Nonlinear Conductivity and Collective Charge Excitations in the Lowest Landau Level

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For weakly disordered fractional quantum Hall phases, the nonlinear photoconductivity is related to the charge susceptibility of the clean system by a Floquet boost. Thus, it may be possible to probe collective charge modes at finite wave vectors by electrical transport. Incompressible phases, irradiated at slightly above the magnetoroton gap, are predicted to exhibit negative photoconductivity and zero resistance states with spontaneous internal electric fields. Nonlinear conductivity can probe composite fermions' charge excitations in compressible filling factors.

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Fractional quantum Hall (FQH) phases exhibit exotic ground states and many-body collective modes. For example, the magnetoroton (MR) mode in Laughlin states was predicted by Girvin, MacDonald, and Platzman (GMP) [1] to have a minimal gap Δ_0 at wave vector k_0 (see Fig. 1). This mode was seen by Raman scattering [2], resistivity activation gap with phonon pulses [3], and microwave absorption with surface acoustic waves [4].

Some of these probes are based on the powerful relation between conductivity $\sigma_{xx}(\omega)$ and charge excitations in weakly disordered quantum Hall gases [5–7]:

$$\sigma_{xx}(\Omega) = \frac{nc^2}{\hbar B^2}\frac{1}{A}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}}^2 k_y^2 \frac{\chi_0''(\boldsymbol{k},\Omega)}{\Omega} + \mathcal{O}(V^4). \quad (1)$$

$V(\boldsymbol{r}) = A^{-1/2}\sum_{\boldsymbol{k}} V_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ is the random disorder potential taken from a Gaussian ensemble with fluctuation spectrum $\langle V_{\boldsymbol{k}}^* V_{\boldsymbol{k}'}\rangle = \Gamma_{\boldsymbol{k}}^2 \delta_{\boldsymbol{k}\boldsymbol{k}'}$, $\chi_0''(\boldsymbol{k},\Omega)$ is the charge susceptibility of the "clean" ($V=0$), but fully interacting, system, at wave vector $\boldsymbol{k}$ and frequency Ω. B, c, n, and A are the perpendicular magnetic field ($\boldsymbol{B} = B\hat{z}$), speed of light, electron density, and system area, respectively.

In this Letter we extend Eq. (1) to the *nonlinear* current versus field response. We propose new and independent experiments to probe the collective charge excitations, which would complement information given by the linear response probes. Our derivation incorporates strong electric fields into the current response function. To go beyond the linear Kubo formula, we use a Floquet boost transformation [8,9]. Our results retain the full many-body correlations in χ_0'', which make them suitable for the strongly correlated FQH phases.

We propose to measure the dc photoconductivity $\sigma_{xx}^{\text{photo}}(\Omega)$ in the presence of an ac radiation field at frequency Ω. Within GMP theory, at frequency $\Omega > \Delta_0$, $\sigma_{xx}^{\text{photo}}$ should become singularly large and negative. As a consequence, the system will be unstable toward a "zero resistance state" (ZRS) [10], where internal electric field domains are spontaneously created [11]. ZRS phases, and spontaneous internal fields, were previously observed at weak magnetic fields (high Landau levels) [12–14]. The photoconductivity has been calculated by various methods [9,15–17]. At high Landau levels, the dark conductivity is metallic, and negative photocurrent can be induced by strong microwave power, which is positively detuned from the cyclotron frequency ω_c. In contrast, we show that the ZRS in the lowest Landau level (LLL) arises from different phenomena. It involves intra-Landau Level excitations of the MR mode which is expected at $\Omega \ll \omega_c$. In addition, negative photoconductivity appears already at leading order in radiation power, since the dark conductivity is zero.

For the compressible phases, such as around $\nu = \frac{1}{2}$, the dark nonlinear current $\boldsymbol{j}_x^{\text{dc}}[E_x]$ is expressed in terms of χ_0''. Thus, predictions of composite fermion theory for $\chi_0''(\boldsymbol{k},\omega)$ could be tested by transport measurements. We conclude by discussing experimentally relevant frequency and field scales, and the regime of validity of the weak disorder expansion.

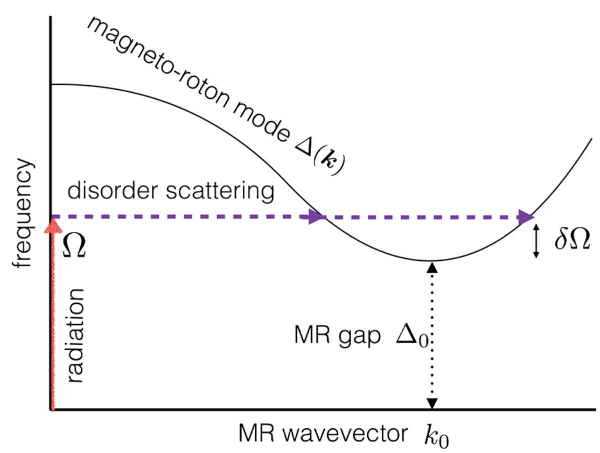

FIG. 1. Magnetoroton collective excitation in an incompressible Laughlin phase following GMP [1]. Radiation above threshold frequency $\Omega > \Delta_0$, combined with scattering by disorder yields $\sigma_{xx} > 0$, by Eq. (1).

Quantum Hall Hamiltonian.—Two-dimensional electrons in a uniform magnetic field, and a time-dependent vector potential $\boldsymbol{a}(t)$, are described by the Hamiltonian $H = H_0 + V$, where

$$H_0[\boldsymbol{a}] = \frac{1}{2m}\sum_{i=1}^{N}\left[\boldsymbol{\pi}_i - \frac{q}{c}\boldsymbol{a}(t)\right]^2 + \sum_{i<j}\Phi(\boldsymbol{r}_i - \boldsymbol{r}_j) \qquad (2)$$

and $V = A^{-1/2}\sum_{\boldsymbol{k}} V_{\boldsymbol{k}}\rho_{\boldsymbol{k}}$. N is the number of particles of charge q, and Φ are the two-body interactions. The density operator is $\rho_{\boldsymbol{k}} = \sum_{i=1}^{N} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_i}$, and the cyclotron momentum for particle i is $\boldsymbol{\pi}_i = \boldsymbol{p}_i - (qB/2c)\hat{\boldsymbol{z}}\times\boldsymbol{r}_i$, obeying $[\pi_i^\alpha, \pi_j^\beta] = i\mathrm{sgn}(q)\hbar^2\ell^{-2}\epsilon^{\alpha\beta}\delta_{ij}$, where $\ell = (\hbar c/|q|B)^{1/2}$ is the Landau length. For electrons, $q = -e$. The current density at time t is

$$\boldsymbol{j}(t) = \frac{q}{mA}\sum_{i=1}^{N}\langle U^\dagger(t)\boldsymbol{\pi}_i U(t)\rangle - \frac{nq^2}{mc}\boldsymbol{a}(t), \qquad (3)$$

where $n = N/A$, and $\langle\cdots\rangle$ denotes thermal and disorder averaging at $t = 0$. $U(t)$ is the time evolution operator.

The calculation of the nonlinear photocurrent generalizes the single electron approach of Auerbach and Pai [9], while using the underlying Galilean symmetry of H_0. We proceed in two steps.

(i) Floquet boost.—We decompose $U(t)$ according to

$$U(t) = W(t)\mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^t dt'\tilde{H}(t')\right), \qquad (4)$$

where $W(t) \equiv \prod_i e^{-i\boldsymbol{g}(t)\cdot\boldsymbol{\pi}_i/\hbar}$ is the unitary Floquet boost. To cancel $\boldsymbol{a}(t)$ from $H_0[\boldsymbol{a}]$, $\boldsymbol{g}(t)$ must satisfy

$$\dot{\boldsymbol{g}} + \omega_c\hat{\boldsymbol{z}}\times\boldsymbol{g}(t) = -\frac{\omega_c}{B}\boldsymbol{a}(t), \qquad (5)$$

where $\omega_c = eB/mc$ and $\boldsymbol{g}(0) = 0$. $\boldsymbol{g}(t)$ can be readily solved for any $\boldsymbol{a}(t)$.

$\tilde{H} = W^\dagger H W - i\hbar W^\dagger\dot{W}$ is the Hamiltonian in the boosted frame, given by $\tilde{H} = \tilde{H}_0 + \tilde{V} + f(t)$, with

$$\tilde{H}_0 = \sum_i \frac{\boldsymbol{\pi}_i^2}{2m} + \sum_{i<j}\Phi(\boldsymbol{r}_i - \boldsymbol{r}_j),$$
$$\tilde{V} = A^{-1/2}\sum_{\boldsymbol{k}} V_{\boldsymbol{k}}\rho_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{g}(t)}, \qquad (6)$$

and $f(t)$ is an irrelevant c number.

(ii) Expansion in the disorder potential.—The evolution operator is $U(t) = U_0\mathcal{T}\exp[-(i/\hbar)\int_0^t dt' V(t')]$, where $U_0 = W(t)e^{-i\tilde{H}_0 t/\hbar}$ is the clean evolution operator, and $V(t) = A^{-1/2}\sum_{\boldsymbol{k}} V_{\boldsymbol{k}}\rho_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{g}(t)}$, where $\rho_{\boldsymbol{k}}(t) \equiv e^{i\tilde{H}_0 t/\hbar}\rho_{\boldsymbol{k}}e^{-i\tilde{H}_0 t/\hbar}$.

The evolution of the global Landau operator $\Pi^\dagger = \sum_i(\boldsymbol{\pi}_i^x - i\boldsymbol{\pi}_i^y)$ for the disorder-free Hamiltonian is given by Kohn's theorem:

$$U_0^\dagger(t)\Pi^\dagger U_0(t) = \exp(i\omega_c t)\Pi^\dagger - \frac{iN\hbar^2}{\ell^2}g^*(t), \qquad (7)$$

where $g(t) = g_x(t) + ig_y(t)$. For the clean system, by setting $U \to U_0$ in Eq. (3), it is easy to see from Eq. (7) that for a dc field $\boldsymbol{E}$, (i) the longitudinal current vanishes at all times, and (ii) the Hall current is the Galilean result $\boldsymbol{j} = (nqc/B)\hat{\boldsymbol{z}}\times\boldsymbol{E}$.

The longitudinal current (3) is expanded in powers of the disorder $(V)^n$. The leading order is for $n = 2$, whose Fourier transform is given by

$$j_x(\omega) = \frac{qn}{m\hbar}\frac{1}{A}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}}^2\frac{k_y}{\omega+\omega_c} \times\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}R_{-\boldsymbol{k}}(\omega-\omega')\chi_0''(\boldsymbol{k},\omega')R_{\boldsymbol{k}}(\omega'). \qquad (8)$$

The electric field factors are given by

$$R_{\boldsymbol{k}}[\boldsymbol{E}(t),\omega)] \equiv \int_{-\infty}^{\infty}dt' e^{i\boldsymbol{k}\cdot\boldsymbol{g}[\boldsymbol{E}(t')]}e^{i\omega t'}. \qquad (9)$$

χ_0'' is the dynamical charge susceptibility of the clean system,

$$\chi_0''(\boldsymbol{k},\omega) = \frac{1}{NZ_0}\mathrm{Re}\int_0^\infty dt e^{i\omega t}\mathrm{Tr}\{e^{-\tilde{H}_0/T}[\rho_{-\boldsymbol{k}}(t),\rho_{\boldsymbol{k}}(0)]\}, \qquad (10)$$

where $Z_0 = \mathrm{Tr}e^{-\tilde{H}_0/T}$, and T is the temperature. Equation (8) applies (to second order in V) for an arbitrary time-dependent electric field.

dc photocurrent.—We consider an electric field with two components: a dc field $E_x\hat{\boldsymbol{x}}$, and a circularly polarized ac field $\mathcal{E}$ of frequency Ω, such that $\boldsymbol{a}(t) = -c\mathcal{E}[\sin(\Omega t)\hat{\boldsymbol{x}} - \cos(\Omega t)\hat{\boldsymbol{y}}] - cE_x t\hat{\boldsymbol{x}}$. Solving Eq. (5) yields

$$\boldsymbol{g}(t) = \frac{cE_x}{\hbar B\omega_c}(\hat{\boldsymbol{x}} - \omega_c t\hat{\boldsymbol{y}}) - \frac{c\mathcal{E}}{B(\omega_c-\Omega)}[\cos(\Omega t)\hat{\boldsymbol{x}} + \sin(\Omega t)\hat{\boldsymbol{y}}] \qquad (11)$$

and

$$R_{\boldsymbol{k}}(\omega) = \sum_{l=-\infty}^{\infty}e^{i\phi_{\boldsymbol{k}}}J_l(k\lambda_{\mathcal{E}})\delta\left(\omega + l\Omega - \frac{ck_y}{B}E_x\right), \qquad (12)$$

where $k = |\boldsymbol{k}|$, $J_l(x)$ is a Bessel function of order l, and $\lambda_{\mathcal{E}} = c\mathcal{E}/B\Omega$ measures the radiation field strength. $\phi_{\boldsymbol{k}}$ is an irrelevant phase.

The nonlinear dc photocurrent as a function of electric fields is

$$j_x^{\mathrm{dc}}(\Omega,\mathcal{E},E_x)=\frac{nc}{B\hbar}\frac{1}{A}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}}^2k_y\sum_{l=-\infty}^{\infty}|J_l(k\lambda_{\mathcal{E}})|^2\times\chi_0''\left(\boldsymbol{k},l\Omega+\frac{ck_y}{B}E_x\right). \tag{13}$$

In the following, we apply Eq. (13) to the incompressible and compressible FQH phases.

Incompressible Laughlin states.—At zero temperature, filling fractions $nhc/|eB|=\nu_m=1/m$, for odd m, are incompressible. In the strong magnetic field limit $\omega_c\gg\Delta_0$, and at frequencies $\omega\ll\omega_c$, the charge susceptibility can be computed *within* the LLL, i.e., using projected density operators $\rho\to\bar\rho_{\boldsymbol{k}}=P\rho_{\boldsymbol{k}}P$, where P is the LLL projector. The projected structure factor $s(\boldsymbol{k})=N^{-1}\langle\bar\rho_{\boldsymbol{k}}\bar\rho_{-\boldsymbol{k}}\rangle$, was computed by GMP [1] using a Monte Carlo simulation of the classical two-dimensional one component plasma. At low $\boldsymbol{k}$, $s(\boldsymbol{k})\sim k^4$. GMP also computed the oscillator strength $f(\boldsymbol{k})=\frac{1}{2}N^{-1}\langle[\bar\rho_{\boldsymbol{k}},[\bar H,\bar\rho_{-\boldsymbol{k}}]]\rangle$ for the Laughlin state with the LLL-projected Coulomb Hamiltonian. The single mode approximation (SMA) *assumes* that the spectral weight is exhausted by a single collective mode, i.e.,

$$\chi_0''(\boldsymbol{k},\omega)\simeq\pi e^{-k^2\ell^2/2}s(\boldsymbol{k})\delta(\omega-\Delta(\boldsymbol{k})), \tag{14}$$

where $\Delta(\boldsymbol{k})=f(\boldsymbol{k})/s(\boldsymbol{k})$ is the magnetoroton dispersion. GMP's calculation is conveniently parametrized by a Taylor expansion about k_0

$$\Delta(\boldsymbol{k})=\Delta_0[1+a_2(\delta k\ell)^2+a_3(\delta k\ell)^3+\cdots], \tag{15}$$

where $\delta k=k-k_0$, and $a_2>0$.

Using Eq. (14), the photocurrent (at radiation powers $\lambda_{\mathcal{E}}/\ell\ll1$) is

$$j_x^{\mathrm{dc}}(\Omega,E_x)\simeq\frac{nc|\lambda_{\mathcal{E}}|^2}{\hbar B}\int_0^\infty dke^{-\frac{1}{2}(k\ell)^2}k^4\Gamma_{\boldsymbol{k}}^2s(k)\times\int_0^{2\pi}d\theta\sin\theta\delta\left(\Omega-\Delta(k)+\frac{E_x}{B}ck\sin\theta\right). \tag{16}$$

The detuning frequency is defined as $\delta\Omega=\Omega-\Delta_0$. The dc electric field defines the electric frequency scale

$$f_x=ck_0E_x/B. \tag{17}$$

In the regime f_x, $|\delta\Omega|\ll\Delta_0$, the dc photoconductivity is

$$\sigma_{xx}^{\mathrm{photo}}\equiv\left.\frac{dj_x^{\mathrm{dc}}(\Omega)}{dE_x}\right|_{E_x=0} \tag{18}$$

$$\simeq|\lambda_{\mathcal{E}}|^2\left(\frac{nc^2}{\hbar B^2}\right)\int_0^\infty dk\Gamma_{\boldsymbol{k}}^2k^5s(k)\frac{d}{d\Omega}\delta[(\Omega-\Delta(k)]$$

$$\simeq-|\lambda_{\mathcal{E}}|^2\left(\frac{nc^2\Gamma_{k_0}^2k_0^5s(k_0)}{4\hbar\ell B^2\sqrt{a_2\Delta_0}}\right)|\delta\Omega|^{-3/2}\Theta(\delta\Omega), \tag{19}$$

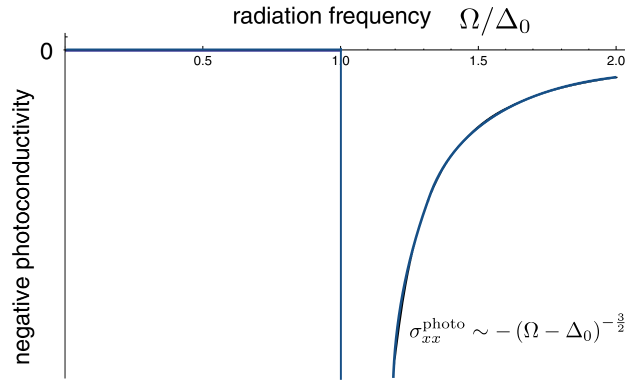

FIG. 2. Photoconductivity in an incompressible Laughlin phase, given by Eq. (19). Δ_0 is the MR gap depicted in Fig. 1. Ω is the radiation frequency. At $\Omega>\Delta_0$, large negative conductivity is predicted.

The dependence of $\sigma_{xx}^{\mathrm{photo}}$ on radiation frequency is depicted in Fig. 2.

At finite driving field $E_x>0$, the photocurrent is

$$j_x^{\mathrm{dc}}=j_0h\left(\frac{\delta\Omega}{|f_x|}\right)\mathrm{sgn}(f_x), \tag{20}$$

where the current scale is

$$j_0\equiv\frac{2|\lambda_{\mathcal{E}}|^2n\ell k_0^4s(k_0)\Gamma_{k_0}^2}{\sqrt{a_2\Delta_2}}|\delta\Omega|^{-\frac{1}{2}}, \tag{21}$$

and $h(u)$ is the universal function,

$$h(u)=|u|^{1/2}\int_{(u-1)_+}^{u+1}\frac{ds}{\sqrt{s}}\frac{s-u}{\sqrt{1-(u-s)^2}}, \tag{22}$$

where $(u-1)_+=\max(0,u-1)$. Photocurrent versus electric field at fixed detuning frequency is depicted in Fig. 3, for both positive and negative detuning frequencies.

ZRS spontaneous field.—Negative uniform conductivity signals a thermodynamic instability toward formation of a ZRS state [10] with spontaneous internal electric fields. For a homogeneous quantum Hall phase these fields are fixed by the minima of the Lyapunov functional condition [11] $j_x^{\mathrm{dc}}(\Omega,\boldsymbol{E}^*)=0$. Near the detuning threshold, we see in Fig. 3 that the current vanishes $f_x^*=1.6\delta\Omega$, which by Eq. (17) yields the magnitude of the spontaneous electric fields as,

$$E_x^*[\mathrm{Volt/cm}]=1.6\times\delta\Omega\frac{B}{ck_0}\Theta(\delta\Omega). \tag{23}$$

Notice that the spontaneous field is an independent measure of the MR wave vector k_0.—For inhomogeneous systems with spatially varying Hall conductivity [18], there is no Lyapunov functional. As a result, spontaneously generated electric fields can fluctuate in time [19], which

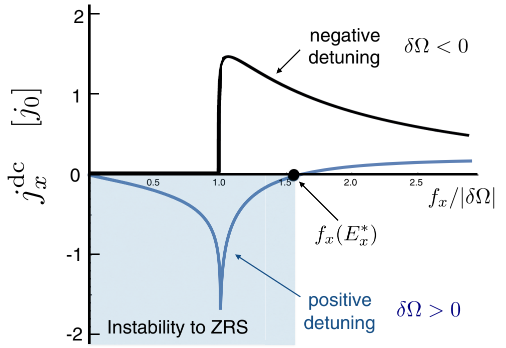

FIG. 3. Nonlinear photocurrent for an incompressible Laughlin phase, given by Eq. (20). f_x is the electric frequency defined in Eq. (17). For positive detuning $\delta\Omega > 0$, (blue curve), the negative photocurrent is unstable in the shaded region toward a ZRS state with spontaneous electric field E_x^* given by Eq. (23).

may be the source of the experimentally observed telegraph noise [14].

Compressible FQH phases.—At $\nu \simeq \frac{1}{2}$, the electronic states have been described by composite Fermions (CF) [20–23], which see an effective weak magnetic field. Their effective "Fermi energy scale" ϵ_F^* is determined by intra-LLL Coulomb interactions. Read [24] computed the long wavelength dynamical charge susceptibility to go as

$$\chi''_{\rm CF}(\boldsymbol{k},\omega) \sim e^{-k^2\ell^2/2}\frac{\hbar^2\omega}{(\epsilon_F^*)^2(k\ell)^3}. \qquad (24)$$

Near $\nu = \frac{1}{2}$, Park [25] has proposed to search for ZRS by photoconductivity above the CF effective Landau level spacing. Since the dark system is metallic, negative conductivity could be achieved at high radiation power if CF Landau levels are sharp modes at finite wave vectors.

Since the excitations are gapless, we can use the nonlinear current as a probe to the low energy excitations. In the "dark" DC case, i.e., $\boldsymbol{E}(t) = E_x\hat{\boldsymbol{x}}$, Eq. (5) yields $\boldsymbol{g}^{\rm dc}(t) = (cE_x/B\omega_c)(\hat{\boldsymbol{x}} - \omega_c t\hat{\boldsymbol{y}})$. Thus,

$$R_{\boldsymbol{k}}^{\rm dc}(\omega) = 2\pi\exp\left(\frac{ick_xE_x}{B\omega_c}\right)\delta\left(\omega - \frac{ck_y}{B}E_x\right), \qquad (25)$$

and Eqs. (8) and (25) yield the dark nonlinear longitudinal current,

$$j_x^{\rm dc} = \frac{nc}{\hbar B}\frac{1}{A}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}}^2 k_y\chi_0''\left(\boldsymbol{k}, \frac{ck_y}{B}E_x\right). \qquad (26)$$

The low field limit yields the linear conductivity

$$\sigma_{xx}(0) = \frac{d}{dE_x}j_x^{\rm dc} = \frac{nc^2}{\hbar B^2}\frac{1}{A}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}}^2 k_y^2\frac{d}{d\omega}\chi_0''(\boldsymbol{k},0), \qquad (27)$$

which coincides with the dc limit of Eq. (1).

The characteristic wave vectors that dominate $\Gamma_{\boldsymbol{k}}$ in the LLL are of the order of ℓ^{-1}. The dark dc conductivity at the compressible filling fraction ν is of the scale

$$\sigma_{xx}(0) \sim \frac{\nu q^2}{h}\frac{\Gamma^2}{(\epsilon_F^*)^2}[1 + \mathcal{O}(\Gamma/\epsilon_F)]. \qquad (28)$$

Away from $\nu = \frac{1}{2}$ the composite fermion theory predicts resonances in $\chi_0''(\omega)$ corresponding to the spectrum of "CF Landau levels." Such resonances should appear as oscillations in the current by Eq. (26).

Experiments.—Photoconductivity measurements in FQH samples were carried out some time ago [26,27]. Oscillatory magnetoresistance was reported without comparison to theoretical calculations. The published data did not show indications of ZRS effects in the Laughlin phases. However, the frequency scale may have been below the magnetoroton threshold: For filling fraction $\nu = 1/3$ at carrier density $n = 7.6 \times 10^{10}$ cm^{-2}, the magnetic field is $B = 9.5$ T, and $\hbar\omega_c \sim 100$ K. The resistivity activation gap [26], which is expected to be similar to Δ_0, was about 210 GHz, which was higher than the microwave frequencies used in these experiments. Recent advances in terahertz spectroscopy may open the door to photoconductivity measurements in the FQH regime. It would be instructive to compare, e.g., the magnetoroton gap by photoconductivity to the activation gap of resistivity.

For the compressible phases, we propose to measure Eq. (26). The electric field $\bar{E}_x$ that corresponds to a charge excitation frequency $\bar{\omega}_{\rm charge}$ is

$$\bar{E}_x = B\ell\bar{\omega}_{\rm charge}/c$$
$$= 0.26\bar{\omega}_{\rm charge}[{\rm GHz}](B[{\rm T}])^{\frac{3}{2}}[{\rm Volt/cm}], \qquad (29)$$

where we have used $\ell = 26\ {\rm nm}\sqrt{B[{\rm Tesla}]}$.

Validity of weak disorder expansion.—Equations (8), (13), and (26) are the second order expansion of the current in the disorder potential V. Note that these expressions have finite LLL limits for $\omega_c \to \infty$, keeping $\nu < \infty$ [28]. The weak disorder expansion of the longitudinal current is valid under the following conditions:

(1) $\sigma_{xx} = 0$ in the clean limit. Note that this condition fails for an ordinary clean metal at zero field.

(2) Landau level broadening. In the absence of interactions the Landau levels are infinitely degenerate. The degeneracy is lifted by Coulomb interactions that introduce the intra-Landau level charge excitation scales $\bar{\omega} = \Delta_0$, $\epsilon_F^*/\hbar$ we have seen above. These energy scales control the higher-order corrections by powers of $\Gamma/\hbar\bar{\omega} \ll 1$.

(3) Density matrix is at equilibrium. It is expected that strong time dependent electric fields modify the density matrix at long times, and produce strong effects on the dc photocurrent [17]. Our analysis above did not take non-equilibrium effects on the density matrix into account,

which is justified in cases of rapid thermalization. In other words, we assume short nonradiative relaxation time by phonons $\tau_{\rm inel} \ll \tau_{\rm tr}$, where at zero magnetic field the transport time $\tau_{\rm tr} = \sigma_{xx} m/e^2 n$ is long in the weak disorder limit.

(4) By applying the derivation of Eq. (26) to the transverse current $j_y(E_x)$, the Hall conductivity $\sigma_{xy}^{\rm dc} = nqc/B$ gains no corrections at any finite order in V. Thus, plateaus of $\sigma_{xy} = \nu e^2/h$ in the incompressible phases are necessarily nonperturbative effects in V such as nucleation of localized quasiparticles or motion of domain edges in the long range potential landscape.

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