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Photonic time crystals: from fundamental insights to novel applications: opinion

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Abstract: In this opinion article, we briefly outline some historical highlights and the most recent developments in the novel and exciting field of photonic time-crystals and present the challenges, disruptive opportunities and potential impact on both the fundamental science of light and on photonic technologies.

The ability to achieve strong and rapid modulations of the electromagnetic (EM) properties of materials has significant implications. Large in magnitude and ultra-fast changes in the EM response even in a single-step manner can lead to profound effects [1–3], including time-refraction and time-reflection. These processes seem similar to refraction and reflection from an interface in space, but in fact, they are fundamentally different from their spatial counterparts. At a spatial interface between two dielectric media the energy (frequency) is conserved; whereas an abrupt temporal change in the refractive index of a homogeneous material leads to the frequency (energy) change while the momentum (wavevector $\mathbf{k}$) is conserved. Specifically, if the refractive index changes from n_1 to n_2 then the new frequency of the refracted and reflected waves ω_2 is given by $\omega_2 = \omega_1 \frac{n_1}{n_2}$, where ω_1 is the frequency of the original wave. Also, causality implies that time-reflections cannot go back in time (unfortunately; people tried – so far in vein), but instead, are back-reflected in space with their phase being conjugated (as demonstrated in water waves [4], in RF [5] and microwaves [6,7] and ultracold atoms [8] as well as in synthetic space [9]). The time-refracted wave and the time-reflected wave have the same wavenumber as the original wave; consequently, both phenomena result in spectral translation: a red-shift is observed for an increase in the refractive index while a blue-shift occurs for a decreased refractive index (Fig. 1).

In time-varying materials, periodic concatenation of time-modulations can lead to the formation of a so-called Photonic Time-Crystal (PTC) as was proposed in 2009 by Peter Halevi [3]. In order to achieve a PTC at optical frequencies, the dielectric permittivity of the material, $\varepsilon(t)$, must undergo periodic variations at time scales of an optical wave cycle. Strong, periodic modulations in the refractive index causes multiple time-reflections and time-refractions, which interfere and lead to dispersion relation organized in bands separated by bandgaps in the momentum ($\mathbf{k}$) rather than in frequency (Fig. 2). In PTCs the energy is not conserved (as time-translation symmetry is broken by the modulation), and the states residing in the momentum gap exhibit exponentially increasing or decaying amplitudes. This significantly impacts the physics involved. For example, when a wave is incident on a (spatial) photonic crystal with its frequency residing within the photonic bandgap - the wave is fully reflected, but when a pulse is propagating within a PTC medium with momentum associated with the PTC bandgap - its group velocity goes to zero, the pulse stops and its amplitude grows exponentially, drawing its energy from the modulation.

While various aspects of wave propagation in time-varying media have been studied over the years [1,10–23], the experimental observations in the optical range are still challenging. Importantly, the realization of photonic time-crystals relies on having sizeable time-reflections and time-refractions. Generally, time-refraction is always present – even when the change in

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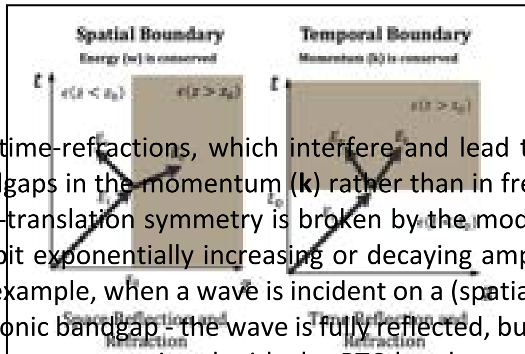

Fig. 1. Reflection by a spatial interface vs reflection by a temporal interface. A monochromatic plane wave incident on a spatial boundary vs a plane wave experiencing an abrupt change in permittivity.

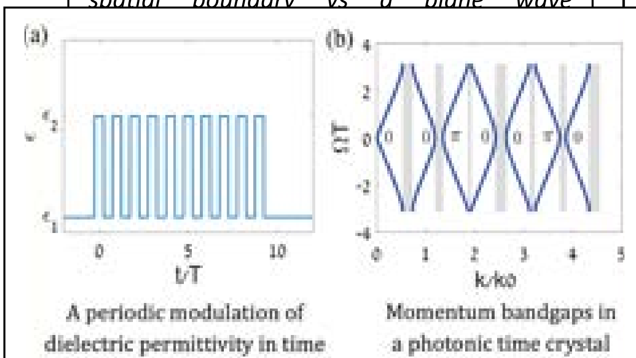

Fig. 2. Schematic of a photonic time-crystal concept where the dielectric permittivity of the material is modulated periodically in time (a), giving rise to dispersion relation characterized by bands separated by significant gaps in the momentum **k** (b).

permittivity is slow, hence time-refraction was observed also at optical frequencies [24–30]. On the other hand, observing time-reflection requires a strong ε(t) modulation occurring at the time-scale of a single cycle, otherwise time-reflection is extremely weak and PTCs become impossible altogether. Traditional nonlinear optics effects (e.g., the optical Kerr effect) is ultrafast, but orders of magnitude too weak to observe time-reflection and PTCs, while other known mechanisms with larger nonlinear responses typically require transport of some sort (particles, charges, etc.) and thus are too slow. For this reason, experiments showing time-reflection at optical frequencies remain a challenge.

So far, the only mechanism that might give rise to sizeable time-reflections at optical frequencies is exciting many electrons at rates comparable to one optical cycle (a few fs), which can lead to index variation on the order of unity. It has been argued that such process is impossible at optical frequencies, but recent experiments have shown otherwise: time-refraction within a single optical cycle was very recently demonstrated [29,30]. This major advancement was possible due the emergence of transparent conducting oxides (TCOs) as a platform for the realization of low loss, ultrafast, tunable and all-optically switchable photonics.

TCOs display large light-induced index changes at ultrafast rates [24–27,31–34] where significant variations in $\varepsilon(t)$ can be induced by laser pulses of few-fs duration and probed near their epsilon near zero (ENZ) point. TCOs exhibit a relatively broad spectral range where the real part of the dielectric permittivity is nearly zero (ENZ), with the imaginary part being relatively small, resulting in a near-zero refractive index (NZI). Such materials enable extreme optics

requirement for the normal component of the displacement field causes the electric field to c the interface between "normal" (epsilon ~ 1) and ENZ materials. This unique behavior of ba characteristics enables extreme optics, including dramatically enhanced nonlinear responses. electrons can be driven between the bands and within the conduction band to higher energies by laser pulses. Since such process relies on optical excitation, which is an induced transit instantaneous, and the response can be as fast as the excitation pulse duration. In this sch

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where both phase velocity and wavelength are anomalously large, ideally diverge, while the group velocity approaches zero. The continuity requirement for the normal component of the displacement field causes the electric field to diverge at the interface between "normal" (epsilon ∼ 1) and ENZ materials. This unique behavior of basic optic characteristics enables extreme optics, including dramatically enhanced nonlinear responses. In TCOs, electrons can be driven between the bands and within the conduction band to higher energies by ultrafast laser pulses. Since such process relies on optical excitation, which is an induced transition, it is instantaneous, and the response can be as fast as the excitation pulse duration. In this scheme, the excitation pulse serves as a "modulator" for a relatively weak probe wave propagating in the medium that experience the large ultrafast time-interfaces induced by the modulator beam. Past experiments at optical frequencies in TCOs employed modulator pulses of tens to hundreds fs, hence were not in the single-cycle regime and could not observe time-reflections. The recent TCOs developments are a step towards the observation of time-reflections at optical frequencies thus paving the way to the realization of the first PTC.

While PTCs at low radio frequencies were demonstrated already in 2015, there has been a recent resurgence of interest in this area due to their intriguing connection with truly unique and novel regimes for light-matter interactions [15,35–41].

PTCs draw parallels with spatial Photonic Crystals (PCs). Even though waves in periodic structures were known for a long time, the field was transformed by Eli Yablonovitch's pivotal paper on the inhibition of spontaneous emission in PCs [42]. This seminal work catalyzed many innovative concepts spanning thresholdless lasers, nonlinear optics in PCs, and beyond. In a similar vein, recent investigations have focused on light-matter interactions in PTCs, as well as in a broader class of time-varying media. Notably, in 2018, a theoretical study on wave propagation in PTCs demonstrated – in simulations - that launching a pulse associated with the momentum gap in PTC slows the pulse down to a complete halt, and its amplitude grows exponentially, extracting energy from the modulation [14]. This paper not only established a link between PTCs and topological phenomena but also identified the topology of the bands and predicted topological edge states within the temporal domain. The subsequent paper on PTCs [20] explored PTCs incorporating random variance in the temporal modulation, showing that any pulse entering a disordered-PTC comes to a complete halt and is amplified exponentially, with nontrivial relation to the phenomenon of Anderson localization. The predictions on PTC containing disorder and on disorder in the time domain [20,43,44] were proved by experiments with water waves [45]. Several theoretical papers on PTC-related physics followed including the theory of interaction between free electrons and their radiation in PTCs, with both semi-classical and quantum theories of this Cherenkov-like interaction [35]; as well as the classical and quantum theories of radiation emission by classical dipoles and by 2-level atoms in PTCs [15]. One important prediction is the concept of a laser based on radiation emission in PTCs, amplified through the modulation of the refractive index within the medium. This novel type of laser emits coherent light without relying on any atomic resonance holding the promise for achieving broad tunability and pulsed emission, with spectral and temporal characteristics dictated by the temporal profile of the refractive index modulation. Some PTC-research highlights include the concept of spatio-temporal photonic crystals where a dielectric medium is modulated in both time and space exhibiting bandgaps in both frequency (energy) and momentum has also been presented [36,37]. The work of [36] suggests a link between PTCs and non-Hermitian Photonics, PT-symmetry and exceptional points. The theory of nonlinear PTCs including the existence of spatio-temporal solitons has also been reported [46]. The flurry of PTC-related research is also witnessed by the workshop on waves in time-varying media [47].

The next steps for the area of PTCs are to demonstrate time-reflection at optical frequencies and then realize a PTC by constructing several time-reflecting boundaries. This hinges on the availability of tailorable and dynamically tunable optical materials such as TCOs including

Indium Tin Oxide (ITO), Aluminum- and Gallium-doped Zinc Oxide (AZO/GZO) and other materials that exhibit ENZ and, when losses are small, NZI behavior. Recently, very large refractive index changes (~0.2) have been observed in TCOs occurring at single-cycle time scale ~5 fs [29]. When illuminating TCOs with an ultrashort laser pulse at 800 nm (mean) wavelength, the induced change in the refractive index is ultrafast and broadband, thus enabling measurements of large frequency translations associated with strong time-refraction. Namely, because of the momentum conservation in time-varying media, increasing the refractive index abruptly leads to time-refraction where the spectrum of all waves propagating in the medium is red-shifted, and subsequently blue-shifted when the refractive index relaxes (decreases) back to its original value. These recent results are the first-time observation of the red-shifted and blue-shifted time-refracted light in the single-cycle regime (Fig. 3). Remarkably, the $\varepsilon(t)$ relaxation to its original value happens in ~10-20 fs [29], defying anticipated interactions with phonons that happen on the time scale of ~200 fs [22]. These encouraging findings pave the way to time-reflection measurements, constituting a significant milestone toward PTC realization.

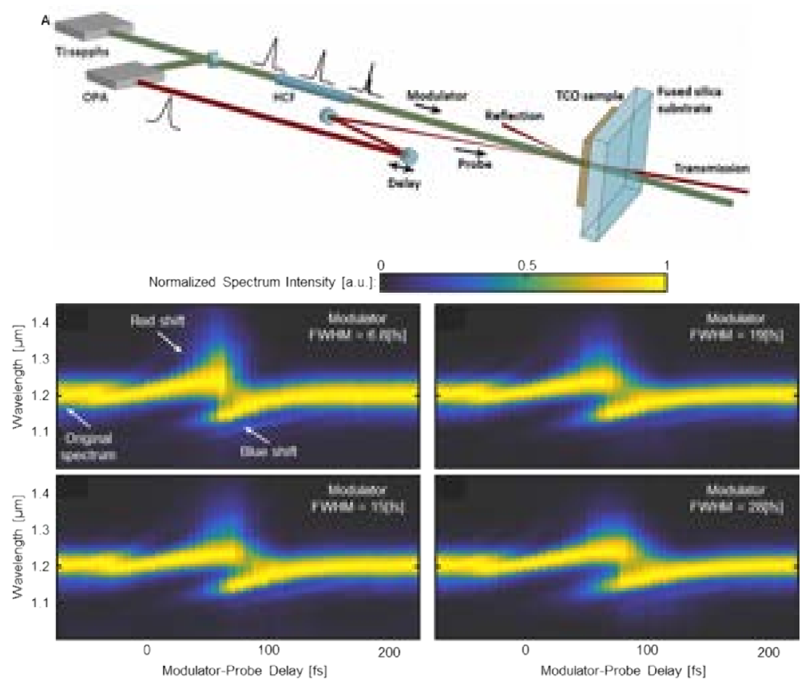

Fig. 3. (Top) Experimental modulator–probe setup for measuring time-refraction in the single-cycle regime. The modulator is a pulsed optical beam centered at 800 nm, compressed by a hollow core fiber system. The probe beam is a down-converted 40 fs pulse at 1200 nm. The modulator and probe pulses are synchronized and arrive to TCO sample at controlled relative delay times. The intensity and spectrum of the transmitted probe and the intensity of the reflected probe are measured. (Bottom) Transmission spectrograms of 44 fs probe pulses, for modulator pulses of various temporal widths (FWHM). As the modulator pulse shortens (down to ~6 fs), the range of red-shifted delay points decreases. Similarly, the range of blue-shifted delay points also decreases with the temporal width of the modulator. The maximal magnitude of the spectral shift increases as the modulator pulse width is shortened.

The recent experiments [29] give rise to fundamental questions concerning physics occurring within such few-fs time frames, challenging prevailing "two-temperature" model [25,48]. These findings defy conventional explanations, signifying a regime where events occur faster than the decoherence processes associated with collisions in materials. While the earlier reported

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skepticism [22] questions the material's ability to respond rapidly and recover after each light cycle, the recent findings showed that the material can both respond within the cycle duration and also relax to its initial state within the same timeframe. These observations hint nearly instantaneous processes, which could be due to virtual interband transitions as recently suggested [49].

Since the recent experimental observations [29] revealed dynamics that surpass traditional theories such as the Drude model, Ohm's law, or the two-temperature model, this opens the door to studying new quantum effects. The exiting new phenomena are expected when processes unfold more rapidly than the standard relaxation processes that typically disrupt classical and, most importantly, quantum coherence and entanglement needed to enable the unique quantum properties. This unparalleled regime could lead to novel interactions and phenomena, as it operates on a timescale far faster than what conventional relaxation theories can accommodate. We anticipate shedding new light on such meta-interactions and observing genuinely unprecedented phenomena.

PTCs promise to advance technology significantly by enabling new radiation sources, novel lasers, new quantum states of light such as cluster states and entangled photons as well as other quantum devices such as detectors of entangled states and beyond.

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