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Letter to the Editor

Asteroids as reference stars for high resolution astronomy

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Abstract. Adaptive optics and speckle imaging can resolve faint objects with the aid of a reference source to measure atmospheric turbulence. Natural bright stars and artificial laser guide stars were proposed for this method. However, the former are not sufficient in number, and the latter are very complex and expensive. Asteroids and other solar system objects extend the number of reference sources and their coverage of the sky. Using partial correction with adaptive optics, asteroids allow resolution of compact sources up to m_V=25 and m_K=21. In the same manner, stars can serve as references for measuring solar system objects in near-occultation.

Key words: adaptive optics – high angular resolution – observational methods – isoplanatism – asteroids and moons

1. Reference guide stars

One of the severe problems in adaptive optics is the difficulty to measure the aberrated wave front to a high accuracy, where this measurement is used to adjust the correcting deformable mirror. Most interesting astronomical objects do not emit enough photons for the wave front sensor to operate to its full power. The problem can be alleviated by using a bright star in the immediate vicinity of the astronomical object of interest (hereafter referred to as the target). The sampled wave front of the reference star is very similar to that of the target, the similarity dropping with distance between the two (Fried 1982, Chassat et al. 1989). Unfortunately the number of such stars and thus the sky coverage is rather small (Rigaut & Gendron 1992). Another solution is to shine a laser guide star in the same direction as the target, with more photons now available (Fugate et al. 1991, Foy & Labeyrie 1985). However, the laser beacon is a result of resonant backscattering from the mesospheric sodium layer at 90 km and its returned light cannot sample the same atmospheric path as a natural guide star at infinity. The method of speckle imaging, where the correction is applied after the measurement, is also in dire need for such a reference source.

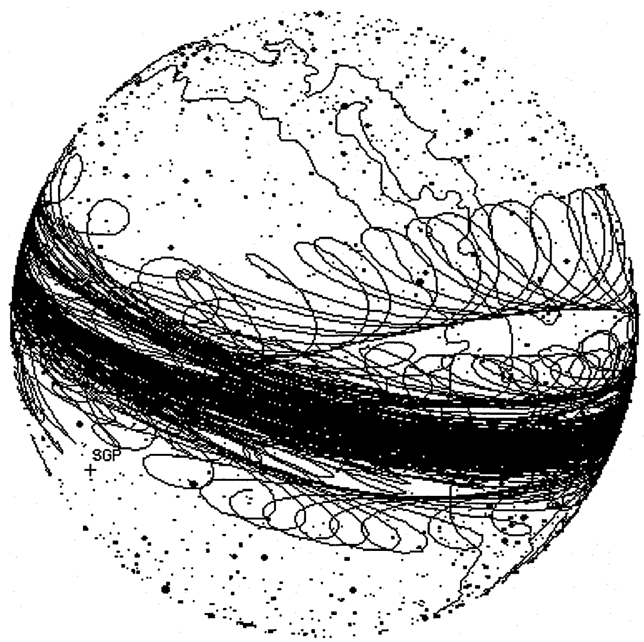

Fig. 1. Paths of the first listed asteroids (1)Ceres to (16)Psyche over 40 years. Notice the coverage relative to the southern galactic pole (marked at lower left), galactic center (hidden, lower right), and the milky way (outlined).

It is proposed here to use objects from the solar system as reference sources (see Léna 1993, Léna 1994). While the larger planets might not be as useful, the other objects, essentially asteroids and outer planets, turn out to be of the right qualities for this purpose: (1) they are bright enough, (2) they are numerous, (3) they cover a large area of the sky as they move in their orbits, (4) they move at a rate

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Table 1. Number of asteroids $N(V)$ in the magnitude range $[V(1,0)-1,\ V(1,0)]$ and cumulative number of asteroids $C(V)$ up to magnitude $V(1,0)$ (Compiled from Binzel et al. (1989)).

V(1,0)	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
N(V)	2	1	11	27	100	235	362	584	764	719	561	66	17	10	5	1
C(V)	2	3	14	41	141	376	738	1292	2056	2775	3216	3282	3299	3309	3314	3315

which allows many minutes of observation (or scanning across a wide target), (5) they are far enough to sample the higher elevations of the atmosphere, and (6) they do not require any special modification to the telescope. The drawbacks are (I) that they do not reach the ecliptic polar area, (II) their orbits are not fully predictable, and (III) unlike laser guide stars, they do not allow extension to wide field-of-view for adaptive optics (Baharav et al. 1994).

The brighter planets have to be ruled out for two main reasons: they scatter too much light in their vicinity, and they have large angular diameters which make the wave front sensor less efficient. Uranus and Neptune are the only planets which could be used for this purpose. From here on we shall refer only to asteroids, keeping in mind these other planets, as well as moons of the brighter planets.

2. Asteroid magnitudes and spatial distribution

For our purpose, we consider a low order adaptive optics system (twenty corrected modes) dedicated to red and near infrared observation on a 4 m class telescope. The current experimental limit on the magnitude of a guide star is approximately 12.5^m (Roddier 1994), and the quality of correction drops with magnitude. Table 1 provides a histogram of magnitudes at opposition. $V(1,0)$ is the magnitude of the asteroid at opposition for Sun-asteroid and Earth-asteroid (hypothetical) distances of 1 AU. When translated to average m_V in the $[-30°, 30°]$ bracket about opposition, these asteroids are fainter by 4.3^m; in the narrower $[-15°, 15°]$ bracket they are 3.8^m fainter. Thus we are able to use today between 200 and 290 asteroids correspondingly.

The orbits of the asteroids are scattered about the ecliptic. Hence regions near the *galactic* poles, some 30° away, are covered, as well as large parts of the Milky Way and the Galactic center (Fig. 1). The scatter of inclinations i about the ecliptic is large enough: while the average inclination is $\langle i \rangle = 9.5°$, there are asteroids with inclinations four times as much. Projected into orbits on our sky, they cover a wider range of ecliptic latitudes, albeit at decreasing density towards the ecliptic poles. It is expected that the distribution will grow more even as a result of an on-going search for asteroids in collision orbits with earth (Cunningham 1988, Liège 1992).

In order to estimate the actual density of asteroids, let us assume simply that all asteroid orbits are circular, and their radii are the same: $a = 2.7$ AU. Hence their period is $T = 4.44$ yr. If we take a sphere of radius a about the sun, then all asteroids draw great circles on this sphere, whose angles with the ecliptic plane are their inclinations i. As an asteroid moves in its orbit, its ecliptic latitude θ on this sphere changes with time as

$$\sin\theta = \sin(2\pi t/T)\sin i\ , \tag{1}$$

where $t = 0$ corresponds to the ecliptic crossing. Taking differentials we get

$$\frac{\mathrm{d}t}{T} = \frac{\cos\theta \mathrm{d}\theta}{2\pi\sqrt{\sin^2 i - \sin^2\theta}}\ ; \quad 0 \le |\theta| < i \le \pi/2\ . \tag{2}$$

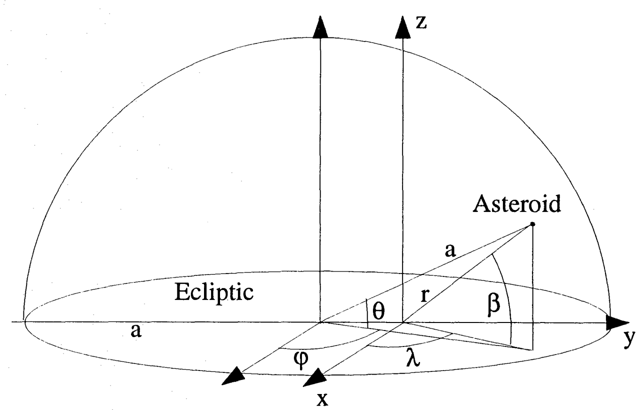

Fig. 2. Assuming the average asteroid has a circular orbit of radius a, we can draw all orbits as great circles on a sphere of the same radius centered on the sun. We average over all orbit inclinations to the ecliptic and over all solar longitudes ϕ to get the average distribution as a function of solar latitude θ. This distribution is translated to earth ecliptic coordinates, shifted by 1AU along the y-axis. Finally we average over longitude λ in a bracket [-30°,+30°] about opposition to achieve the asteroid distribution as a function of earth ecliptic latitude β.

This is the fraction of time an asteroid spends in the latitude band $\mathrm{d}\theta$ centered on θ. Hence it is equal to the fraction of the number of asteroids of inclination i in this band (van Houten et al. 1970).

How many asteroids are there with an ecliptic inclination i ? A normalized distribution function $\nu(i)$ can be approximated from a histogram of the inclinations of all asteroids (Binzel et al. 1989). This histogram was found to resemble a Rayleigh distribution, and the best fit was to

$\nu(i) = pi \exp(-qi)$, with $p = 130.2 \pm 9.5$, $q = 11.41 \pm 0.42$ (i in radians). Assuming there is no correlation between the brightness of an asteroid and its inclination, the distribution of the cumulative number of asteroid up to magnitude $V(1,0)$ is then $C(V,i) = C(V)\ \nu(i)$. We multiply Eq. 2 by $C(V,i)$ and integrate over all inclinations to get the number of asteroids in latitude θ,

$$\begin{aligned}\tau(\theta, V)\ \mathrm{d}\theta &= \frac{1}{2\pi}\int_{|\theta|}^{\pi/2} \mathrm{d}i\, C(V,i)\ \frac{\cos\theta \mathrm{d}\theta}{\sqrt{\sin^2 i - \sin^2\theta}} \\ &\approx C(V)\, k \exp(-l\,\theta^S)\ . \end{aligned} \quad (3)$$

The approximation is a rather good fit, using numerical integration, with $k = 4.0 \pm 0.2$, $l = 10.0 \pm 0.4$ and $S = 1.135 \pm 0.005$.

The next step is to transfer this distribution to earth ecliptic longitude and latitude β, λ. We choose the sun to be at $(0, -1, 0)$ Astronomical Units in earth cartesian coordinates (Fig. 2). The solar latitude θ is given by $\sin\theta = z/\sqrt{x^2 + (y+1)^2 + z^2}$, where $x = r\cos\beta\cos\lambda$, $y = r\cos\beta\sin\lambda$, $z = r\sin\beta$, are earth ecliptic coordinates. Since all asteroids are assumed to be on the surface of the sphere, we can write $a^2 = x^2 + (y+1)^2 + z^2 = r^2 + 2y + 1$, whence

$$r = \sqrt{\cos^2\beta \sin^2\lambda + a^2 - 1} - \cos\beta\sin\lambda\ . \quad (4)$$

The relation between solar latitude and earth longitude and latitude can now be recovered through

$$\begin{aligned} z &= a\sin\theta = r\sin\beta \\ &= [\sqrt{\cos^2\beta\sin^2\lambda + a^2 - 1} - \cos\beta\sin\lambda]\sin\beta\ . \end{aligned} \quad (5)$$

The distribution function $\tau(\theta)$ can now be expressed in earth coordinates. Since we are interested in its dependence on ecliptic latitude β, we integrate on the longitude

$$\begin{aligned}\tau(\beta, V)\mathrm{d}\beta &= C(V)\mathrm{d}\beta \int_{\pi/2-\Lambda}^{\pi/2+\Lambda} \mathrm{d}\lambda\ k\ \exp\{-l \times \arcsin^S[\\ &\quad (\sqrt{\cos^2\beta\sin^2\lambda + a^2 - 1} - \cos\beta\sin\lambda)\frac{\sin\beta}{a}]\} \\ &\approx C(V)\, m\, \exp(-n\,\beta^S)\,\mathrm{d}\beta\ , \end{aligned} \quad (6)$$

where Λ is the bracket about opposition where the asteroid is bright enough. To provide specific values we chose a longitude bracket $\Lambda = \pi/6$; the corresponding limiting magnitude is $V(1,0) = 8.2$, which sets the number of asteroids $C(V) = 200 \pm 10$ (Table 1). The last line is again a numerical solution which was well fitted with $m = 2.62 \pm 0.13$, $n = 6.23 \pm 0.25$ and $S = 1.135$. $\tau(\beta)$ is in asteroid·radian^{-1} and is symmetrical about the ecliptic plane $\beta = 0$. It is plotted in Fig. 3 (left scale), expressed in asteroid·degree^{-1}. Notice that by taking narrower brackets around opposition and thus more asteroids (because they are brighter), we can increase the value of the prefactor $C(V)\ m$ by a small amount.

The sky area covered by all asteroids can now be estimated. We start by defining an adaptive optics cross section: this is the sky area around the beacon within which it can serve as a reference for the target (Chassat et al. 1989). The radius of this cross section is $\rho_V \approx 10$ arcsec in the visible (low order correction only) and $\rho_K \approx 30$ arcsec at the K-band (nearly full correction with the same number of corrected modes). The instantaneous coverage by an asteroid is a disk of angular area $\pi\rho^2$. The average coverage over one year is a band swept by the asteroid around the sky. The band length is $2\pi/T$ (ignoring the significant contribution of retrograde motion), and its width is 2ρ. Since asteroids are bright enough only when close to opposition in a range of $\pm\Lambda$, we have a total annual coverage of $(2\pi/T) \times 2\rho \times (2\Lambda/2\pi) = 4\Lambda\rho/T$. In Eq. 6 we calculated $\tau(\beta)\ \mathrm{d}\beta$, the number distribution of asteroids on the sky; the instantaneous and annual area distributions are $\pi\rho^2\tau(\beta)\ \mathrm{d}\beta$ and $4\Lambda\rho\tau(\beta)\ \mathrm{d}\beta/T$ correspondingly.

What is the fraction of sky covered by asteroids at each ecliptic latitude in a year? To find this value we need to divide the absolute coverage by the area of each ecliptic latitude band $\mathrm{d}\beta$. This area is $2\pi\cos\beta\ \mathrm{d}\beta$. Thus we get a relative sky coverage

$$R(\beta) = \frac{4\Lambda\rho\tau(\beta)\ \mathrm{d}\beta/T}{2\pi\cos\beta\ \mathrm{d}\beta} = \frac{2\Lambda\rho\tau(\beta)}{\pi T\cos\beta}\ . \quad (7)$$

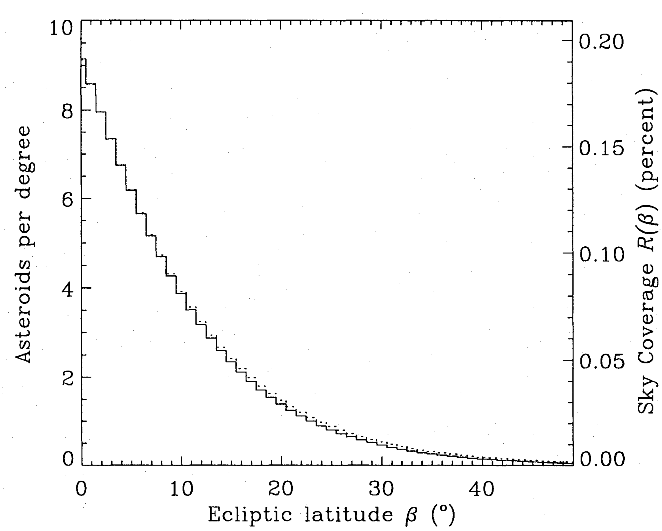

Fig. 3. Average number of asteroids per degree (left axis, solid line) and average annual coverage per degree of latitude (right axis, dotted line) as a function of the earth ecliptic latitude β. The average coverage is for the case of observation in the visible, considering an adaptive optics field of view radius of $\rho_V = 10$ arcsec and a limiting magnitude of $m_V = 12.5$ for the guide asteroid (K-band coverage is $\rho_K/\rho_V = 3$ times as large). The instantaneous (as opposed to annual) distribution is smaller by a factor of $\pi\rho_V T/4\Lambda = 0.00033$.

Figure 3 (right scale) shows this distribution for the case of correction in the visible (the K-band coverage is three times as large). We see that at zero ecliptic latitude, for a cross section $\rho_V = 10$ arcsec and maximum opposition angle $\Lambda = 30^o$, the annual sky coverage is 0.2%. This value is comparable to what is obtained by using natural reference stars (Rigaut & Gendron 1992).

3. Utilization in astronomy

Consider an astronomical program which concentrates on a general type of object to be resolved (galaxies, AGNs, planetary nebulae, YSOs, etc.) In order to appreciate the availability of beacons, correlation must be made between all the objects to be measured and all the asteroids bright enough to serve as guide stars. This correlation is calculated in time and in space (or rather, on the available night sky). Once there is a chance for a near occultation, the exact elements of the orbit are to be calculated (or measured), due to the slightly chaotic movement of some asteroids. Finally times and sky locations have to be optimized for efficient observing. An example of this procedure is given in Bus et al. (1994).

The attainable limiting magnitude on the scientific target can be derived in the following way: The average overlap time of the moving asteroid cross section with a scientific target is $(\rho/2\pi)T \approx 30$ mn in the visible and 108 mn in the K-band. Suppose that our telescope is 3.6 m in diameter, and we wish to correct only twenty modes (a low order adaptive optics system) using an asteroid of $m_V = 12.5$. We assume an overall throughput (including detector efficiency) of 0.3, a pixel size of 0.1 arcsec, and 70% of the energy concentrated on 3×3 pixels. The sky emission in the visible and the infra red is $m_V = 20.5$ and $m_K = 12.6$ per square arcsec. This allows to resolve enough details (signal-to-noise ratio ≥ 6) on most objects of $m_V = 25$ or $m_K = 21$ and brighter. More data could be collected and finer details resolved on multiple passes of asteroids, on brighter objects, and with brighter and slower asteroids (or further planets and moons).

As the asteroid moves across the field, the adaptive system follows it, so the quality of correction slightly changes with distance from a fixed target. Another mode of operation is to lock the telescope and adaptive optics system on the moving asteroid. Now the target will drift across the field. It is then possible to use a scanning CCD to register the target, where the scanning speed is opposite that of the asteroid. This scheme is particularly suited to very large objects, where only a band can be resolved near the asteroid path.

Finally, the reverse is also true: in order to observe asteroids, it is possible to use nearby stars, planets or other asteroids as beacons. Some measurements of solar system bodies have recently been carried out, using the object itself as a guide source for adaptive optics (Saint-Pé *al* 1993). Unfortunately, not all asteroids are bright enough and small enough to allow such self-referencing. If we wish to resolve fine details on large, low albedo asteroids, comets, moons, and planets, we can use field stars or other solar system bodies as references (see also comment by Saint-Pé 1992). Occultation methods (Cunningham 1988, Bus et al. 1994) have long been used to get the profile of such objects. We propose here to use near-occultation for true high-resolution imaging. Following this line, at least one planet was measured by an adaptive optics system using its own moon as a beacon (Drossart, in preparation).

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