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Superfluid stiffness renormalization and critical temperature enhancement in a composite superconductor

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The motivating question

How can we design a higher T_c superconductor ?

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What limits T_c in a superconductor ?

Two necessary ingredients for superconductivity

Pairing | **Phase Coherence**

Complex Order Parameter:

$$¢(r) = j¢(r)je^{i\mu(r)}$$

Amplitude: Pairing Gap

Pairing Temp.

$$T_p \approx \frac{1}{2}\Delta_0$$

Phase: Condensate Phase

Phase Ordering Temp.

$$T_\theta \approx \frac{1}{2}\frac{\hbar^2 n_s}{m^*}\xi^{d-2}$$

$$T_c \cdot \min[T_p; T_\mu]$$

The BCS superconducting transition

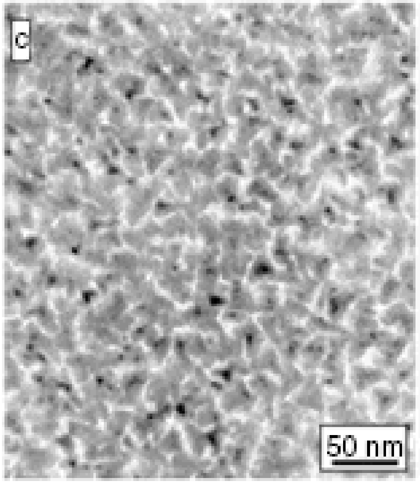
In conventional superconductors pairing precipitates order

$$T_\theta \gg T_p \approx T_c$$

Material	T_p[K]	T_θ[K]	T_c[K]
Pb	7.9	6×10^5	7.2
Nb_3Sn	18.7	2×10^4	17.8

Pairing and phase coherence may occur separately

Example: Granular superconductors

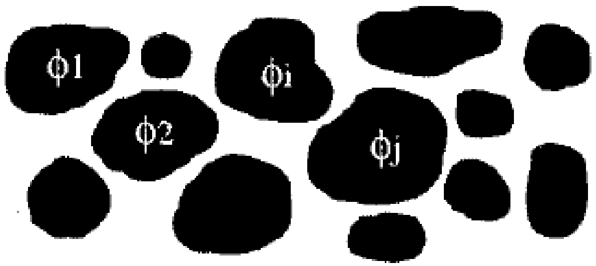


Granular niobium film

Pairing
is established on each grain at bulk T_c

The film's T_c is determined by inter-grain
Phase Ordering

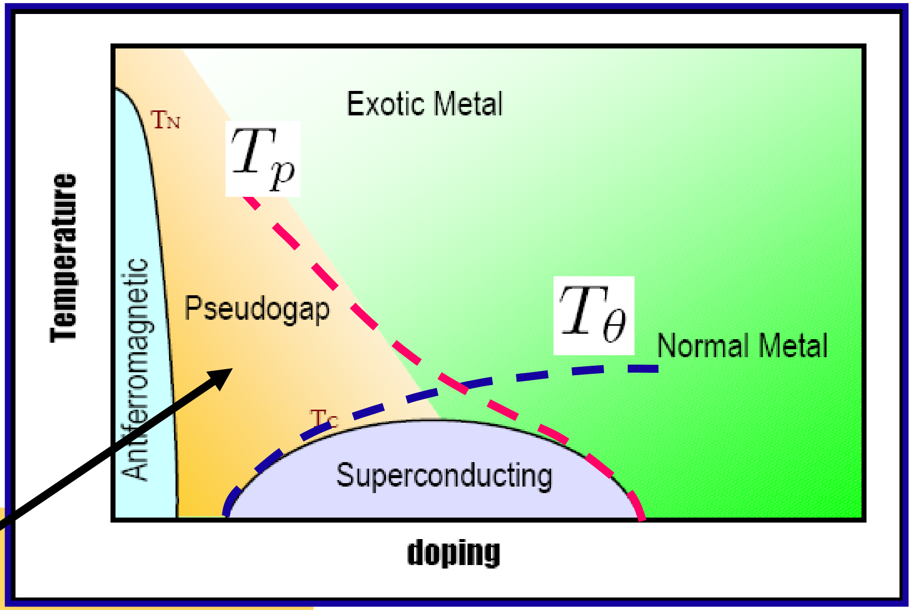
$$T_p > T_c \approx T_\theta$$



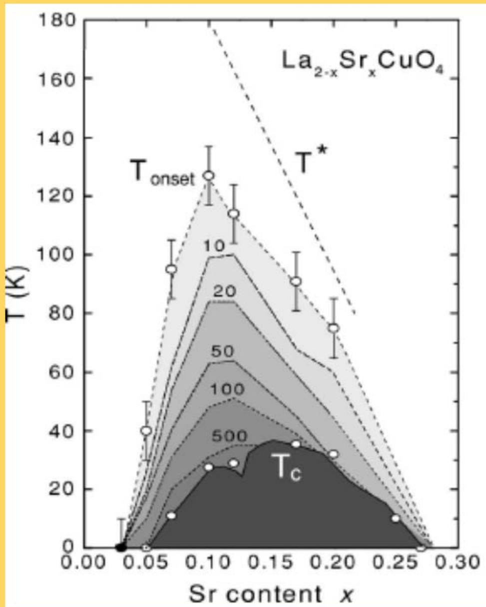
Pairing and phase ordering in the HTSC

$$T_p > T_{c,\max}$$

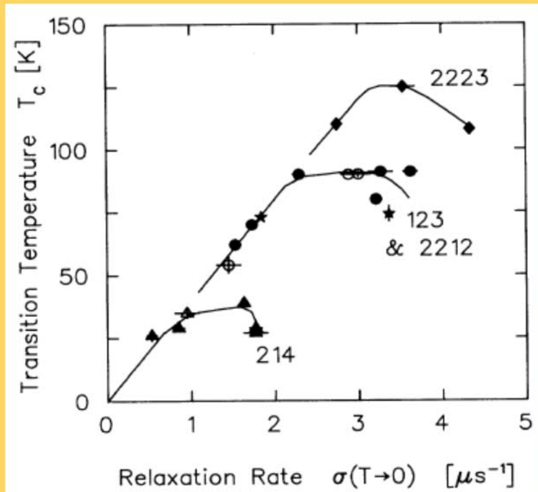
Material	T_p [K]	T_θ [K]	T_c [K]
LSCO (ud)	75	47	30
LSCO (op)	58	54	38
LSCO (od)	30	90	34



Emery and Kivelson (Nature 1995)



Wang, Li, Ong. (PRB 06)



Uemura *et al*. (PRL 89)

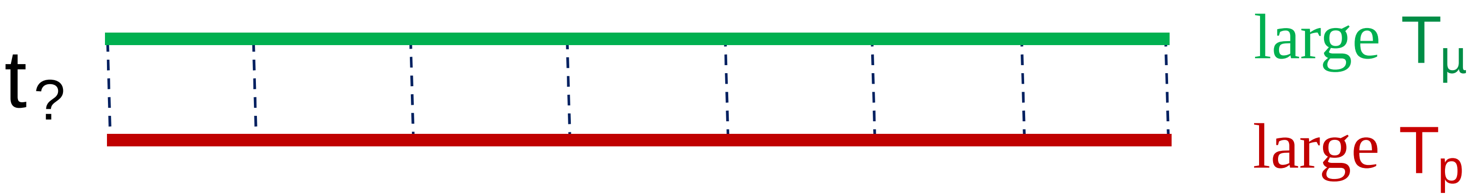
Statement of the problem

Given a system with a high pairing scale ¢$_0$ *but with* T_c *reduced by phase fluctuations, can one design a composite system in which*

$$T_c \rightarrow T_p = \Delta_0 / 2 ?$$

The basic idea

Couple the strong-pairing superconductor to a metal with a large phase stiffness



The superconductor's phase stiffness is enhanced via Josephson tunneling through the metal

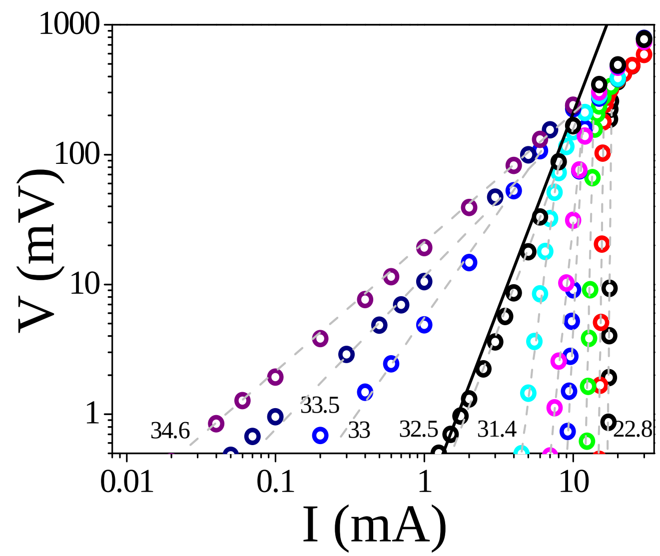
Pairing is suppressed in the superconductor owing to the same delocalizing events (proximity effect)

Best or worst of both worlds?

What is the optimal $t_?$? What is the optimal T_c ?

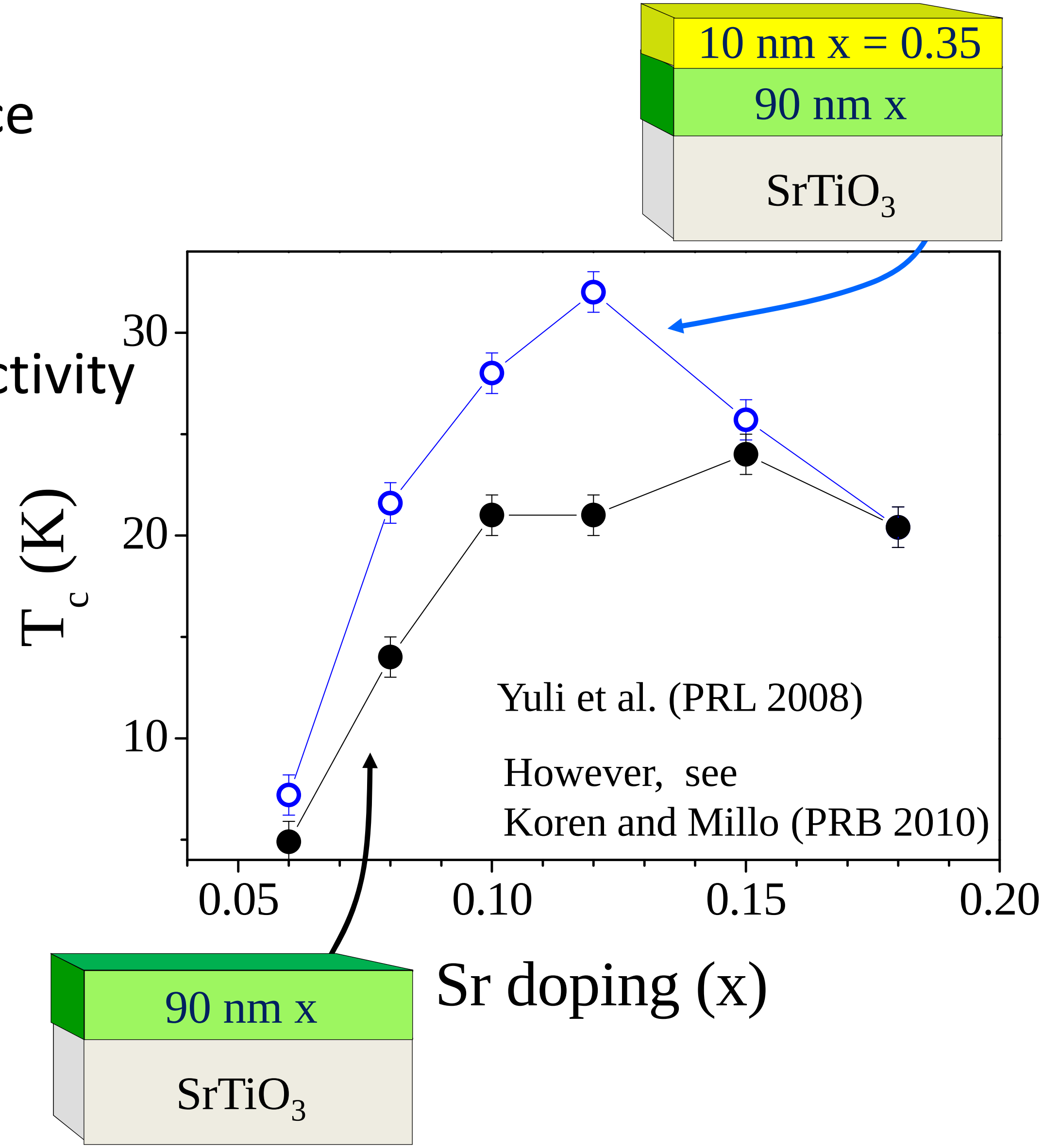
Realization in cuprate bilayers

- The enhancement takes place in the underdoped regime.
- No Meissner effect above bulk T_c: Surface superconductivity

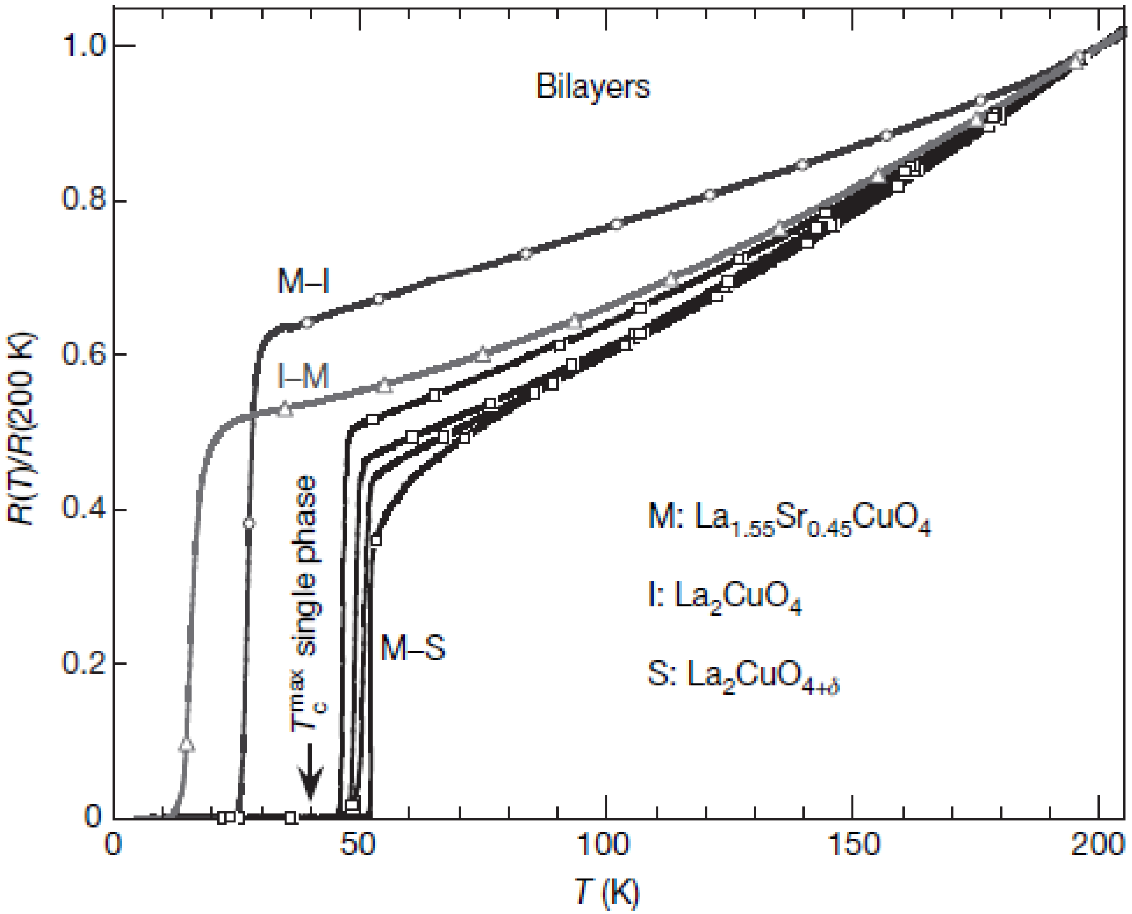


$V / I^{a(T)}$

$a(T_{BKT}) = 3$



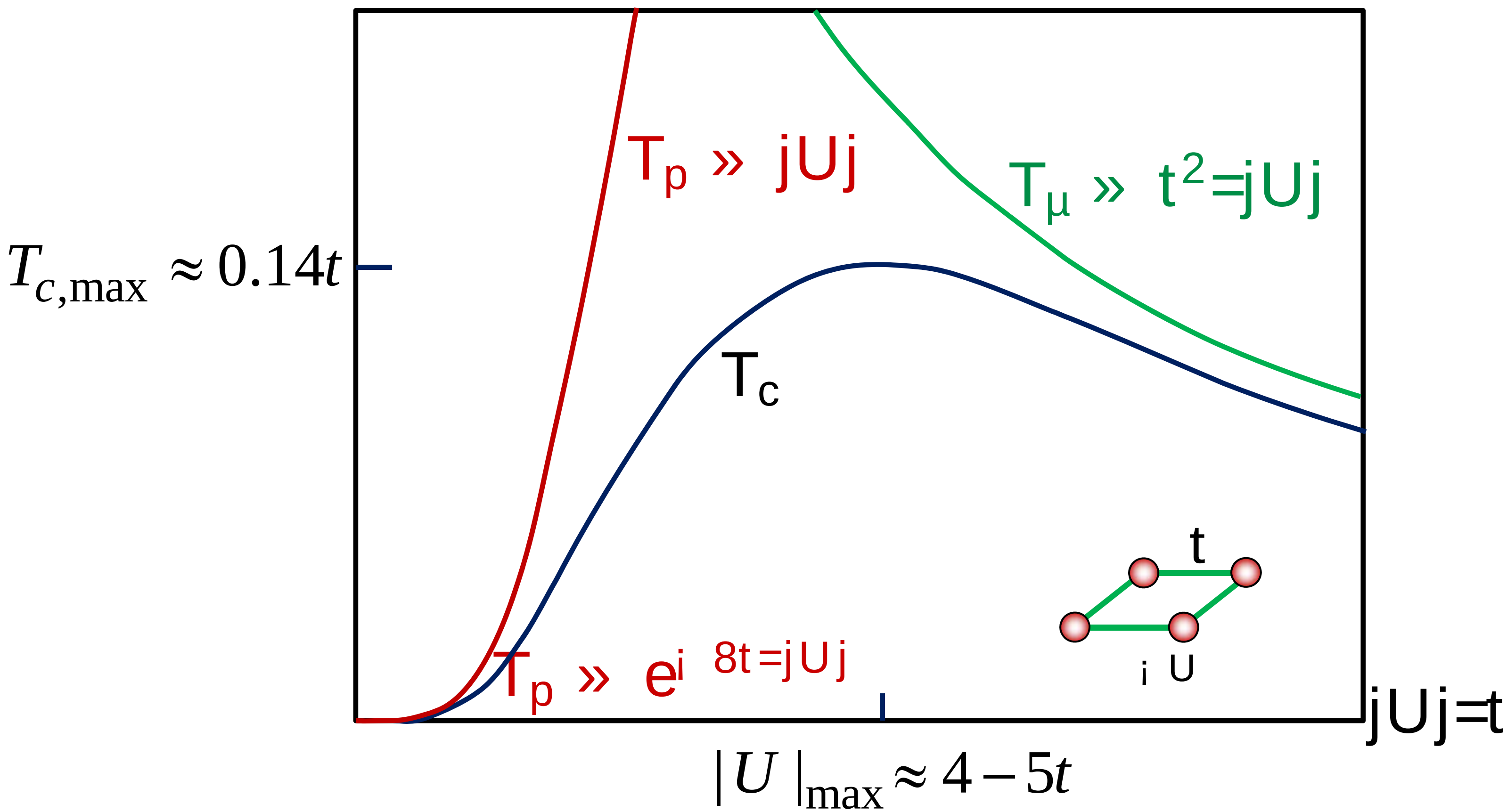
T_c Enhancement in metal-insulator bilayers



Role of charge transfer, strain?

Gozar et al. (Nature 2008)

The negative-U Hubbard model



BCS
Pairing limited

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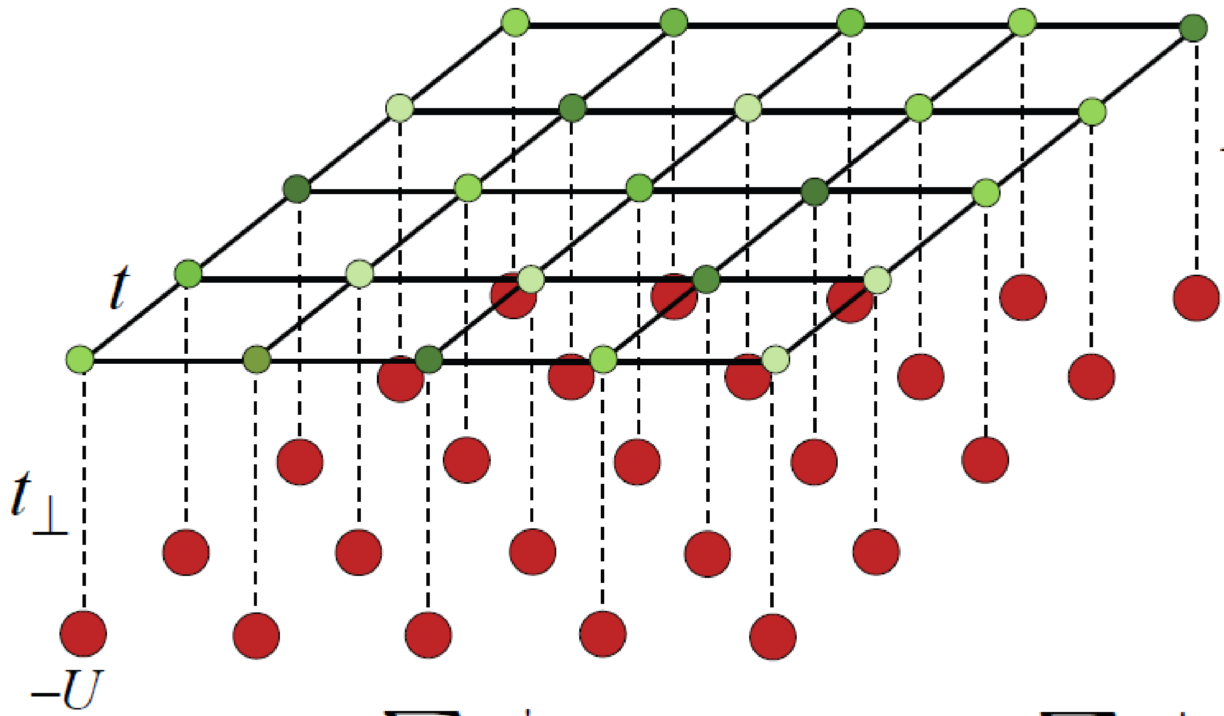
BEC
Phase-stiffness limited

A toy bilayer model

$$\sum_{i,\sigma} c^\dagger_{i\sigma} \left(\partial_\tau - \mu + \epsilon + V_i\right) c_{i\sigma} - t \sum_{\langle i,j\rangle,\sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.}\right)$$



$$- t_\perp \sum_{i,\sigma} \left(c^\dagger_{i\sigma} f_{i\sigma} + \text{H.c.}\right)$$

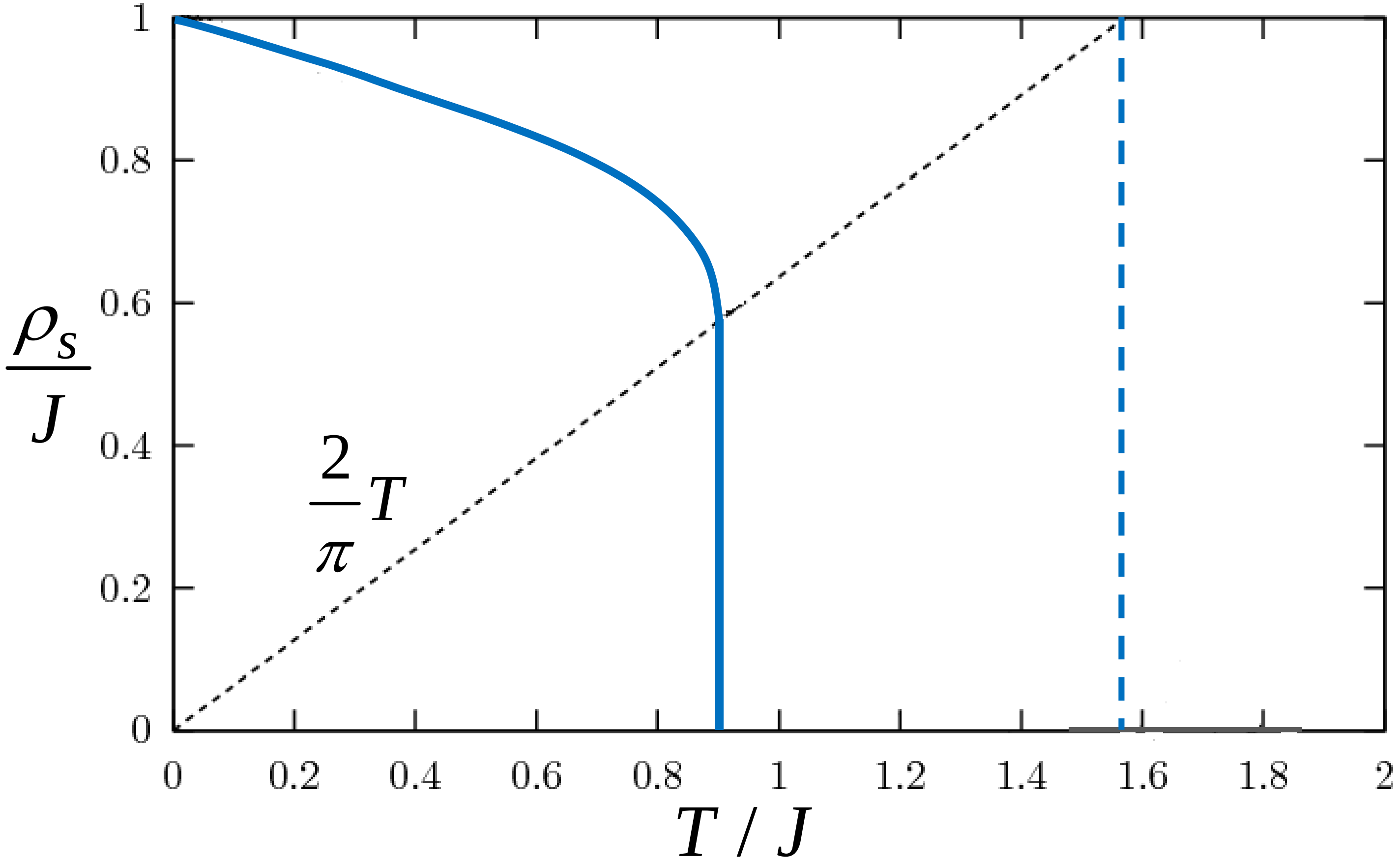
$$\sum_{i,\sigma} f^\dagger_{i\sigma} \left(\partial_\tau - \mu\right) f_{i\sigma} - U \sum_i f^\dagger_{i\uparrow} f_{i\uparrow} f^\dagger_{i\downarrow} f_{i\downarrow}$$

Berg, Orgad and Kivelson (PRB 2008)

Determining T_c from the phase stiffness

$$\rho_s = \frac{1}{L^2} \frac{\partial^2 F}{\partial \phi^2}\bigg|_{\phi=0}$$

$$H_{XY} = -J \sum_{\langle i,j \rangle} \cos\left[\theta_i - \theta_j + \left(x_i - x_j\right)\phi\right]$$



Mean field calculation

- Decouple the interaction:

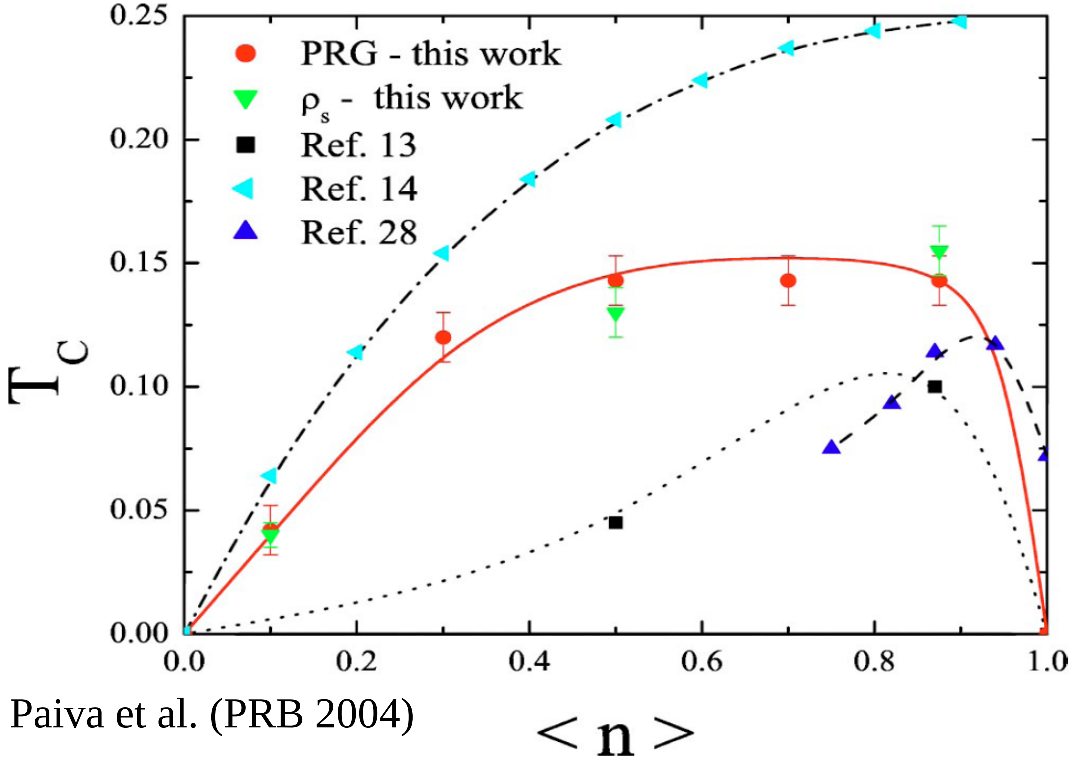
$$-U f_{i\uparrow}^\dagger f_{i\uparrow} f_{i\downarrow}^\dagger f_{i\downarrow} \to -\Delta_i^* f_{i\uparrow} f_{i\downarrow} - \Delta_i f_{i\downarrow}^\dagger f_{i\uparrow}^\dagger, \qquad \Delta_i = U\langle f_{i\uparrow} f_{i\downarrow}\rangle$$

- Solve the BdG equations in the presence of twisted boundary cond.

$$-t \sum_{\langle i,j\rangle,\sigma} c_{i\sigma}^\dagger c_{j\sigma} \to -t \sum_{\langle i,j\rangle,\sigma} e^{iq_x(x_i-x_j)/2a} c_{i\sigma}^\dagger c_{j\sigma}$$

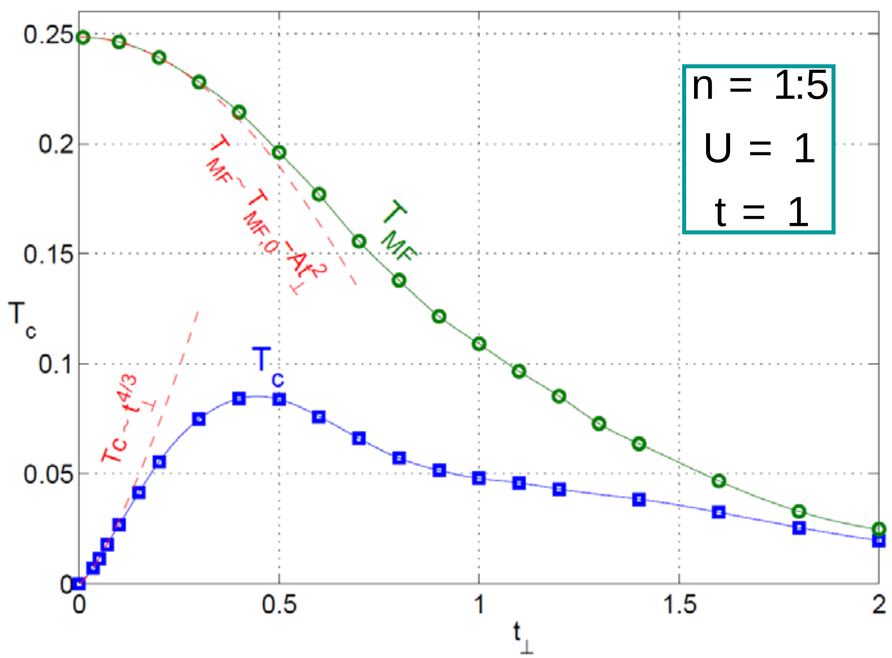
- Calculate $\rho_s^0(T) = \frac{1}{L^2} \frac{\partial^2 F}{\partial q_x^2}\Big|_{q_x=0}$

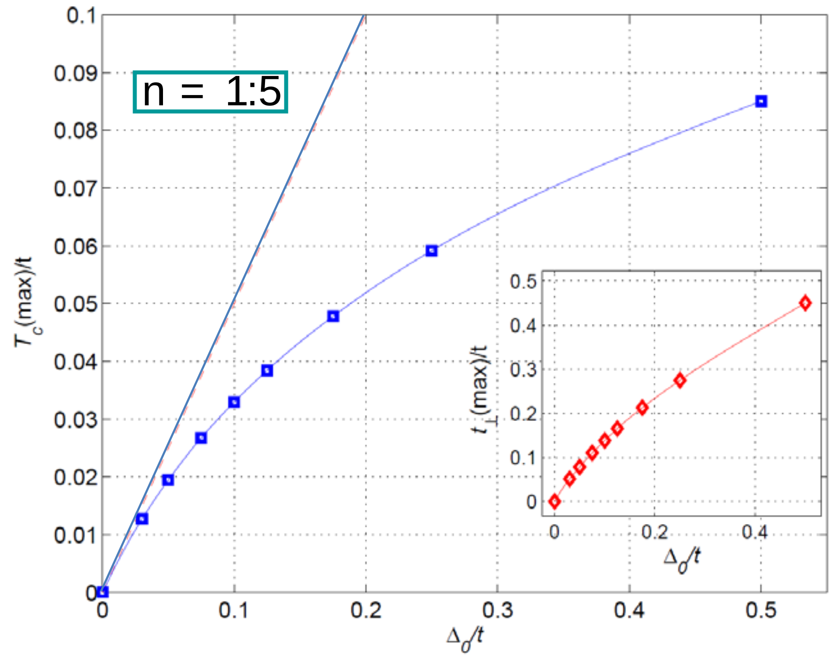
- Use it to determine T_c



Paiva et al. (PRB 2004)

Mean field results





- Competition between phase stiffness and the proximity effect
- T_c reaches a maximum for $t_?^{max}$ ¼ ¢ $_0$
- As t=¢ $_0$! 1 ; the maximal T_c approaches the full pairing scale:

$$T_c^{max} \; ! \; \frac{¢_0}{2}$$

Berg, Orgad and Kivelson (PRB 2008)

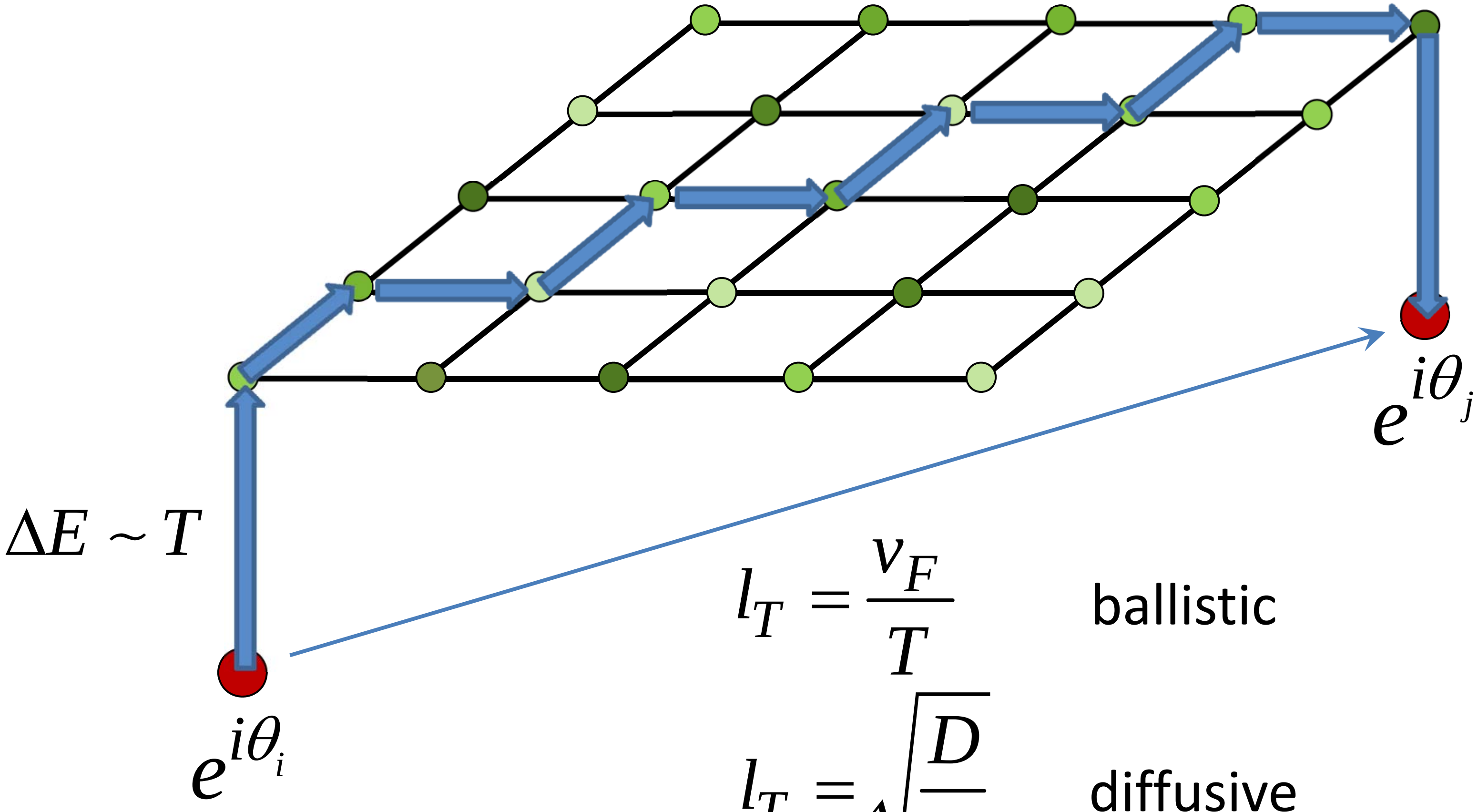
Effective phase action for small $t_\perp$

- Assume $t_\perp^2/t, T \ll \Delta_0 \Longrightarrow |\Delta| = \Delta_0 \approx U/2$
- Integrate out the fermions

$$S_\theta = \frac{1}{8\Delta_0}\int_0^\beta d\tau \sum_i (\partial_\tau \theta_i)^2$$
$$- \int_0^\beta d\tau d\tau' \sum_{i,j} K(r_{ij}, \tau - \tau') \cos[\theta_i(\tau) - \theta_j(\tau')]$$

Decays exponentially for r_{ij} larger than the thermal length and as $1/r_{ij}^2$ for shorter separations

Effective phase action for small $t_\perp$



$$l_T = \frac{v_F}{T}$$ ballistic

$$l_T = \sqrt{\frac{D}{T}}$$ diffusive

Thermal phase fluctuations

$$H = -\frac{J}{2}\sum_{i,j}\left(\frac{a}{r_{ij}}\right)^2 e^{-r_{ij}/\lambda}\cos(\theta_i - \theta_j)$$

$$J \sim t_\perp^4 N_F a^2/\Delta_0^2$$

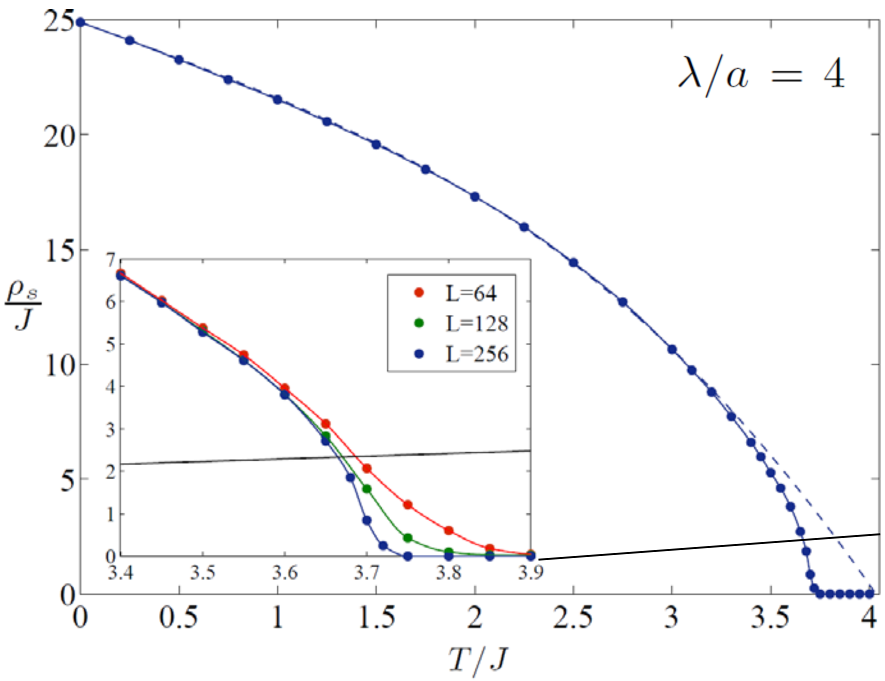
$$\lambda \sim l_T$$

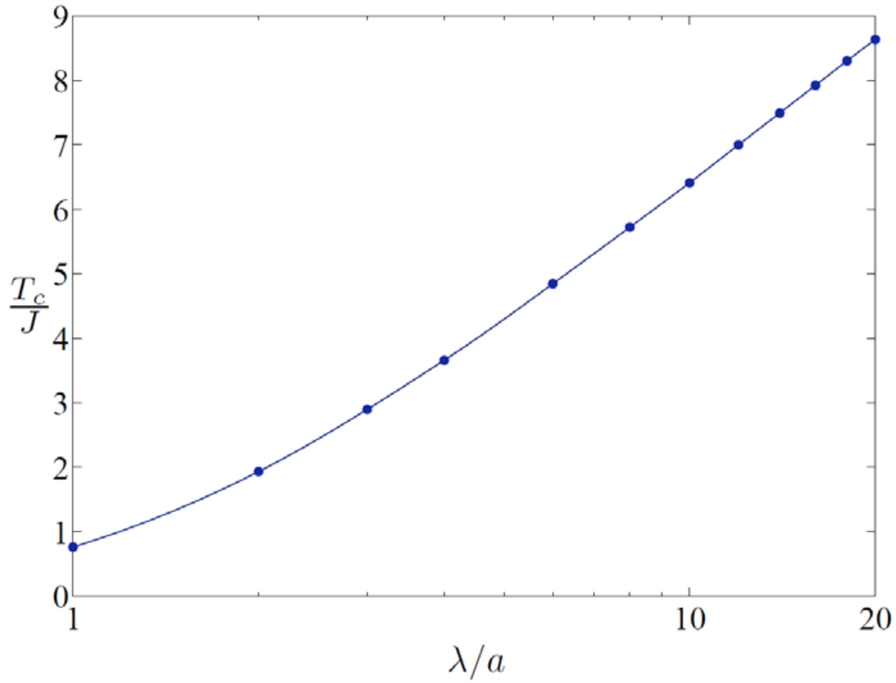
The bare phase stiffness $\rho_s^0 = \frac{\pi}{2}\left(\frac{\lambda}{a}\right)^2 J$

Using it in the BKT criterion $\rho_s(T_c) = \frac{2}{\pi}T_c$ gives $T_c \sim \left(\frac{\lambda}{a}\right)^2 J$

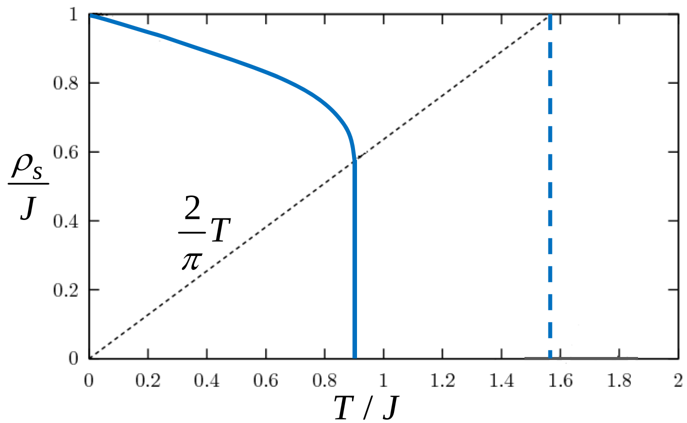
thus implying in the clean limit $T_c \sim t_\perp^{4/3}$

Thermal phase fluctuations



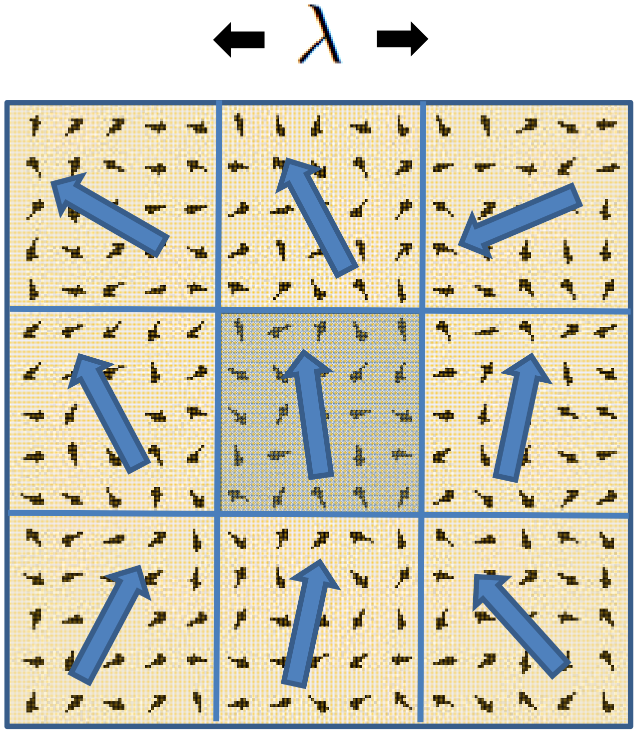


- ρ_s is massively renormalized
- $T_c \sim J \ln\left(\frac{\lambda}{a}\right)$



Wachtel, Bar-Yaakov and Orgad (PRB 2012)

A coarse grained model



$$\bar{H} = -\frac{\bar{J}}{2}\sum_I \mathbf{m}_I^2 - \bar{J}\sum_{\langle I,J\rangle} \mathbf{m}_I\cdot\mathbf{m}_J$$

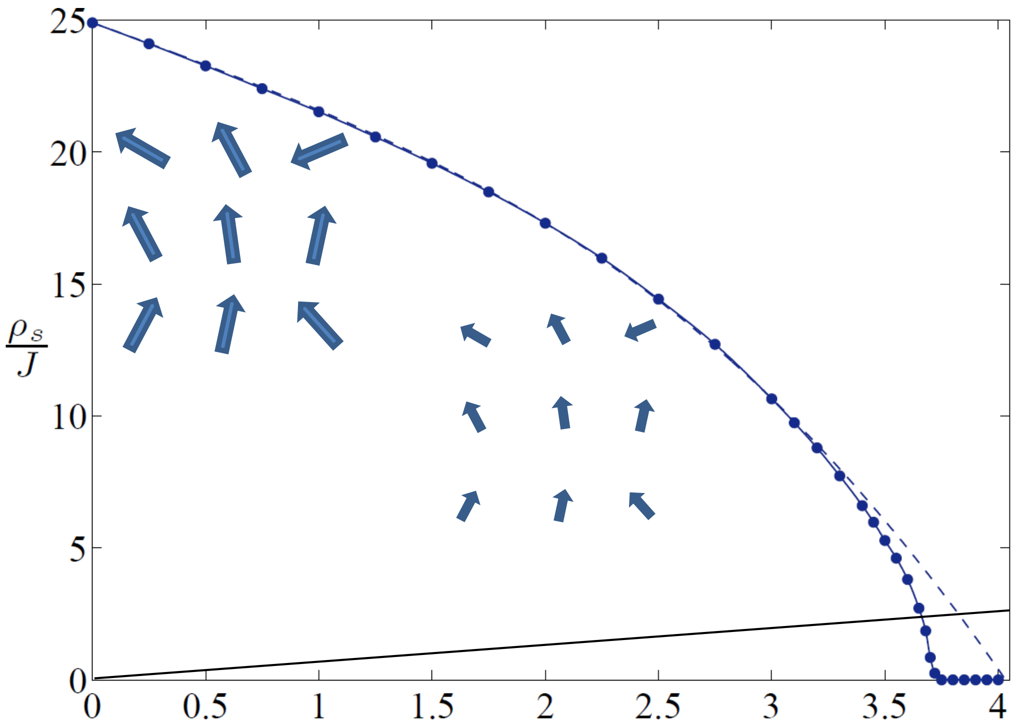
$$\mathbf{m}_I = \sum_i \mathbf{s}_i^I$$

$$\bar{J} \approx J\left(\frac{a}{\lambda}\right)^2 \int_a^\infty d^2r\,\frac{e^{-r/\lambda}}{r^2} \approx J\left(\frac{a}{\lambda}\right)^2 \ln\left(\frac{\lambda}{a}\right)$$

Inter-block mean field approximation:

$$\bar{H}_{MF} = -\frac{\bar{J}}{2}\mathbf{m}^2 - \bar{J}z\mathbf{M}\cdot\mathbf{m} \qquad \mathbf{M} = \langle \mathbf{m}\rangle_{\bar{H}_{MF}}$$

Thermal phase fluctuations



$$T_{c,MF} = \frac{1+z}{2}\left(\frac{\lambda}{a}\right)^2 \bar{J} \sim J\ln(\lambda/a)$$

- ρ_s is rapidly suppressed by fluctuations on scales smaller than λ
- As ρ_s approaches $(2/\pi)T$ vortex fluctuations on scales larger than λ drives the BKT transition.

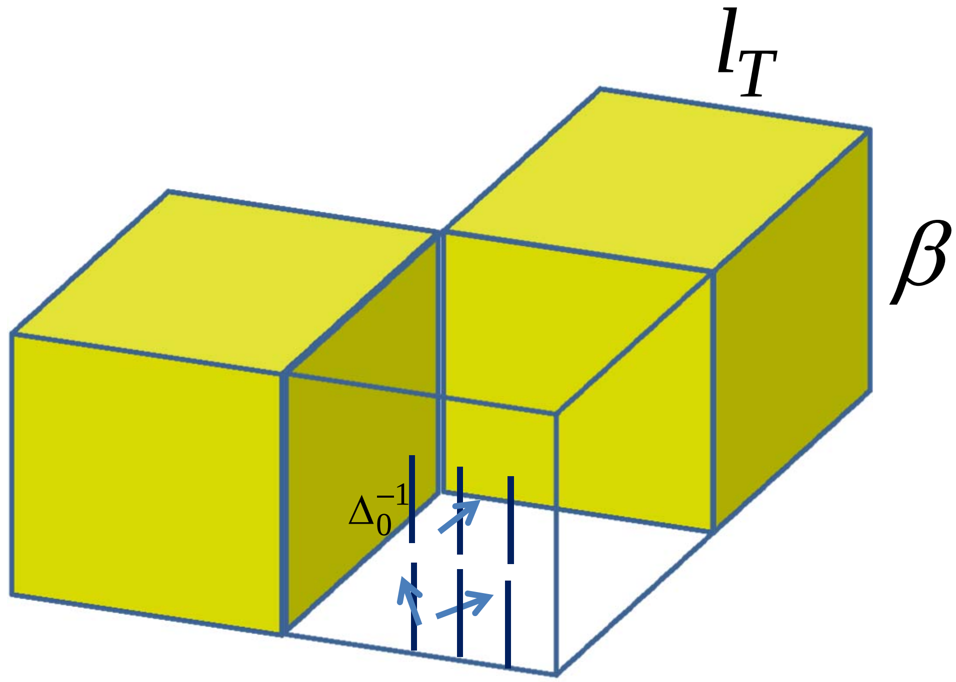
Quantum phase fluctuations

$$S_\theta = \frac{1}{8\Delta_0}\int_0^\beta d\tau \sum_i (\partial_\tau \theta_i)^2 - \int_0^\beta d\tau d\tau' \sum_{i,j} K(r_{ij}, \tau-\tau') \cos[\theta_i(\tau) - \theta_j(\tau')]$$

$$\langle e^{-i[\theta_i(\tau)-\theta_i(\tau')]}\rangle = e^{-2\Delta_0|\tau-\tau'|}$$

Decays as $1/(\tau-\tau')^2$

Coarse grain space-time:



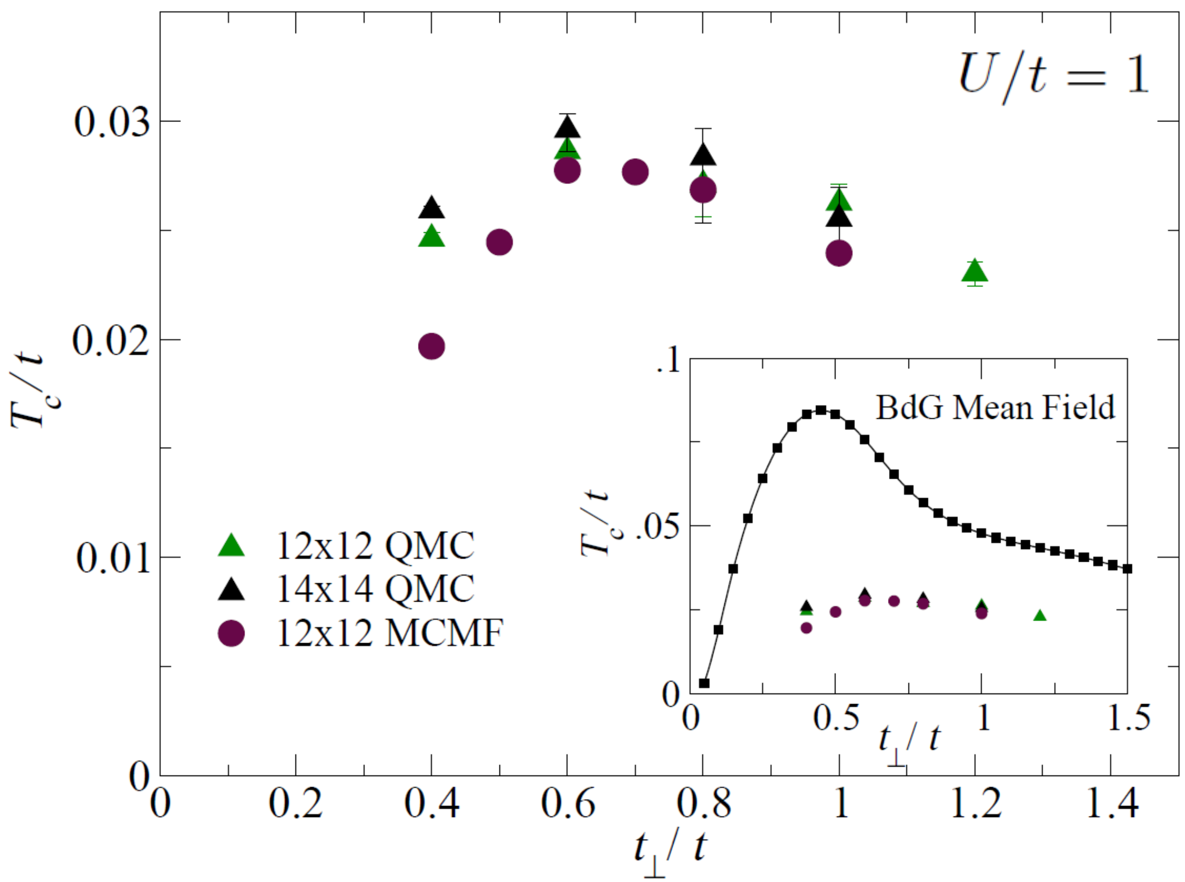
Quantum phase fluctuations

At small $t_\perp$:

$$T_c \sim \min(\Delta_0, \frac{v_F}{a}) \exp\left[-\frac{2\Delta_0^3}{(1+z)t_\perp^4 N_F a^2}\right]$$

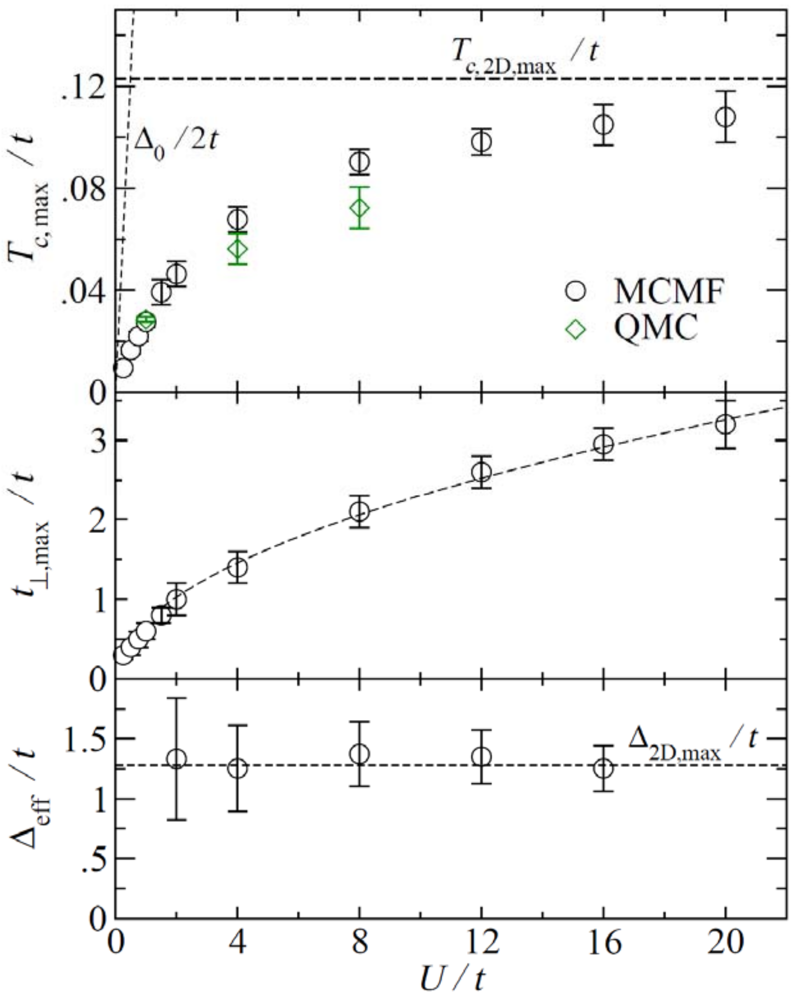
- Quantum phase fluctuations exponentially suppress T_c from its value based on thermal fluctuations only.
- Disorder has no effect on T_c

Intermediate $t_\perp$: numerical results



- No sign problem: "exact" Quantum Monte Carlo
- At lower T and larger U use Monte Carlo Mean Field method: includes thermal but not quantum fluctuations.

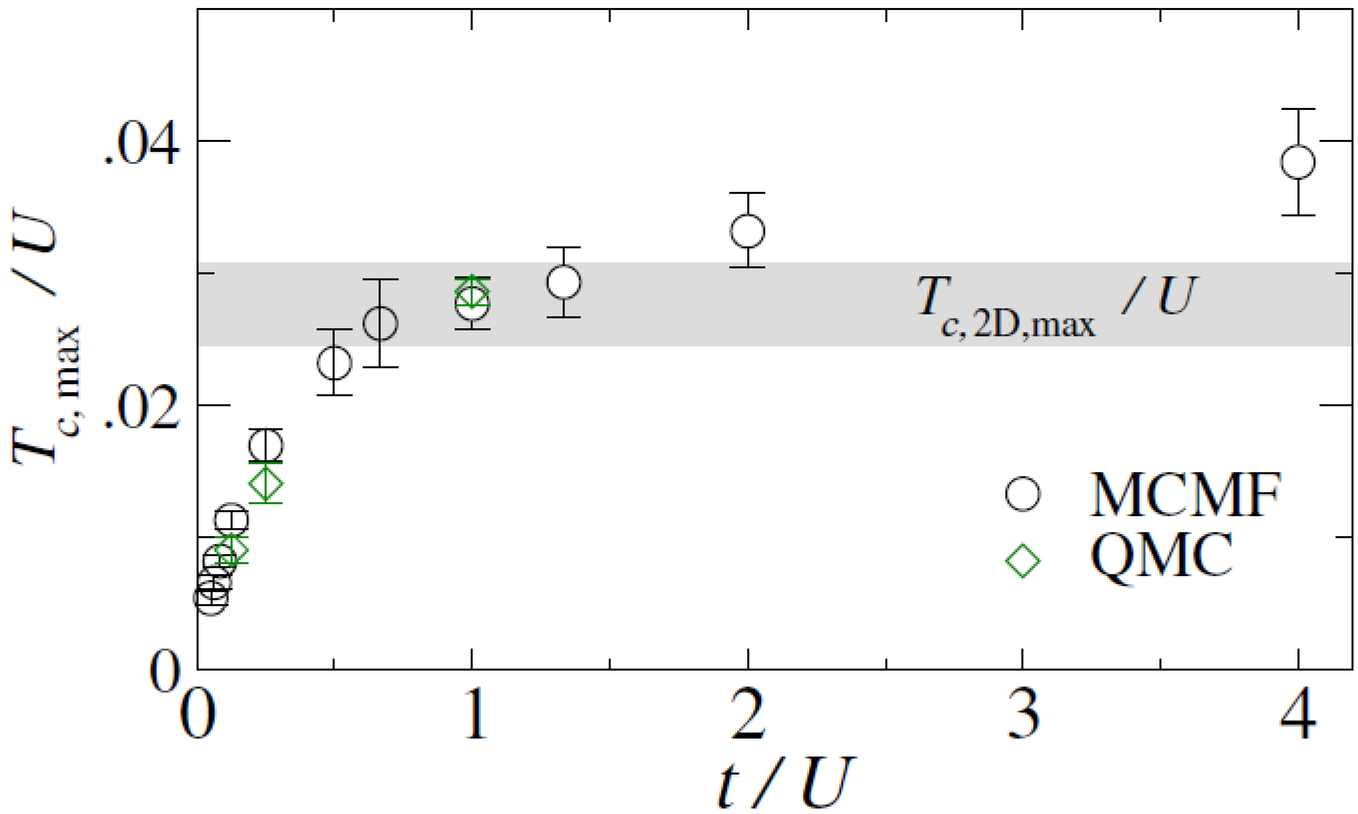
Intermediate $t_\perp$: numerical results



At large U/t near $T_{c,max}$:

- Amplitude fluctuations are negligible
- $T_{c,max}$ is obtained by adjusting t? to a point that maps the bilayer onto a single layer attractive Hubbard model with an optimal U/t ratio.

Intermediate $t_\perp$: numerical results



At large *t/U* :

- $T_{c,max}$ seems to exceed the maximal T_c of the 2d attractive Hubbard model towards the highest possible value $T_p=U/4$.

Conclusions

- Renormalization of the phase stiffness is important in systems with long range phase couplings.

- A system with a large pairing scale ¢ $_0$ but low phase stiffness can form a high-T_c superconductor when coupled to a metal provided that: 1. The coupling is optimal: $t_?^{max}$ ¼ ¢ $_0$
  2. Large metallic bandwidth: W À ¢ $_0$

- The induced phase couplings decay as r^{-d}.
  A thin metallic coating is therefore better than a thick one.

- Disorder has little effect on T_c. Imperfect interface can actually increase the interlayer coupling and benefit the enhancement.