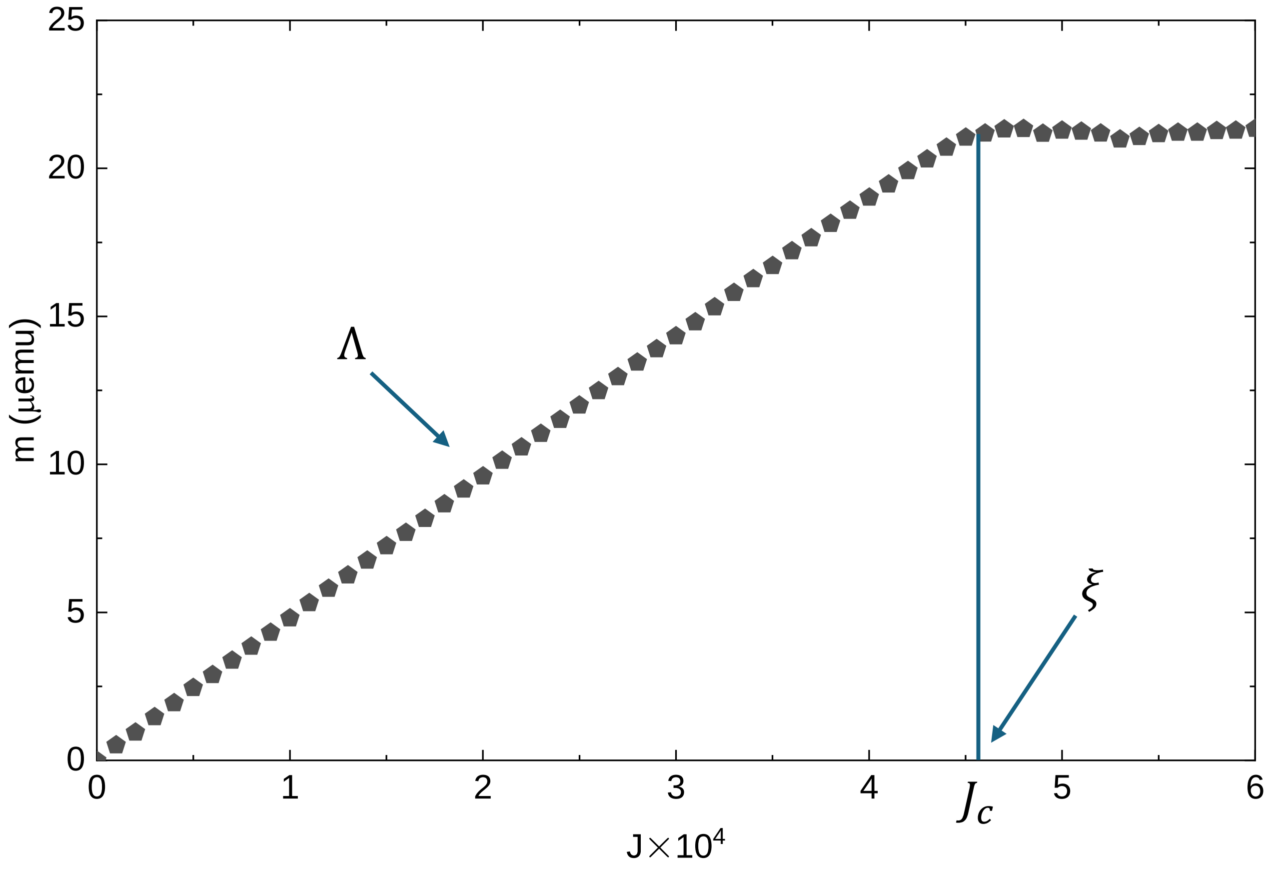
Ring set 2: Inner radius constant

$$J_c = P(r_{in}, r_{out}, \Lambda)\frac{r_{in}}{\xi}$$

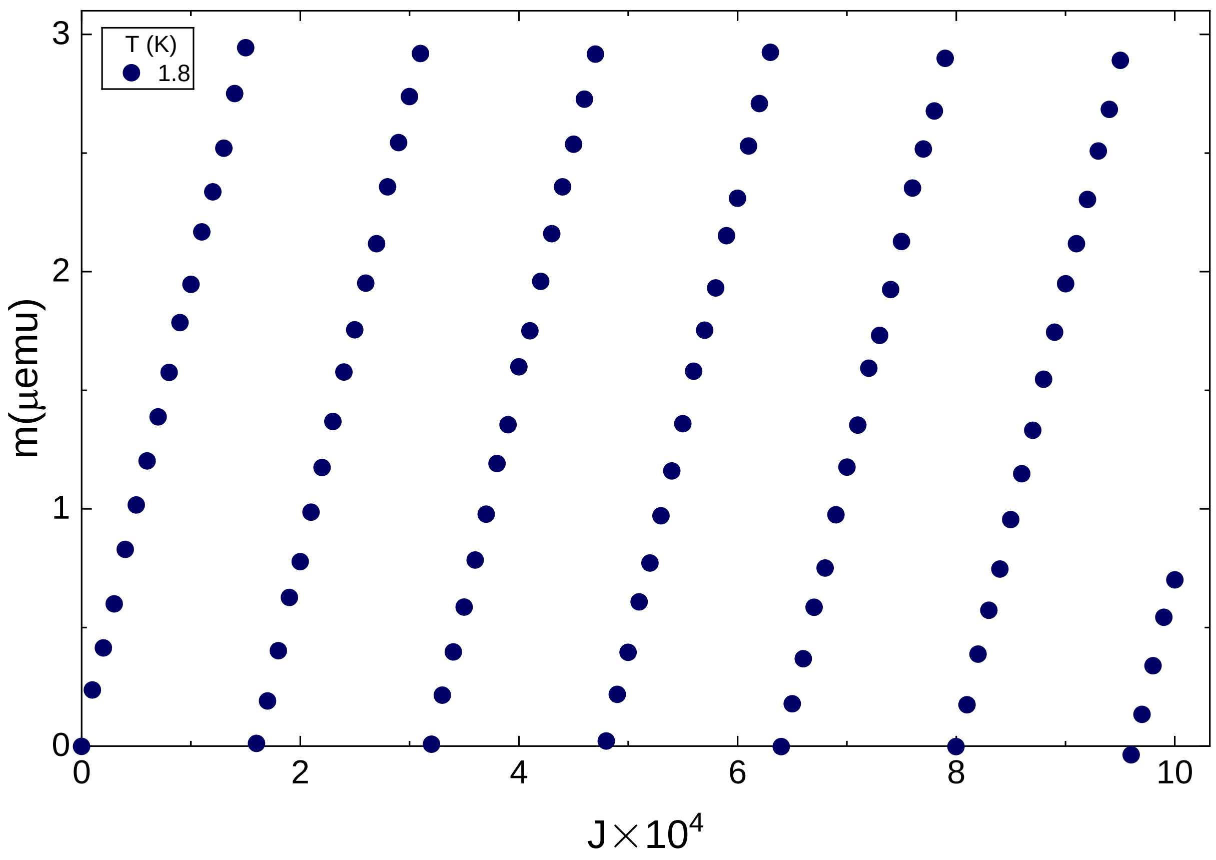
- Again, Λ turns out to be ~ the same for all rings.
- We observed ~ agreement with the theory apart from w=10μm.
- The linear fit through the origin gives ξ=14.1(2) nm.
- We suspect that the 10 μm sample misbehave due to preparation difficulties.



Recall

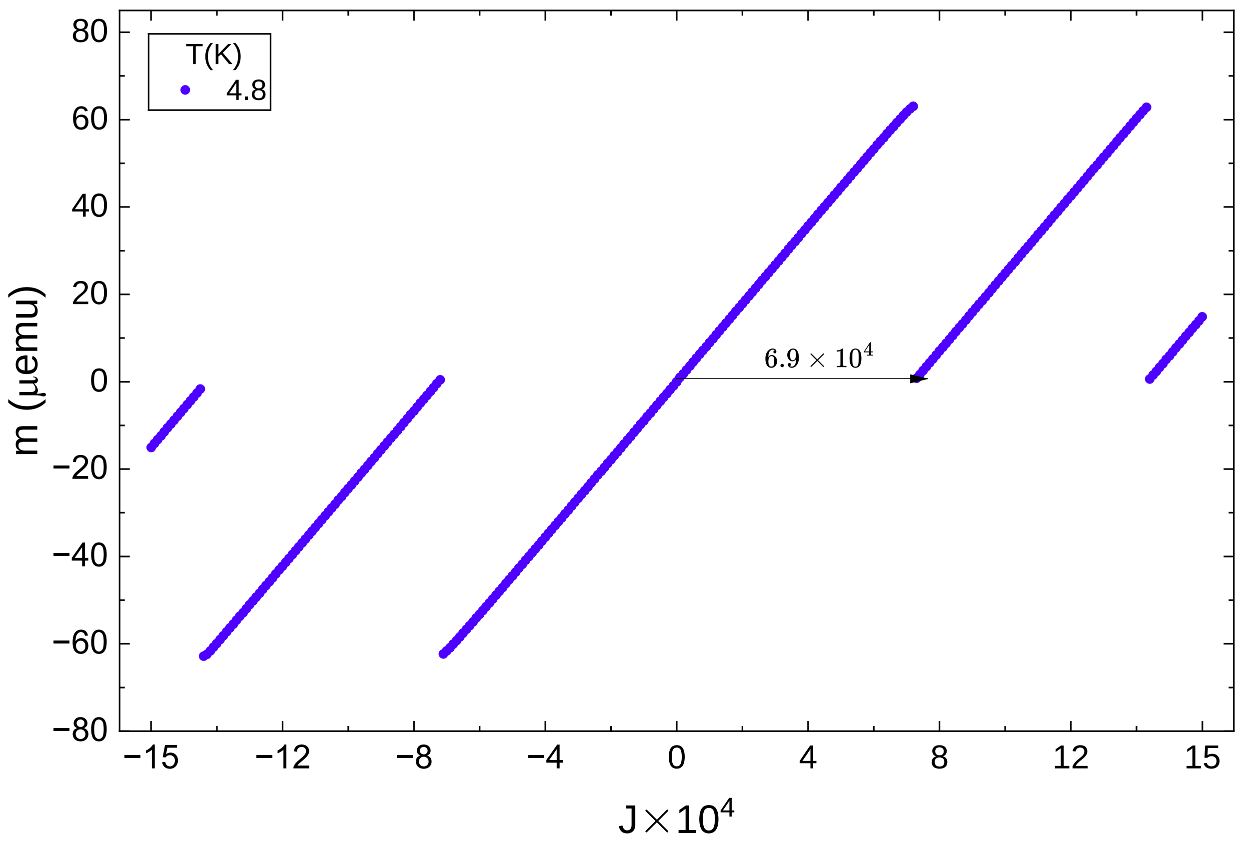


Multiple flux jumps



MoSi ring $d = 50$ nm $r_{in} = 750\ \mu m$ and $r_{out} = 760\ \mu m$

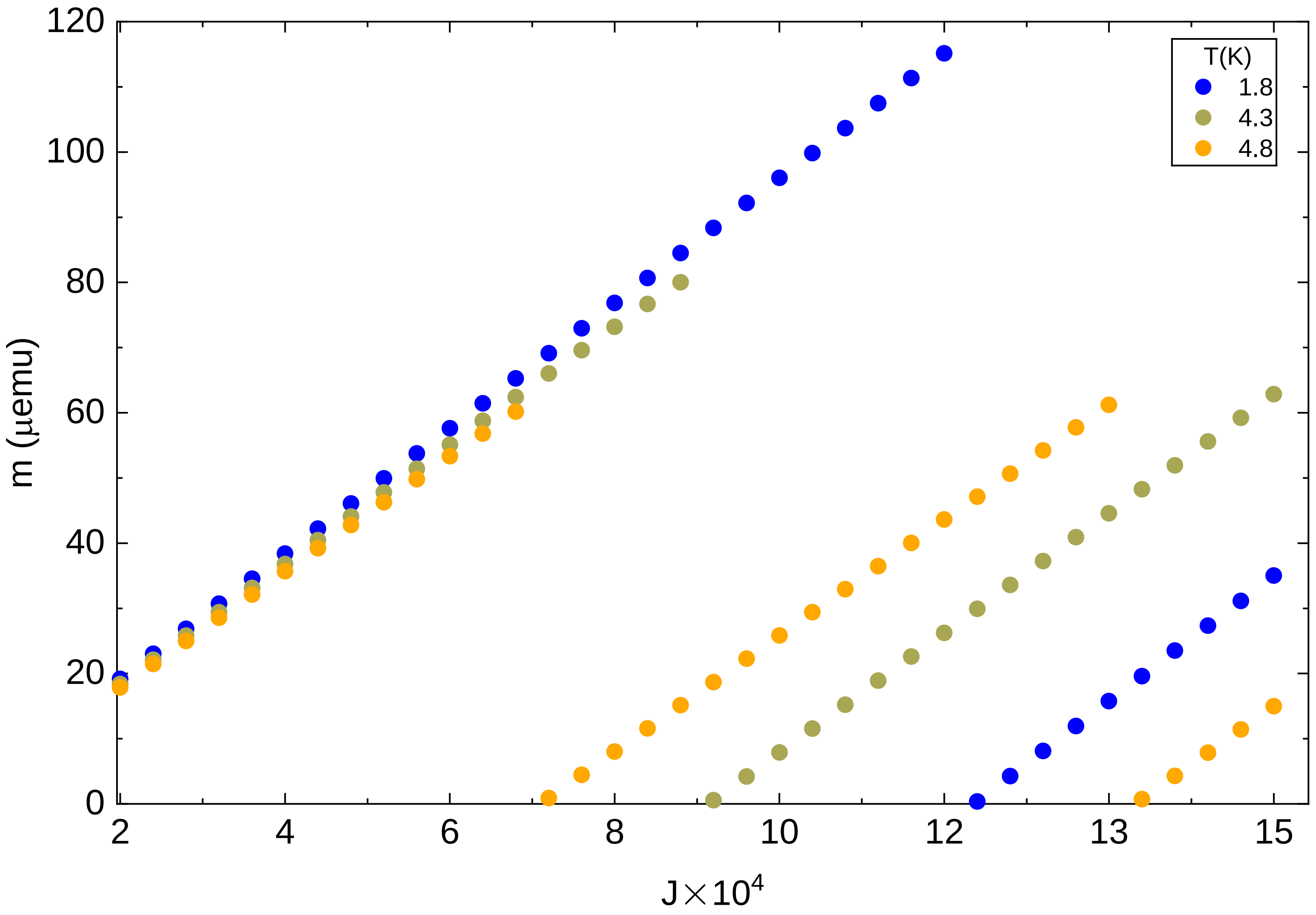
Multiple flux jumps width dependence



MoSi ring $d = 50$ nm $r_{in} = 750\mu m$ and $r_{out} = 850\mu m$

This is reproducible.

Multiple flux jumps vs temperature

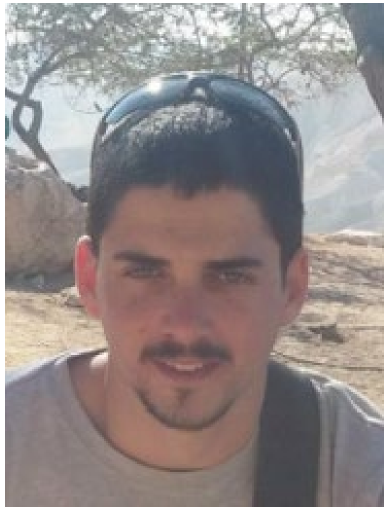


MoSi ring $d = 50$ nm $r_{in} = 750\ \mu m$ and $r_{out} = 850\mu m$

Conclusions

- Inner/outer/Λ determine the critical current via
$$P > \frac{1}{\sqrt{3}} \,.$$
- Roy Rabaglia, AK and Ari Turner theory matched generally for the two sets of rings.
- Several rings showed periodic flux jumps from unknow origin.

Group members



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Thank you !

Questions?