Atmospheric tomography using a fringe pattern in the sodium layer

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We wish to measure and separate the contribution of atmospheric turbulent layers for multiconjugate adaptive optics. To this end, we propose to create a periodic fringe pattern in the sodium layer and image it with a modified Hartmann sensor. Overlapping sections of the fringes are imaged by a lenslet array onto contiguous areas in a large-format camera. Low-layer turbulence causes an overall shift of the fringe pattern in each lenslet, and high-attitude turbulence results in internal deformations in the pattern. Parallel Fourier analysis permits separation of the atmospheric layers. Two mirrors, one conjugate to a ground layer and the other conjugate to a single high-altitude layer, are shown to widen the field of view significantly compared with existing methods.

The small size of the isoplanatic angle (of the order of 2 arcsec) severely limits the benefits of adaptive optics, especially for observation of extended astronomical objects. It is well known that a large number of deformable mirrors could, in principle, provide phase compensation over an extended field of view¹⁻⁷ (FOV). Attempts to extend the FOV involve the creation of multiple guide stars and the solution of a set of equations for the phase gradient of the turbulent layers.³⁻⁵

In this Letter we present a new approach⁸ for wavefront sensing that can provide turbulence information over a wide FOV and permits distinction between the effects of turbulence at different heights. Our approach is based on generating a two-dimensional fringe pattern with a period of 1-5 m, at the sodium layer, by a coherent superposition of three beams from a single laser (Fig. 1). This fringe pattern is imaged (Fig. 2) by the telescope and a lenslet array onto a large-format camera, much like a Hartmann sensor. The information available to us in each subaperture is a section of the fringe pattern (Fig. 1), perturbed by the turbulence effect on the downward propagation. Simple image processing, treating the fringe pattern as a whole image rather than as multiple guide stars, permits extraction of the wave-front errors to be conjugated by deformable mirrors of at least two layers. According to recent observations,^{3,9,10} in many cases the turbulence is indeed concentrated in several layers and not distributed uniformly over the whole atmosphere. In such a case the separation between low- and highaltitude turbulence is clear. However, even without the assumption of a layered structure of the atmosphere, a division into low- and high-altitude turbulence can sometimes be made, represented as two phase screens (a ground layer at h_0 and a much higher one at h_1). Figure 2 shows the turbulence encountered by the fringes within the FOV of neighboring subapertures. The effect of the bottom layer is a shift of the imaged section of the fringe pattern, since $d > r_0^{\text{btm}}$, where d is the diameter of the lenslet and r_0^{btm} is Fried's parameter of the bottom layer (which will always be larger than r_0 for the whole atmosphere). For the high altitude, each subaperture images part of the fringe pattern through a large section of the higher layer; each fringe is imaged through a slightly different section of that turbulence. Therefore the distant turbulent layer induces a distortion of the fringe pattern that is superposed onto the shift induced by the bottom layer. Thus it is possible to extract the contribution of the two layers separately and feed the information to two deformable mirrors conjugated to these equivalent phase screens.

To extract the information from the projected fringe pattern imaged by the lens array, one may use digital Fourier demodulation.^{11,12} In this technique the fringe pattern distortions are assumed to be caused only by reimaging through the top layer, while the average position of the fringe pattern is due to bottom-layer phase gradients. Extraction of the bottom-layer phase gradients is performed (up to a constant) by comparison with the theoretical unperturbed pattern. The remaining constant is found by setting the average tilt of all subapertures to zero, leaving the global tilt to be treated separately with a natural guide star. The top-layer phases are reconstructed separately for each lenslet. These aberrations, represented as phasors, are averaged according to the known (and significant) overlap between the FOV's of the lenslets.

One of the main concerns in the implementation of adaptive optical systems is the budget of required resources and their availability. The most important resources needed, detector size, photon power and computing power, are shown to be within easy reach of currently available technology: For a telescope of 5-m diameter, $M \times M$ subapertures, each with $N \times N$ pixels, and r_0 in the 0.25-m range, we would need $M \approx 20$ subapertures. The number of required pixels is $N \approx 128$ pixels (permitting 20-40 fringes across the diameter of each subaperture). If we assume a continuous camera such as a CCD, it has to be of size 2560×2560 , at a readout rate of 1 ms, as dictated by the atmospheric rate of change. It is expected that detectors of this size will be available



Fig. 1. Part of the fringe pattern in the sodium layer as viewed from the ground (simulation) with the laser beam setup (inset) as viewed from above.



Fig. 2. Projected fringe pattern in the sodium layer reimaged by the telescope and the (conjugate) lenslet array on the large-format camera. The FOV's of adjacent lenslets overlap at higher layers. The dashed-dotted and dotted lines signify FOV's of different subapertures.

in the near future. The information extraction procedure that we use requires $M \times M$ Fourier transforms, each of $N \times N$ size (processed in parallel), followed by inverse transforms of the same size and number. Using available parallel processors can ensure that the computations be completed within 1 ms. Regarding energy requirements, we follow a path similar to that taken by Gardner *et al.*¹³ for an adaptive-optics system with several guide stars and a Hartmann sensor. They calculated the total photon count in a single Hartmann cell to provide an allowed error. Following a similar approach for our system, we assume that the same requirement will be placed on the photon count arising from each fringe in a single Hartmann sensor. Assuming a 45-m-diameter pattern at the sodium layer and a fringe spacing of 2 m, we find that to achieve an error level of $\Delta \phi_{\rm rms} = \lambda/15$ over each subaperture we would need a laser of ~ 600 W of power, Copper-vapor-pumped dye lasers are known to provide 1000 W of power at the sodium D_2 line.¹⁴

Some of the main points regarding the system performance are as follows:

(a) Corrected angular FOV: The maximum corrected FOV is approximately the diameter of the fringe pattern divided by the altitude of the sodium layer. For a fringe pattern 45 m in diameter, the corrected field is approximately 80 arcsec (full angle). A good correction within this field can be achieved, provided that it is not larger than the smaller of the isoplanatic angles of the two layers.

(b) Subaperture size: In conventional, singledeformable-mirror correction, the subaperture size is taken to be of the order of r_0 . In our case the subaperture size should be of the order of r_0^{btm} of the bottom layer, which means we may manage with fewer subapertures. The total number of correcting elements in the two deformable mirrors would therefore amount to less than twice the number of subapertures used in the single-deformable-mirror method.¹

(c) Position of emerging laser beams: The best position for achieving good fringe contrast is above the central obscuration of the telescope or in the immediate vicinity of the telescope. The reason for this is the thickness of the sodium layer. At each slab of the sodium layer the fringe pattern is slightly different, since the angle subtended by the upward rays is slightly different. If the beams do not emerge from the center of the FOV, there will be a gradual smearing of the fringes, up to a complete washout of the contrast.

(d) Atmospheric corruption of the upgoing beams creating the fringe pattern: As long as the distance between the beams is smaller than r_0 , all the beams encounter practically the same turbulence on their way up. As a result, the fringe pattern remains uncorrupted (except for a global shift as encountered when one uses conventional laser guide stars).

(e) Height of the top phase screen: The height to which the second deformable mirror is to be conjugated can be estimated.

(f) Partial correction of a wider field: Even if limited to a single deformable mirror, measurement of the separate layers has a significant contribution. Correction of the worse layer will yield better pointspread functions over a wider field, rather than a properly corrected central field, and worse pointspread functions in the periphery. Moreover a wider FOV permits the use of further and fainter natural guide stars to solve the tilt problem [point (d) above]. If the FOV of each lenslet contains only one r_0^{top} of the top layer, then correcting that layer separately is impossible and unnecessary—it will be measured and corrected together with the bottom layer.

We performed simulations that compared the expected performance of the present method with a conventional system, using a single laser guide star at the sodium layer. Only a vertical slice of the full three-dimensional problem was considered, and no photon noise was added. A geometrical-optics approximation was used,^{3,4} and diffraction effects between the turbulent layers were neglected.⁶

The turbulence is simulated by six Kolmogorov phase screens, at heights of 0, 100, 200, 9900, 10,000, and 10,100 m (grouped into bottom and top turbulent layers). The results presented are for $r_0^{\rm btm} = 0.3$ m for the bottom layer, $r_0^{\rm top} = 1$ m for the top layer, and total $r_0 = 0.28$ m. The isoplanatic angle of the example used in Figs. 3 and 4(a) is



Fig. 3. Phase-front aberrations caused by turbulent layers: single realization. Solid curves, simulated phases; dashed-dotted curves, corrections based on estimation.



Fig. 4. Correction errors: rms phase difference over the telescope aperture. Curves from the top: uncorrected error, after correction by the conventional method, after correction of the bottom layer by the present method, and after correction of both layers by the present method. GS, guide star.

1.5 arcsec. The 5-m telescope has subapertures of diameter d = 0.125 m (a conservative value). Each lenslet images 20 fringes onto a detector that is 100 pixels long. The full FOV of the lenslets is taken to be 80 arcsec. The height of the top phase screen is taken to be $h_1 = 10$ km. For comparison we also calculate the correction according to the conventional single laser guide star method. In this case, each subdetector has only 25 pixels since the FOV of the lenslet is four times smaller.

Figure 3 (top panel) shows the phase shift caused by the bottom layer compared with the correction derived from the conventional, single-guide-star system. The number of correcting elements in the deformable mirror is taken to be the same as the number of subapertures. The middle and bottom panels show the phase shift caused by the two layers (for the same turbulence) compared with the two separate corrections derived by the system described in this Letter. The number of correcting elements for the bottom layer is the same as in the conventional system. For the top layer fewer than half the number of correcting elements are used. All corrections are performed with a segmented mirror after a simple integration of the slopes. This is sufficient since our aim was to compare the two methods.

The rms optical path difference before and after the correction is shown in Fig. 4 as a function of angle from the center of the FOV. Figure 4(a) gives the correction errors derived from the results given in Fig. 3, and Fig. 4(b) gives the correction errors for an average over 100 different realizations. In all curves (including the one labeled Uncorrected) global tilt correction is assumed, with a natural guide star positioned in the center of the FOV.

The simulation results clearly show that for a FOV much larger than the isoplanatic angle the double conjugate system with a projected fringe pattern results in a significantly better correction than does the conventional system.

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