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Photonic time-crystals - fundamental concepts [Invited]

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Abstract: Photonic Time-Crystals (PTCs) are materials in which the refractive index varies periodically and abruptly in time. This medium exhibits unusual properties such as momentum bands separated by gaps within which waves can be amplified exponentially, extracting energy from the modulation. This article provides a brief review on the concepts underlying PTCs, formulates the vision and discusses the challenges.

The recent advances in photonics and material science bring us closer to arbitrarily modulating the macroscopic optical properties of materials in time and space. Much of the work on optical time modulation in the past century in both passive and active systems was done in the framework of weak perturbations or resonant effects. While spatial modulation schemes of materials' properties became very advanced thanks to progress on "photonic crystals" [1] and more recently on metamaterials [2], temporal modulation at ultrafast rates is usually done as very small perturbations such as in optical parametric amplification or other known low-order perturbative nonlinear effects [3]. Microscopically, perturbative nonlinear optics relies on tiny optically induced variations to the wavefunction of the valence electrons, which can yield changes in the refractive index n(t) of 10^{-2} and smaller. Under these conditions, the temporal modulation alters the phase of the propagating wave, but does not induce sizeable time-reflections [4]. On the other hand, when the change of the refraction index n(t) is in the order of 1 and within a single cycle of the electromagnetic field, the index change creates a temporal boundary, in analogy to common spatial boundary described by Fresnel equations (Fig. 1(a),(b)). However, because it is extremely hard to generate a temporal boundary at optical frequencies, it did not receive as much attention [5]. The analogies between boundaries in time and boundaries in space come from the duality of Maxwell equations with regard to space and time [6]. Therefore, whereas energy (frequency) is conserved at interfaces between dielectric media, a time-interface in a homogeneous medium necessarily changes the frequency, while conserving the momentum (wavevector k). In analogy to a spatial boundary, a temporal boundary generates refraction and reflection, which are commonly known as "time-refraction" and "time reflection".

One important difference between a spatial boundary and a temporal boundary is that causality implies that time-reflections cannot go back in time; instead, time-reflections are back-reflected in space. Time reflections are also present in a moving spatial boundary: a spatial boundary that moves at high speeds [6]. In fact, photon generation in various emulations of Hawking radiation and other effects shown in moving boundaries [7,8] are closely related to temporal boundaries. Another distinction is that the material permittivity when described as a function of the microscopic (atomic) polarization of the material, does not have this space-time duality,

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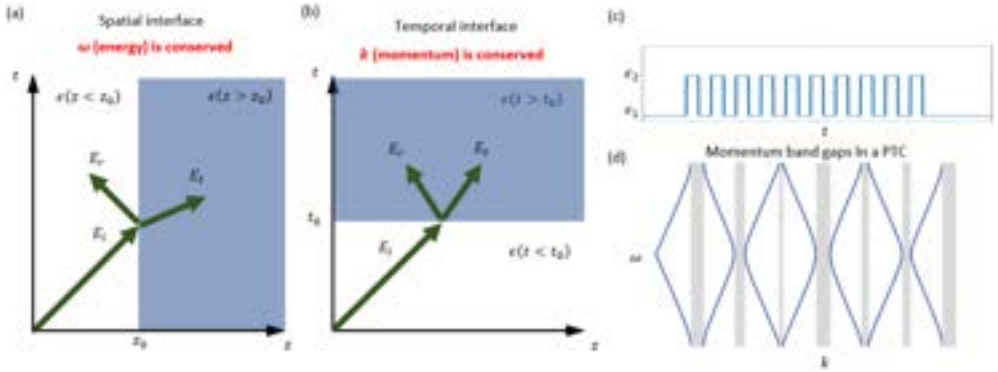

Fig. 1. (a),(b) A plane wave incident on a spatial boundary vs a plane wave experiencing an abrupt change in permittivity (a temporal boundary), respectively. (c) Archetypical model of photonic time crystals. (d) Typical band diagram of a PTC with significant gaps in the momentum k.

which can easily be seen by considering dispersion [9]. However, recent experiments in optics, for example, experiments displaying a broadband frequency shift by modulating the refracting index [10,11], show that the description of the permittivity strictly as a function of time encompasses much of the physics even for modulations slower than single cycle index changes [4].

Over the years, temporal boundaries were studied theoretically in electromagnetism [12] but mostly in fields such as antenna theory [13] in which electromagnetic frequencies are low and therefore such large and abrupt time-boundaries are much more feasible. The study of phase transitions in plasma also incorporated these notions [14,15] and therefore, it raises important questions about the implications on solid state materials [16]. The conservation of momentum (k-vector) implies that the time-refracted wave exhibits a frequency shift: red shift if the index goes up and blue shift if it goes down [5]. The progress with experiments on time-reflection is far more challenging: the amplitude of the time-reflected wave goes down dramatically when the duration of the temporal boundary is longer than a cycle, and when the modulation in the refractive index is small. Consequently time-reflections were first observed with water waves [17], and more recently with microwaves [18], but thus far time-reflection from a temporal boundary has never been observed in at optical frequencies. However, with modern technology, ultrafast pulses can be shaped even on the scale of a single optical cycle [19], and solid materials were found to support dramatic changes of the refractive index [20,21]. In fact, this is the reason why we are now experiencing a growing interest in space-time optics: the study of non-perturbative optical modulation in solids in both time and space [22–27]. In these studies, the general approach is that the modulation can be faster than the group velocity of the propagating pulse, and can even be spatially-homogenous over regions much larger than the wavelength, and therefore momentum is conserved. The velocity of the modulation, which is the velocity of change in the refractive index, is essentially the group velocity of the modulator pulse. The modulation velocity can, in principle, be faster than the group velocity of an optical pulse propagating in the medium, since the modulating pulse and the propagating pulse may have different group velocities. When the modulation of the refractive index is periodic in time $n(t) = n(t + T)$ to, the modulation generates multiple time-reflections and time-refractions, which interfere to yield a band structure: a Photonic Time-Crystal (PTC) (Fig. 1(c)) [6,22,28,29]. This notion is not to be confused with recent work in quantum many-body physics known as "Time-Crystals" [30], where the system forms periodic oscillations in time spontaneously, or, if the many-body system is driven periodically in time – oscillations appear at half the frequency [31,32]. Thus, unlike condensed

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matter "Time-Crystals", PTCs are not spontaneously formed by many-body interactions but emerge as the outcome of an external periodic modulation of the EM properties of the medium.

Since the changes in PTCs are homogenous, they exhibit dynamic Casimir like physics [33–38], but unlike in dynamic Casimir studies, the modulation strength induces multiple wide momentum gaps in the optical regime, with far reaching consequences. The first consequence is that the gaps can be larger than the spectrum of a pulse or the linewidth of an atomic resonance. Thus, launching an ultrafast pulse associated with the momentum gap will result in dramatic increase of energy where the pulse extracts energy from the modulation. This contrasts with the propagation of a pulse associated with a momentum band, which does not result in energy growth. Figure 2 shows an FDTD simulation that shows this phenomenon. In this figure, taken from [4], a PTC of the form shown in Fig. 1(c) is simulated in two cases. In the first case (Fig. 2(a)), the initial condition is a pulse with a wavenumber (k) spectrum that resides in a band (see Fig. 1(d)), and in the second case the wavenumber resides in the band-gap. The figure shows the amplitude of the displacement field as a function of time. In the case of band propagation, light is not amplified and only splits at the start and end of the PTC. The reason for the split is the existence of two Floquet solutions, time reversed to one another, of the wave equation for each k value in the PTC. This occurs at the interfaces in time when the PTC begins and when it ends. Inside the PTC the splitting does not occur, since in the PTC there are actually only two Floquet modes that satisfy the momentum conservation conditions.

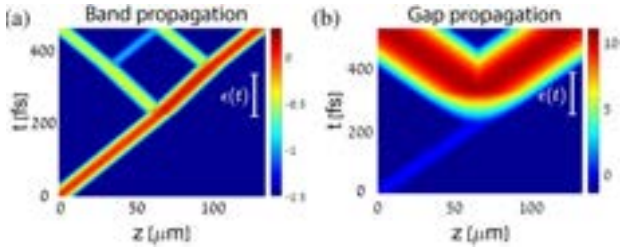

Fig. 2. FDTD simulation of a pulse propagating in a PTC. (a) The amplitude of the displacement field of a pulse (color bar in logscale) when propagating under the influence of a PTC (white marking – spatially homogenous and infinite). The momentum of the pulse is in the momentum band, the pulse splits at the beginning and end of the PTC. (b) Same as (a) but the momentum of the pulse is in the band-gap, resulting in exponential increase of energy. This PTC is of the form depicted in Fig. 1(c) with $\epsilon_1 = 3, \epsilon_2 = 1$ and $T = 2[fs]$.

In the case of gap-propagation, there is splitting when the PTC begins, but now the frequency ω is complex. In Fig. 2(b) the only part of the splitting is the exponentially increasing part (and not the decaying part). Therefore, we effectively see that there is no splitting at the start of the PTC in this case. An exception to this behavior is an edge-state between two PTCs with different topological phases in which the decaying solution decreases the total energy significantly [4]. Interestingly, the end of a PTC always exhibits a splitting, due to the fact that in free space there are no gaps, and therefore each Floquet mode (of the PTC) couples to two eigenstates (plane waves) for each k. Mathematically, the energy enhancement is interpreted as the excitation of modes with complex frequencies in the momentum bandgap, but since in this system energy is not conserved, the light couples to both positive and negative values of the imaginary part of the frequency. Naturally, the solution with exponential growth in energy overshadows the decaying solution. Importantly, the group velocity of the gap modes is zero, hence the energy added to the pulse is not moving as long as the PTC exists [22]. Another consequence of the wide momentum gaps in PTCs is that the amplification is non-resonant, i.e., no phase-matching is required for the amplification process, and there is no condition on the pulse frequency. In this context, imagine a point source (be it an atom or a nano-antenna) which emits light in all directions at some arbitrary

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frequency. Because the emission is to all direction, some of the directions will be exciting gap modes of the PTC, which will be amplified exponentially by the modulation – irrespective of the frequency [39]. This concept of non-resonant amplification occurs also at the quantum level, where pairs of photons associated with gap modes are spontaneously generated by the modulation [39,40].

It is instructive to compare the amplification in PTCs to the parametric gain in optical parametric amplifiers (OPAs). However, OPAs are resonant phenomena in both frequency and momentum: the frequency of the pump equals the sum frequencies of the signal and the idler, and momentum is conserved by virtue of phase-matching. In contrast, PTCs are not resonant in any sense: amplification occurs for any wavevector associated with the momentum gap, and the frequency of the amplified wave in the gap is not necessarily related to the frequency of the modulator. In fact, the best way to view this is to consider photon generation in the momentum gap, where the frequency of the pair of counterpropagating photons does not have to lie on the dispersion curve (see Fig. 1 in [39]). Intuitively, this can be understood by the fact that momentum gaps are parallel to the frequency axis (Fig. 1(d)). Since there is no stringent requirement on the frequency matching, spontaneous emission of photons will always be amplified, regardless of the source, as long as the source is small enough (an atom, a nano-antenna) such that it emits to all direction. The power and momentum bandwidth are determined by the abruptness of the modulations. In a related vein, the modulation can also alter the interaction with free electrons by facilitating radiation emission below the Cherenkov threshold, and by creating interesting effects in the middle of the momentum gap in the form of destructive quantum interference between the photons emitted from the electron and the spontaneously generated photons at mid-gap [40]. Finally, wide momentum gaps also give rise to topological classification of the momentum bandgaps with topological invariants, and their physical implications on the emitted energy [22], in analogy to the topological band theory for frequency bands [41–44].

Of-course no material can sustain exponential increase of energy for extended periods of time, especially if the momentum gaps are wide. But, as in the case for photonic crystals, the main phenomena associated with PTCs also occur with just a few periods of modulation: 4-5 cycles of periodic modulation are sufficient to display a finite increase in energy; the energy increase stops when the modulation stops, and the pulse becomes a propagating mode. Following this logic, the energy increase of PTCs was recently demonstrated in metasurfaces [45] in the microwave regime, but has yet to be shown in optics. This leads to additional gap-related phenomena, such as disorder in time, i.e., when the modulation does not have a constant amplitude but a randomly varying one. In this case light will localize in space and exponentially increase in energy, drawing the energy from the modulation [46]. Some of these effects are a manifestation of Anderson localization in space (all transport stops, group velocity goes to zero, single parameter scaling, etc.), but some are new features – most notably - the increase in pulse energy – which is facilitated by the fact the energy is not conserved in a PTC. This behavior was recently experimentally demonstrated in water waves [47].

Perhaps the most important aspect of PTCs is the quantum light-matter interactions, which merits further discussion. If PTCs occur in regions of the electromagnetic spectrum in which light interacts with atoms or electrons, then momentum bandgaps have dramatic effects on the radiated quantum emission [39,40]. Alongside the enhancement of energy, the momentum bandgap draws coherent radiation from particles which is then enhanced by the modulation. As explained earlier, since the momentum band gaps occur for a broad frequency range, the process is non-resonant and therefore can in principle lead to broadband tunable lasing [39], especially in spectral regions where coherent radiation sources are rare and important, such as THz.

With recent demonstrations in microwaves systems, PTCs are expected to have central importance in the study of metamaterials, and other electromagnetic phenomena, along-side other wave systems such as acoustics, and mechanics. Most of their implications are general to

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all wave systems, and optical realizations are expected in the near future. It is already clear that the most important PTC-related phenomena are related to light-matter interactions, and much is still awaiting to be discovered.

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