

## Atmospheric scintillations for measuring remote wave fronts

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ABSTRACT

Stellar scintillations provide statistical information about the higher atmosphere (7 – 12km). Since each realisation of scintillation is the Fresnel diffraction off high altitude turbulence, it can be inverted separately. Sensors for adaptive optics integrate the wave front error over all layers of turbulence. They measure scintillation for calibration. But this discarded information yields the high wave fronts. Separate correction for low and/or high turbulence widens the repaired field of view. The method requires that the reference star is bright and small, that the middle turbulence (2–7km) is negligible, and that the sensor has good spatio-temporal resolution. Simulations show that the turbulence can be retrieved, with lowest and highest frequencies lost first.

2. INTRODUCTION

A strong limitation for conventional adaptive optics systems is their limited field of view limited of a less than an arcminute. This is because they measure the atmosphere by integration along a cylinder or a cone extending from the telescope aperture to the natural or artificial star. Two or more deformable mirrors could compensate the phase over an extended field<sup>1</sup>, provided one probes a larger volume of the atmosphere using a number of guide stars<sup>2–5</sup>.

The accuracy of wave-front sensing for adaptive optics is compromised by scintillation<sup>6</sup> which is produced by high-elevation turbulence<sup>7</sup>. Is it possible to use the scintillation pattern also to measure the high turbulence<sup>8</sup>? What is its use for adaptive optics in general?

3. FRESNEL DIFFRACTION FROM HIGH ALTITUDE TURBULENCE

The turbulence above astronomical sites was shown to be concentrated in several, rather thin, layers<sup>3,9,10</sup>. It can then be represented by a set of thin phase screens<sup>5</sup> if the thickness and altitude of the layers are such that diffraction effects within the layer are negligible<sup>3,4</sup>. If this is the case, there is hope for multi-conjugate correction of the atmosphere, when each flexible mirror is conjugated to a specific turbulent layer.

A single plane wave (originating from a star) undergoes minute changes as it traverses different thin atmospheric layers. Using either arguments of conservation of energy and geometrical optics, or Fermat's principle<sup>11</sup> we get a direct relationship between scintillation and refractive index variations. Let the refractive index of air be  $n(x, y, z) = 1 + \mu(x, y, z)$ , where  $\mu(x, y, z) \gg 1$ . The natural logarithm of the intensity pattern at ground level ( $z=0$ ) is<sup>11,7</sup>

aperture were taken. This completed the creation of the simulated scintillation pattern (Fig. 2).

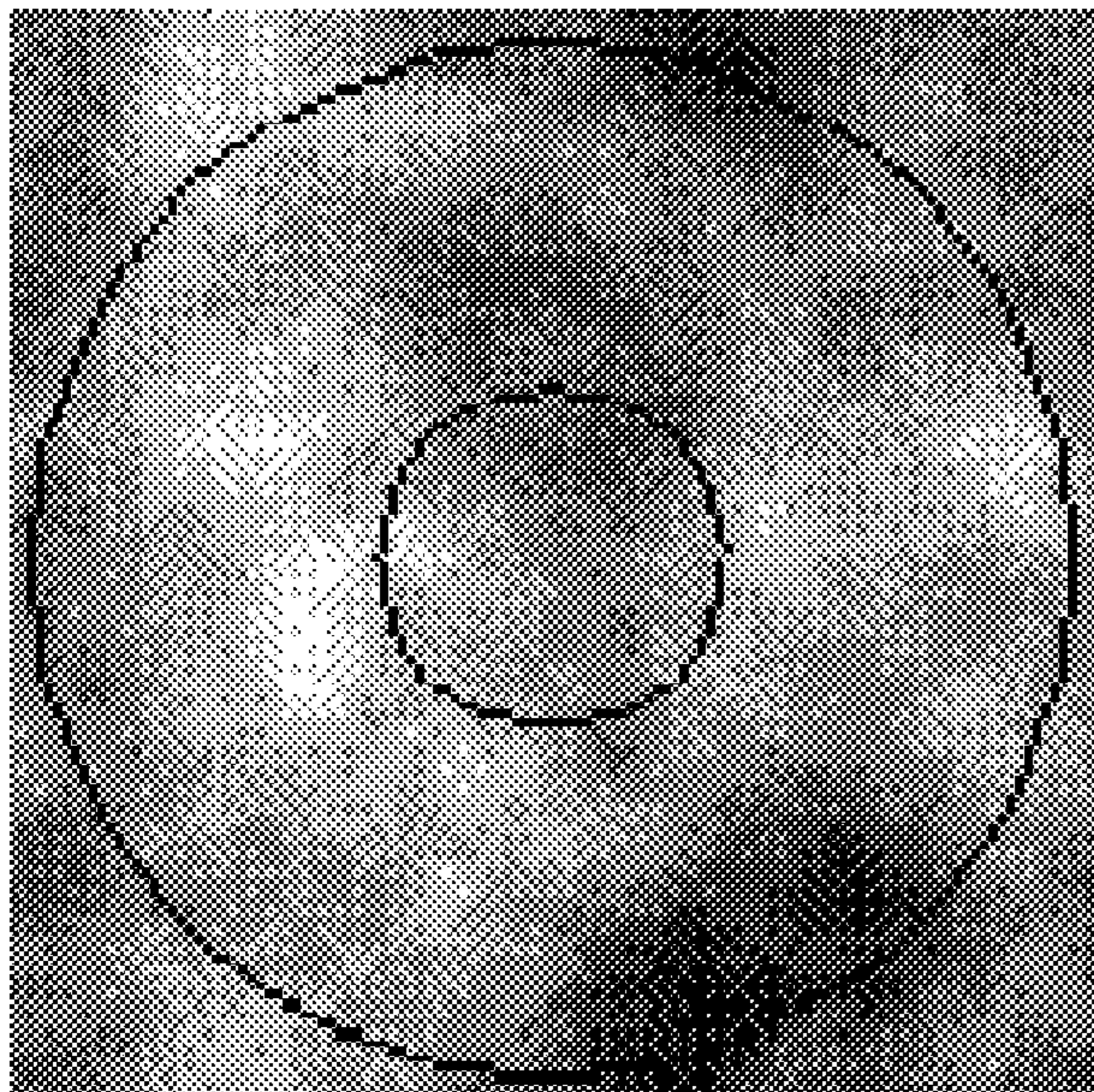


Fig. 1. A single realization of high-altitude wave fronts,  $r_0 = 10$  cm.  $128^2$  array, each element 2.5 cm. The telescope aperture (3 m diameter, 1/3 obscuration) is shown only for reference. This wave front served to create Fig. 2.

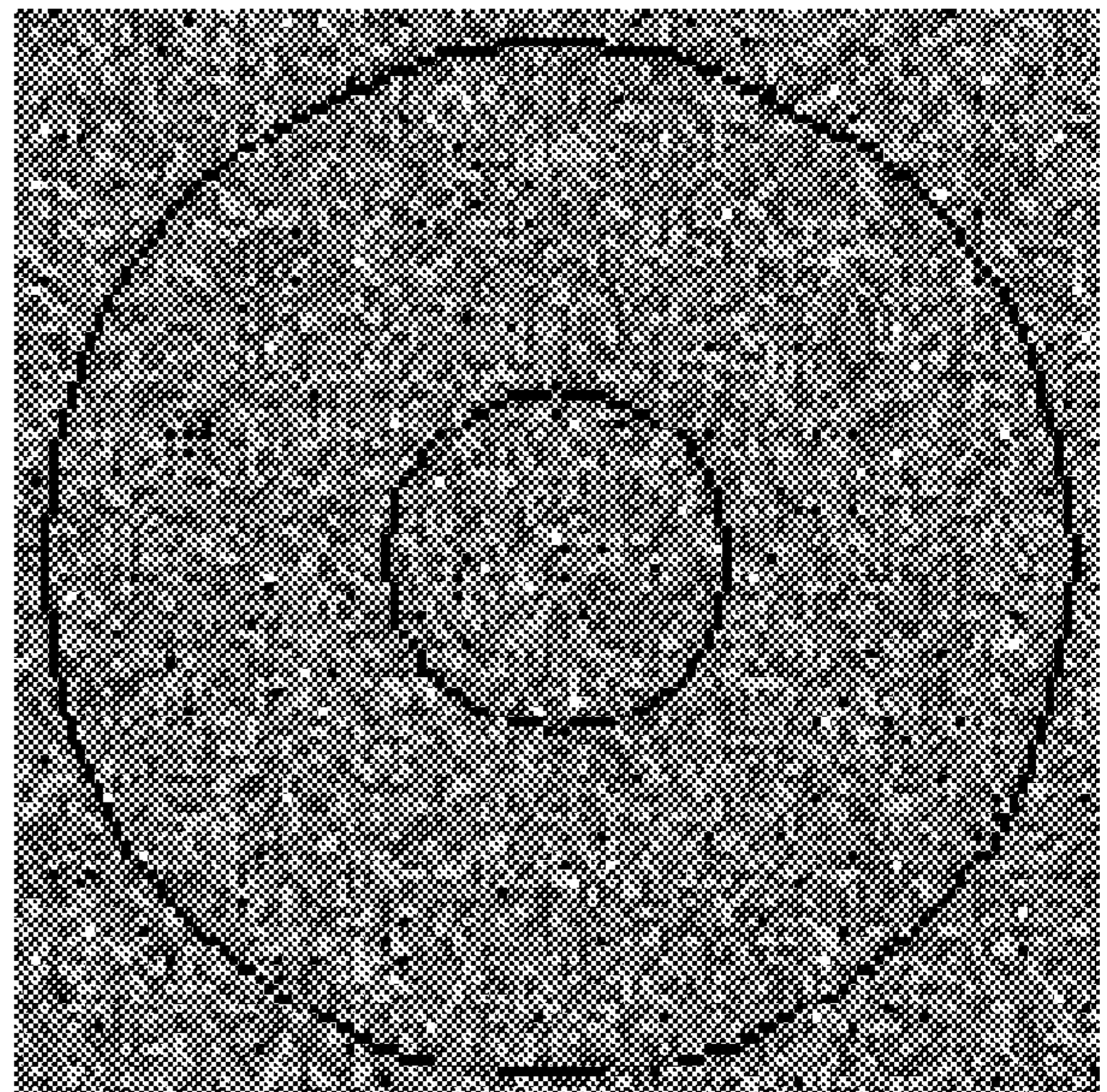


Fig. 2. Wide band scintillation pattern at the telescope aperture resulting from Fig. 1. The average number of photons per pixel is 100, as might be detected from a  $6^m$  star (0.5 total efficiency,  $\lambda = 550$  nm,  $\Delta\lambda = 300$  nm,  $\tau = 3$  ms,  $6.25$  cm<sup>2</sup> pixels).

The original wave front was recovered by a simple deconvolutions of the normalized logarithm of the intensity variations (Fig. 3). Then we tried a Wiener filter (Eq. 6) which included the atmospheric spectrum and the Poisson noise (Fig. 4). The Figures show the results for a low flux; the original wave front was reconstructed faithfully at high intensity ( $\bar{S} > 1000$  photons per pixel) and low turbulence. Lower signal (100 photons) resulted in poorer reconstruction (Fig. 5). A second order effect was due to the lack of boundary conditions and was not felt when the diameter spanned 120 pixels or more.

#### 4. APPLICATION

It is proposed to use scintillation in adaptive optics wave front sensors instead of calibrating it away. Suppose the sensor samples the intensity  $\bar{S}(x, y)$  densely, but is limited by additive detection noise. Its logarithm is  $\bar{\chi}(x, y) = \ln [\bar{S}(x, y) / \bar{S}]$ .  $\bar{S}$  is the average intensity over the aperture and over time, and we estimate<sup>9</sup> the altitude of the turbulence  $\bar{h}$ . Unfortunately, inverting the Fresnel transform is difficult if we only have the intensities (Eq. 4), but inverting the laplacian (Eq. 1) is simpler, and similar to curvature sensing<sup>13</sup>, but different mainly in boundary conditions. The first terms in Zernike expansion are: piston (inconsequential); tip and tilt (can be combined with global tip-tilt correction); curvature (set by the average intensity).

Solving for the wave front from its laplacian can be achieved by filtering the latter with a quadratic filter,  $C(\omega) = -1/4\pi^2\omega^2$ , where  $\omega^2 = u^2 + v^2$  is the Fourier frequency. A better choice is a Wiener-Helstrom filter<sup>16</sup>,  $W(\omega) = C^{-1}(\omega) F_A(\omega) / [C^{-2}(\omega) F_A(\omega) + F_N(\omega)]$ , where  $F_A(\omega)$  and  $F_N(\omega)$  are the power spectra of the wave fronts at the top layer and of the detection noise in the log-intensity  $\bar{\chi}(x, y)$ . These functions can be measured continuously or be given from models. Neglecting inner- and outer-scale effects, the power spectrum of a thin layer is<sup>7,15</sup>

$$F_A(\omega) = 6.9 \cdot 2^{2/3} \sin(5\pi/6) \Gamma^2(11/6) \pi^{-2} r_0^{-5/3} \omega^{-11/3} \equiv \gamma^{-11/3} \quad (5)$$

$r_0$  is Fried's parameter for the high turbulence<sup>10</sup>. Hence  $\gamma = 0.0672k^2 \int_h^\infty C_n^2(z) dz$ .

The power spectrum of the log-intensity is limited by shot noise. If the average intensity  $\bar{S}$  corresponds to  $\nu$  photons, then  $F_{ls}(\nu) \approx \{1.022\nu - 0.967 \log \nu + 8.10\nu e^{-\nu}\}^{-1}$  (accurate to 1% at  $\nu=1$  photon and slightly worse for weaker fluxes). If the scintillation is mild, with intensity variance of  $\sigma^2 < 0.04$ , then, within 1% again,  $F_N \approx (1 + 1.2\sigma^2) F_{ls}$ . The intensity variance is<sup>7</sup>  $\sigma^2 \approx 0.077 C_n^2 k^{7/6} \hat{h}^{11/6}$ . The leading term yields  $F_N(\omega) \approx 1/\bar{S}$ . Thus both scintillation at the same pixel and its covariance with neighbouring pixels are ignored. Assuming a very large telescope aperture, we get a Wiener filter

$$W(\omega) \approx \frac{-4\pi\bar{S}\gamma}{16\pi^2\gamma\bar{S}\omega^2 + \omega^{5/3}} \quad (6)$$

#### 5. SIMULATION

Blind tests were run to assess the method: scintillation was simulated through Fresnel propagation, and the original wave fronts were recovered through inversion of the laplacian. We created fractal wave fronts<sup>17</sup> with  $r_0 = 10$  cm (Fig. 1) and convolved them with the Fresnel kernel (Eq. 3), as simulation of the propagation. The resultant field was squared to yield the intensity at the aperture plane, Poisson noise was applied to the result, and finally only the pixels inside the

$$\chi(x, y) \equiv \ln \frac{I(x, y)}{\bar{I}} = - \int_0^\infty z \nabla^2 \mu(x, y, z) dz \approx - \sum_{i=1}^L z_i \nabla^2 \mu_i(x, y), \quad (1)$$

where  $\nabla = \partial/\partial x + \partial/\partial y$ ,  $\bar{I}$  is a constant average intensity, and we have assumed that the refractive index perturbations ( $\mu \neq 0$ ) occur only in  $L$  layers. Because of its proximity, low turbulence has negligible effect as compared to high turbulence.

The irradiance transport equation<sup>12</sup> derived under the assumption of Fresnel diffraction also ties the propagating intensity with wave-front phase

$$\partial I(x, y, z)/\partial z = -\nabla I(x, y, z) \cdot \nabla \varphi(x, y, z) - I(x, y, z) \nabla^2 \varphi(x, y, z), \quad (2)$$

Roddier has shown this to be equivalent to curvature sensing as implemented by him<sup>13</sup>. The intensity arriving from the source is constant, so  $\nabla I = 0$  and we get an equivalent to Eq. 1.

Rigorous Fresnel diffraction calculations provide us with the same results, especially if we assume that the atmosphere is composed of two main layers, one high above the telescope and one next to it. Phase errors are added to the beams only inside these layers, while the field undergoes Fresnel diffraction between them. The Fresnel approximation is valid above some elevation  $H \gg k^{1/3} \rho^{4/3} / 2$ , where  $k=2\pi/\lambda$  and  $\rho$  is the lateral (here: horizontal) distance between the scattering point and the measurement point. This condition is fulfilled<sup>10</sup> for  $\rho=1\text{m}$ ,  $\lambda=0.5\mu\text{m}$  and  $H \gg 116\text{m}$ . Smaller  $H$  is possible because of the principle of stationary phase<sup>14</sup>.

When a field arrives from a point source at infinity, its phase is constant, so it can be written as  $O(x, y) = F$ . It accumulates a phase angle  $\varphi(x, y)$  as it passes through the top layer. At an altitude  $h$  the field can be described as  $P(x, y) = O(x, y) e^{i\varphi(x,y)} = F e^{i\varphi(x,y)}$ . After free space propagation to the top of the boundary layer, the field can be written as a convolution of the former field and a Fresnel point spread function<sup>14</sup>,

$$Q(x, y) = P(x, y) * \frac{1}{2\lambda h} \exp \left[ ikh \left( 1 - \frac{x^2 + y^2}{2h^2} \right) \right]. \quad (3)$$

Through the turbulent boundary layer the field  $Q(x, y)$  accumulates an additional phase  $\psi(x, y)$  to become, at the telescope aperture,  $R(x, y) = Q(x, y) \exp i\psi(x, y)$ . The intensity is *independent* of the effects of the boundary layer:

$$S(x, y) = |R(x, y)|^2 = |Q(x, y) \exp i\psi(x, y)|^2 = |Q(x, y)|^2, \quad (4)$$

The adaptive optics wave-front sensor measures the phase of this field, which can be written as the sum of the phases of the two layers<sup>7,15</sup>,  $\arg \{R(x, y)\} = \varphi(x, y) + \psi(x, y)$ .

## 6. SUMMARY

If scintillation provides a clue about high turbulence, is it also useful for adaptive optics?

The main limitations are:

- (a) Intensity. For  $\nu$  photons per pixel the shot noise level is  $\sqrt{\nu}$ . Intensity fluctuations equivalent to  $\nu \pm \sqrt{\nu}$  will yield  $\bar{\chi} \approx \chi \pm \nu^{-1/2}$ . This sets the minimal wave front fluctuations to be retrieved. Use can be made to improve on this from the statistics of detection noise (uncorrelated) and turbulence (fractal and predictable<sup>17</sup>).
- (b) Resolution. The pixels in wave front sensors are larger than  $r_0$  because of low flux, and average over the finer scintillation<sup>7</sup>. (The same applies for temporal integration.) The inversion should be carried out by direct least-squares fitting under the constraints of photon and atmospheric noise.
- (c) Color. Fresnel and the atmosphere scattering are wave length dependent (Eqs. 3, 6) but could not be found in laboratory experiments or in the literature<sup>18</sup>.
- (d) Source size. The angular size of objects prone to scintillation is<sup>7</sup>  $\gamma_0 = \sqrt{2\lambda/\pi h} \approx 1.4$  arc-seconds, for  $h \approx 8\text{km}$  and  $\lambda = 0.589 \mu\text{m}$ . Stars, distant planets, bright asteroids<sup>19</sup>, and small sodium beacons are smaller or near this limit.
- (e) Contributions of separate layers. Wave front sensing provides information about all atmospheric layers, and scintillation about the top layer. It is possible to solve simultaneously for both wave fronts in order to reduce their covariance.
- (f) Validity of the model for thick layers. If diffraction occurs inside the layers then Eqs. 1 or 2 can be solved iteratively for the separate contributions of thin sub-layers.

If scintillation is significant, then it is easier to measure and correct. However, since the information exists anyway, it should not be ignored. Apart from adaptive optics, the measurement of distant turbulence is necessary at times. The proposed method, using a star or a small beacon as a source, can be employed.

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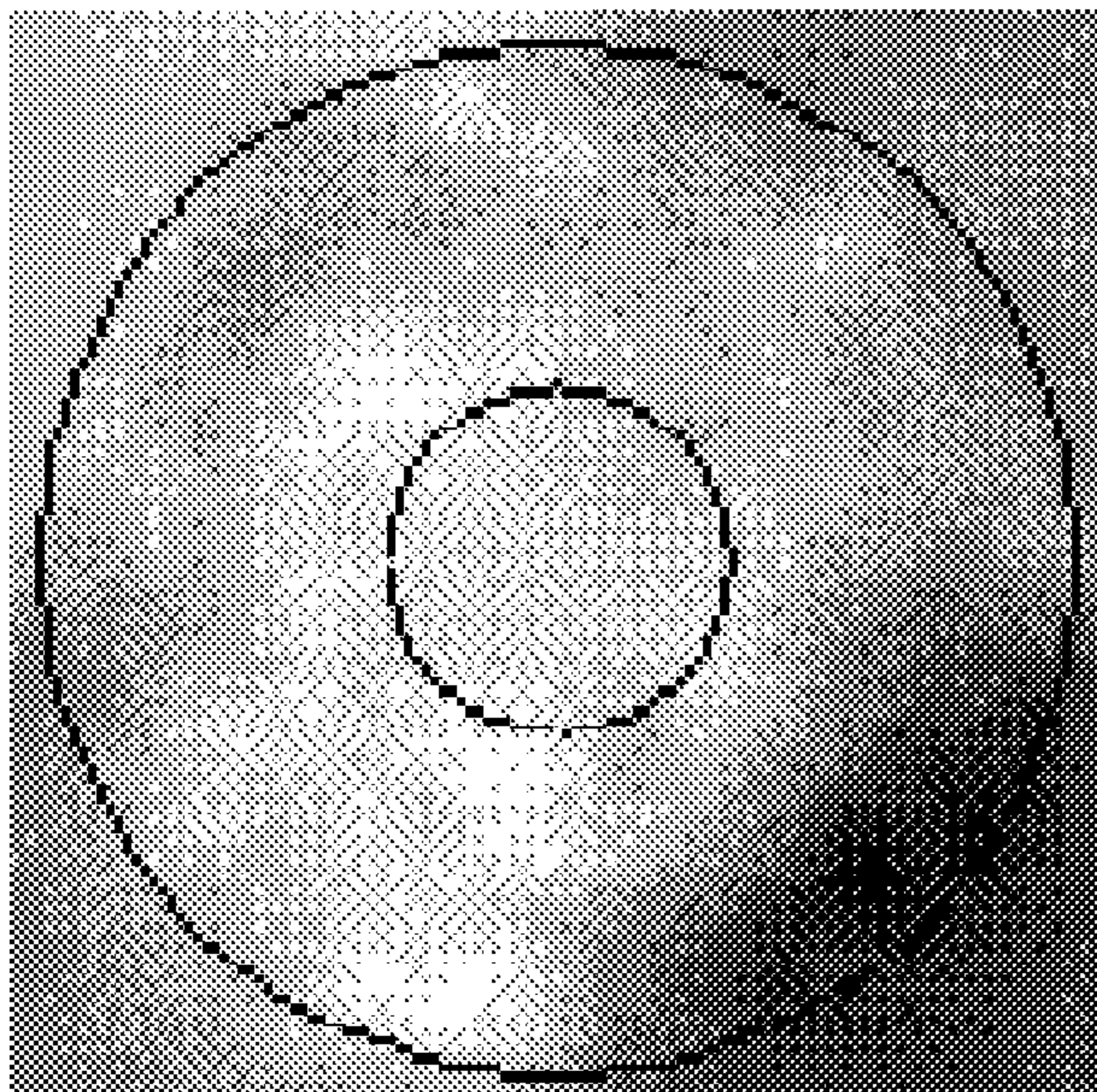


Fig. 3. Direct deconvolution of the laplacian: reconstructed wave fronts using the scintillation pattern *inside* the aperture in Fig. 2. Low and high frequencies are lost: the rms difference between the original and the reconstructed wave front over the aperture, global slope removed, was 0.77 waves.

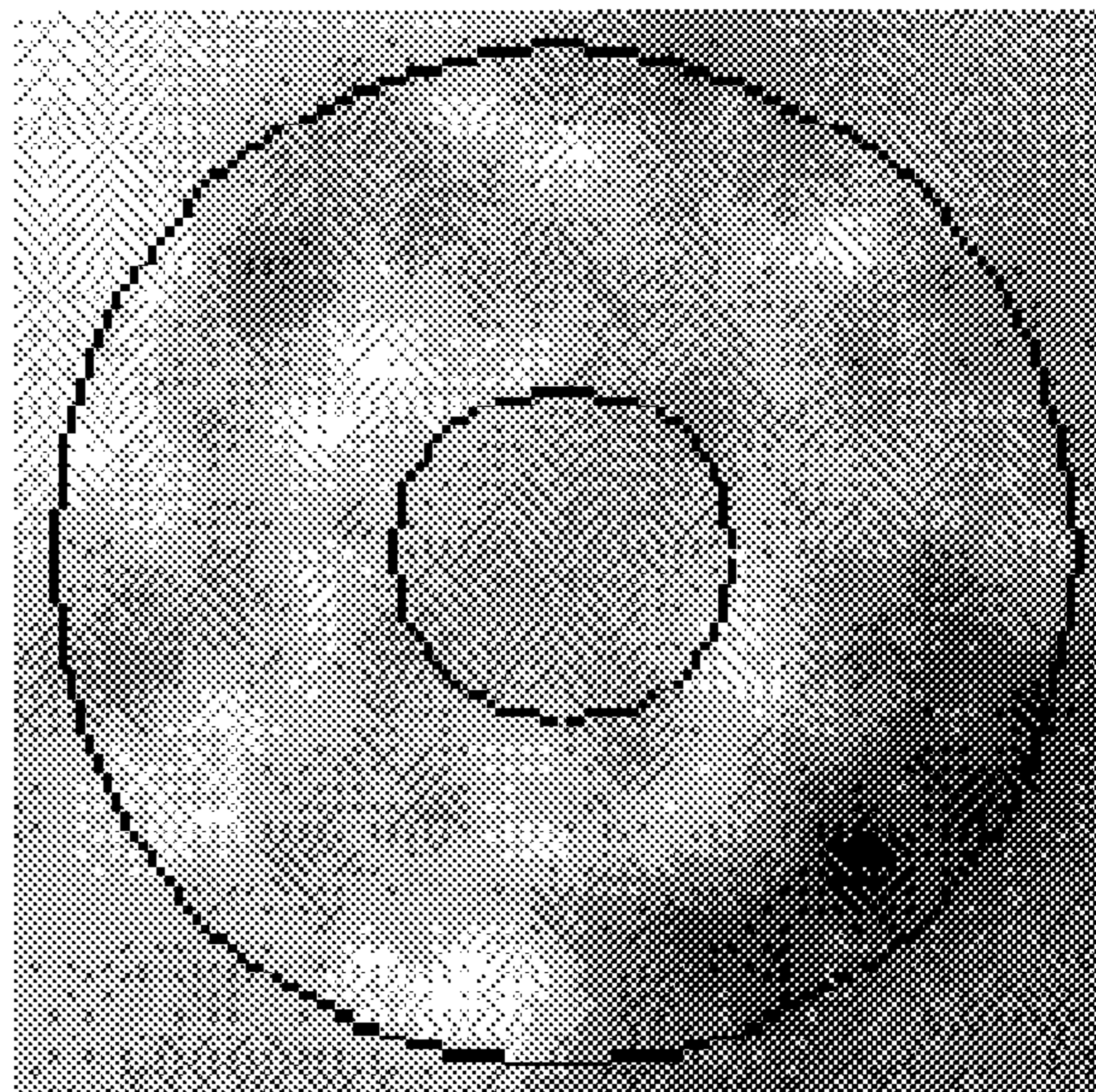


Fig. 4. Wiener deconvolution: reconstructed wave fronts using models of the atmosphere and the noise. The rms difference between the original and the reconstruction is also 0.77 waves. At 1000 photons per pixel (approximately a  $3.6^m$  star) both results were indistinguishable from the original.

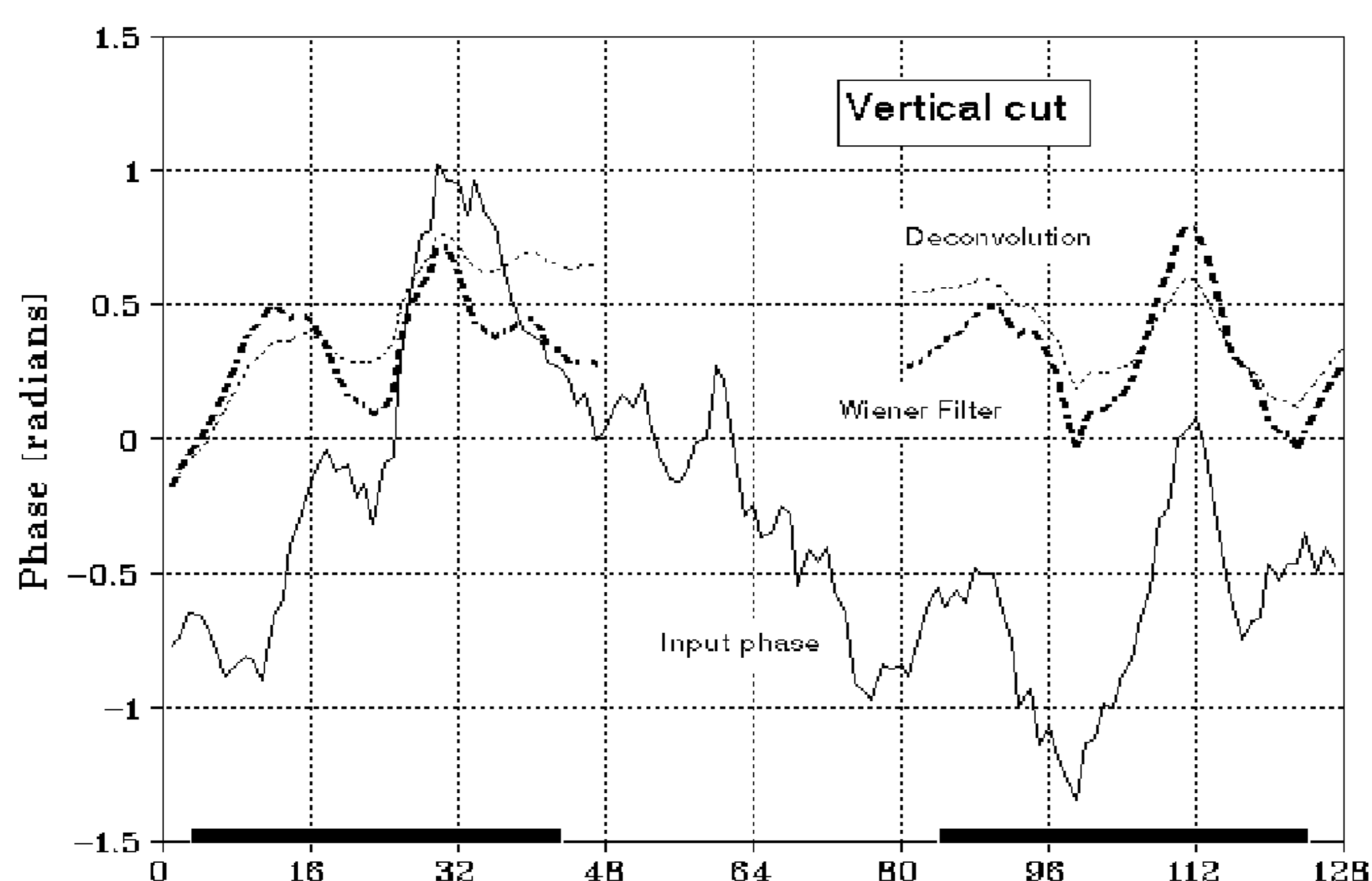


Fig. 5. Cuts across the centers of the input (Fig. 1) and resultant wave fronts using direct (Fig. 3) and Wiener deconvolutions (Fig. 4). The telescope is marked as two bars at the bottom.

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