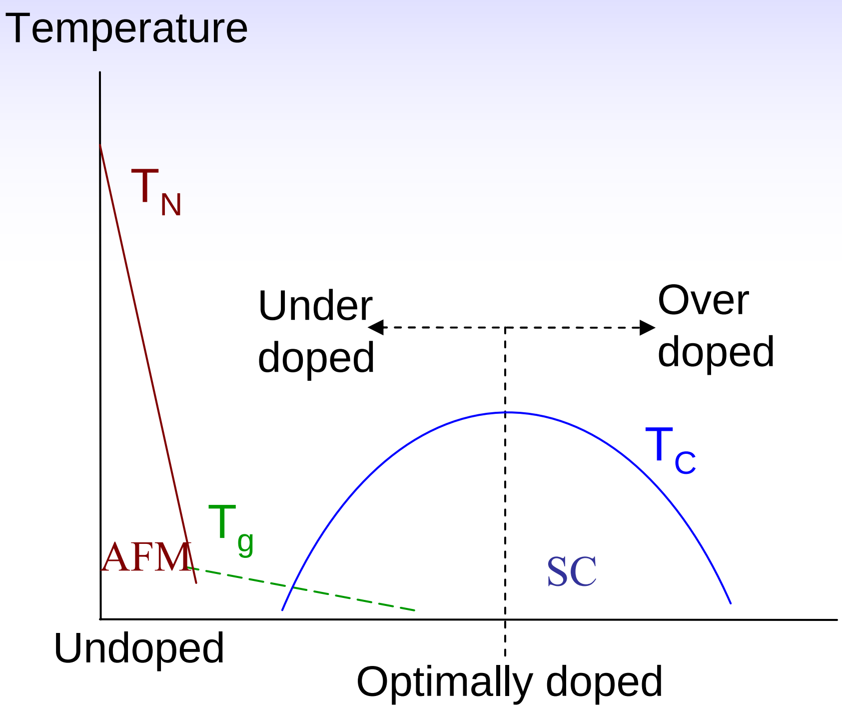
# Correlation Between the Critical Temperatures of Cuprates Superconductors and their Magnetic Interactions

Rinat Assa

Amit Keren

# High Temperature Superconductors




Schematic phase diagram

- The cuprates high $T_C$ superconductors are ceramic compounds all having $CuO_2$ planes.
- At low doping levels the origin of the magnetism lies in the $CuO_2$ planes.
- Superconductivity also occurs in the planes.

# Previous work

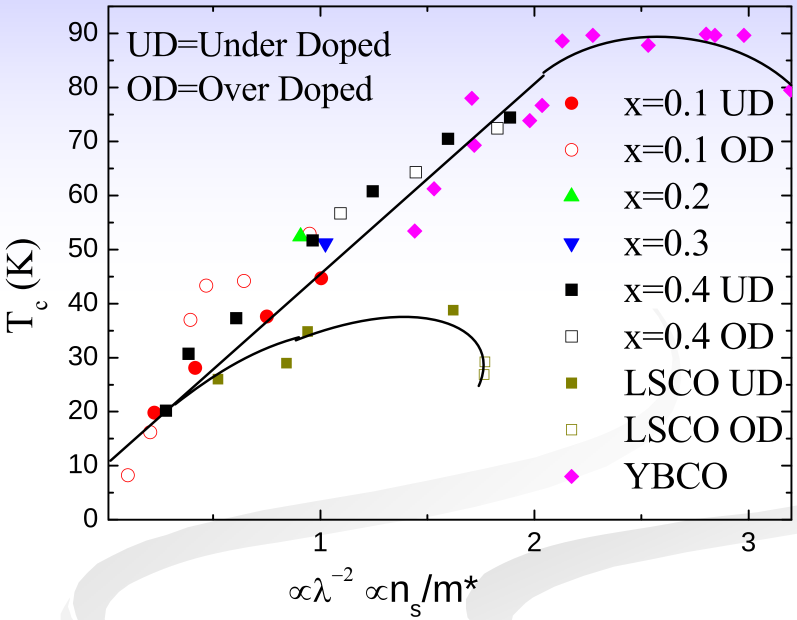
For all underdoped cuprates:

- Uemura:

$$T_C \propto \frac{1}{\lambda^2} \propto \frac{n_S}{m^*}$$

- Kanigel et al:

$$T_C = J_f^s \cdot n_S$$




$J_f^s$ is a magnetic energy scale per family in the SC state.

A. Kanigel et al, Phys. Rev. Lett **88**,137003 (2002).

# Motivation

- Is $J_f^s$ unique for each family?

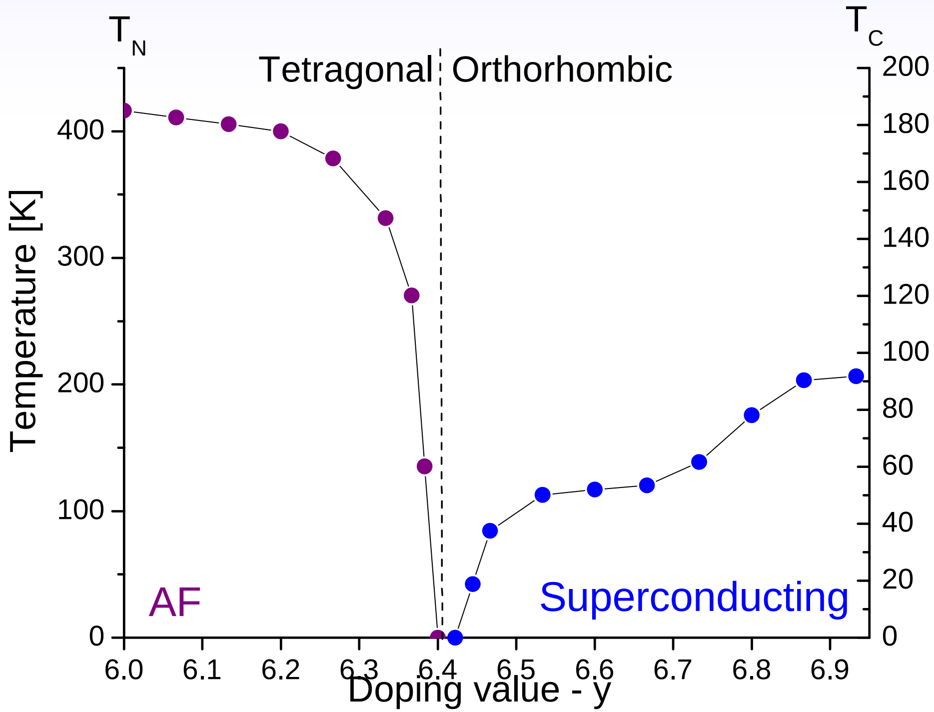
Showing $T_C \propto n_s$ , namely, $m^*$ or $J^s{}_f$ = const, by performing NQR measurements on YBCO.

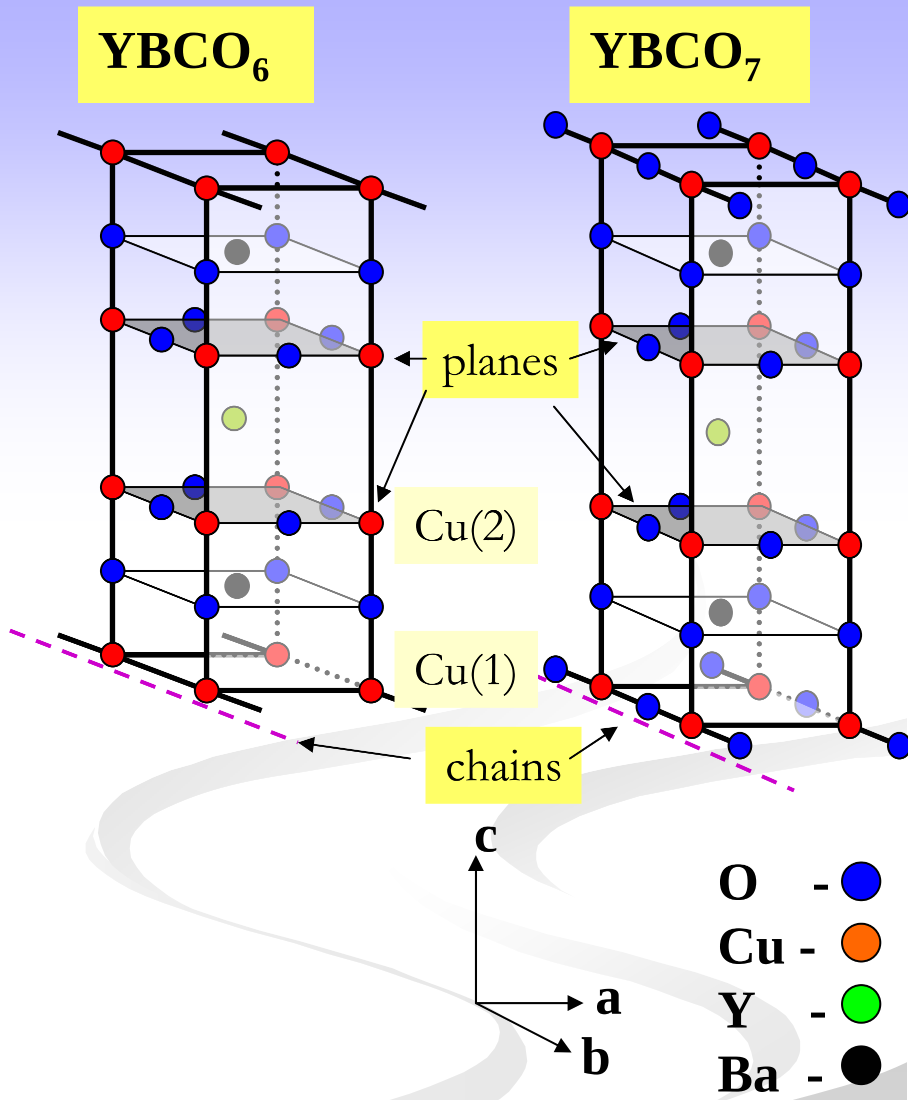
- Is $T_C{}^{max}$ correlated with the Néel temperature $T_N$ of the parent antiferromagnetic compound?

μSR measurements on the CLBLCO family.

# $Y_1Ba_2Cu_3O_y$

Phase diagram



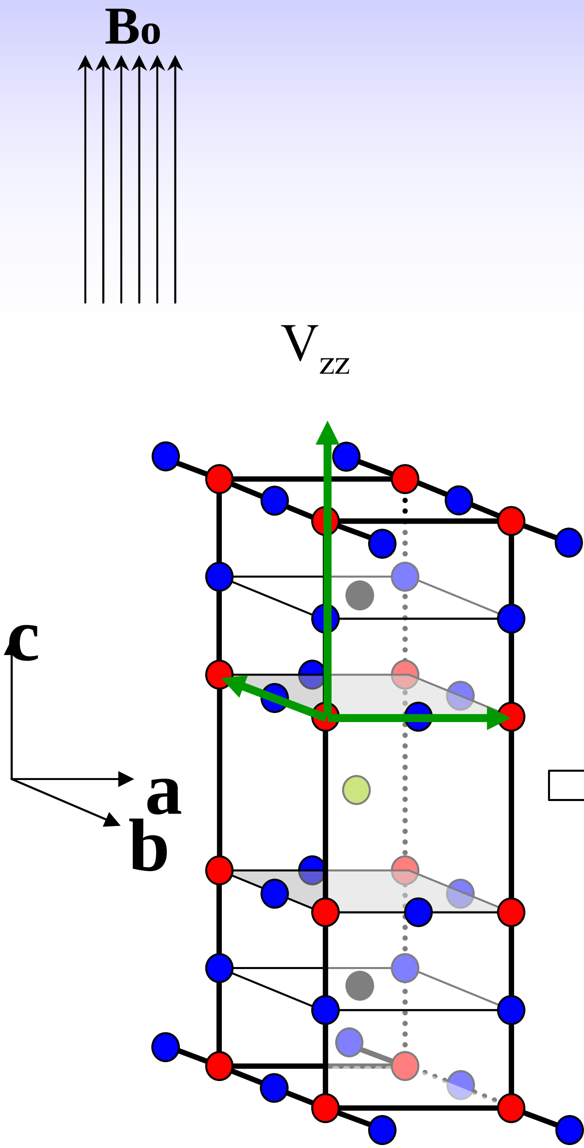




Unit cell

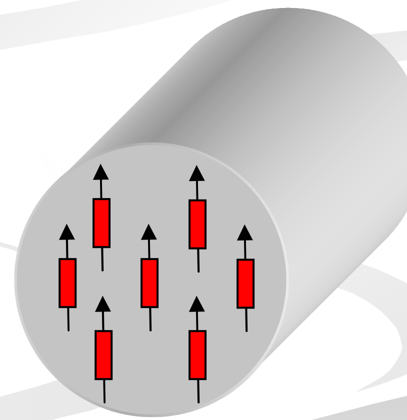
These samples are unique in that they contain a single $Cu^{63}$ isotope and not two.

# Orientation of the YBCO powder

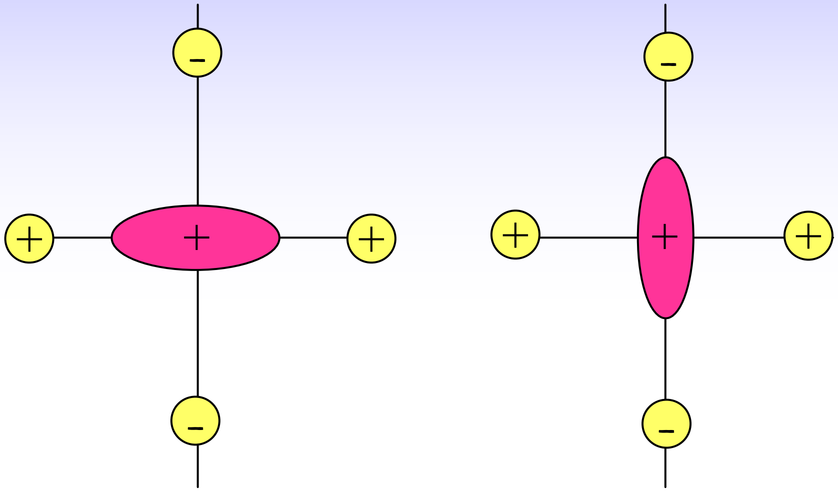
- For better signal intensities the YBCO powder was oriented.




**$B_o \parallel c \parallel V_{zz} \equiv z$**



# Nuclear Quadruple Resonance (NQR)



From the environment:

The EFG (Electric Field Gradient):

$$V(r) \Rightarrow \frac{\partial V}{\partial x_i \partial x_j} \equiv V_{ij} \qquad i, j = x, y, z$$

From the nucleus:

$I_x, I_y, I_z$ - The spin of the nucleus

- To obtain the NQR Hamiltonian we must couple V(r) and I .
- Every spin Hamiltonian contains even powers of I.
- The simplest coupling to write is $$H \propto I_i \frac{\partial V}{\partial x_i \partial x_j} I_j$$

# The NQR Hamiltonian

Choosing directions wisely leaves: $V_{xx}$, $V_{yy}$, and $V_{zz}$.

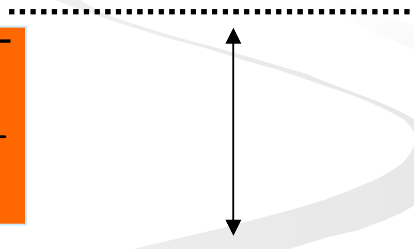
Since $V_{xx}+V_{yy}+V_{zz}=0$ (Laplace)
only two parameters remain:

$$\nu_q \propto V_{zz} \qquad \eta = \frac{V_{xx}-V_{yy}}{V_{zz}}$$

$$\hat{H}_q = \frac{\hbar \nu_q}{6}\left[3\hat{I}_z^2 - \hat{I}^2 + \eta(\hat{I}_x^2 - \hat{I}_y^2)\right]$$

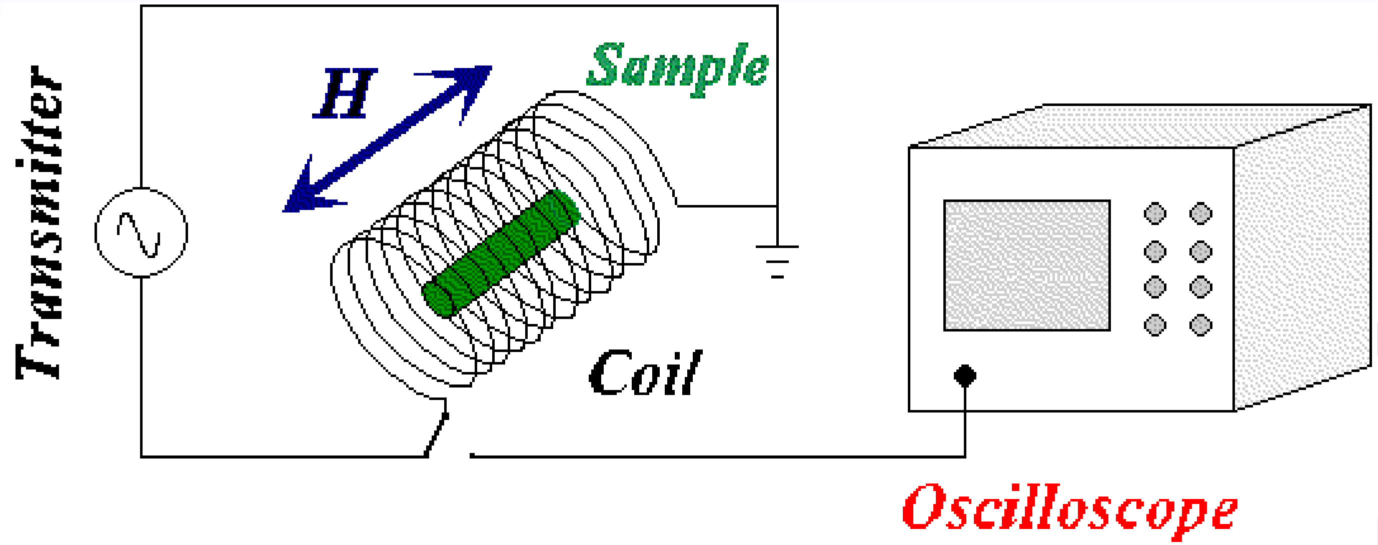
For a spin 3/2 nucleus:

$$f_{NQR} = \hbar \nu_q \sqrt{1+\frac{\eta^2}{3}}$$

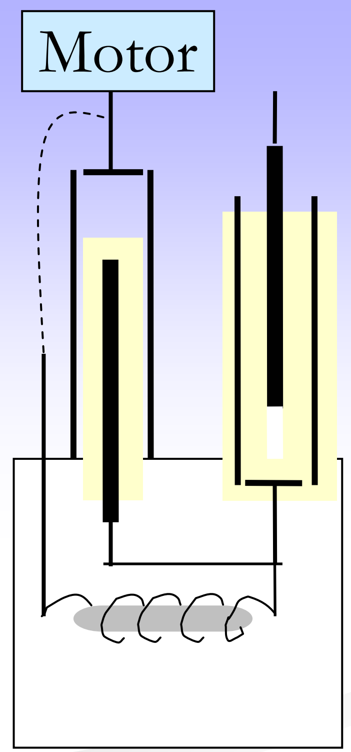


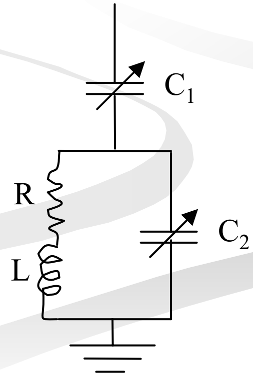
$f_{NQR}$ is determined by the EFG.
A change in $f_{NQR}$ indicates a change in the surrounding charges.

# Technical aspects of NQR

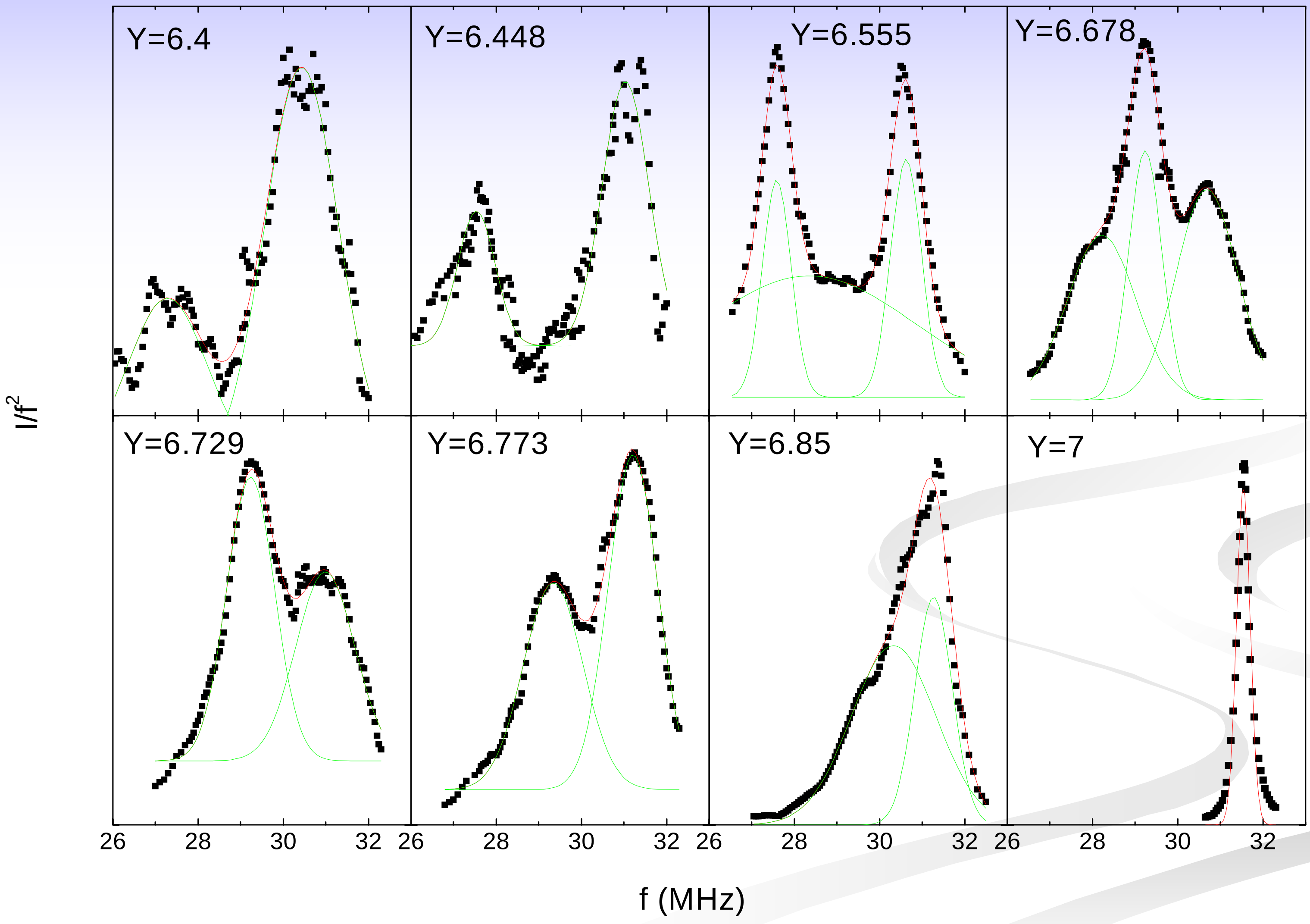


- The impedance and the resonance frequency of the circuit can be changed by the capacitors.
- One of the capacitors is automatically controlled by a motor.
- Our NQR system does an automatic frequency sweep in a wide range of frequencies (~3MHz), while delivering 1KW of power.

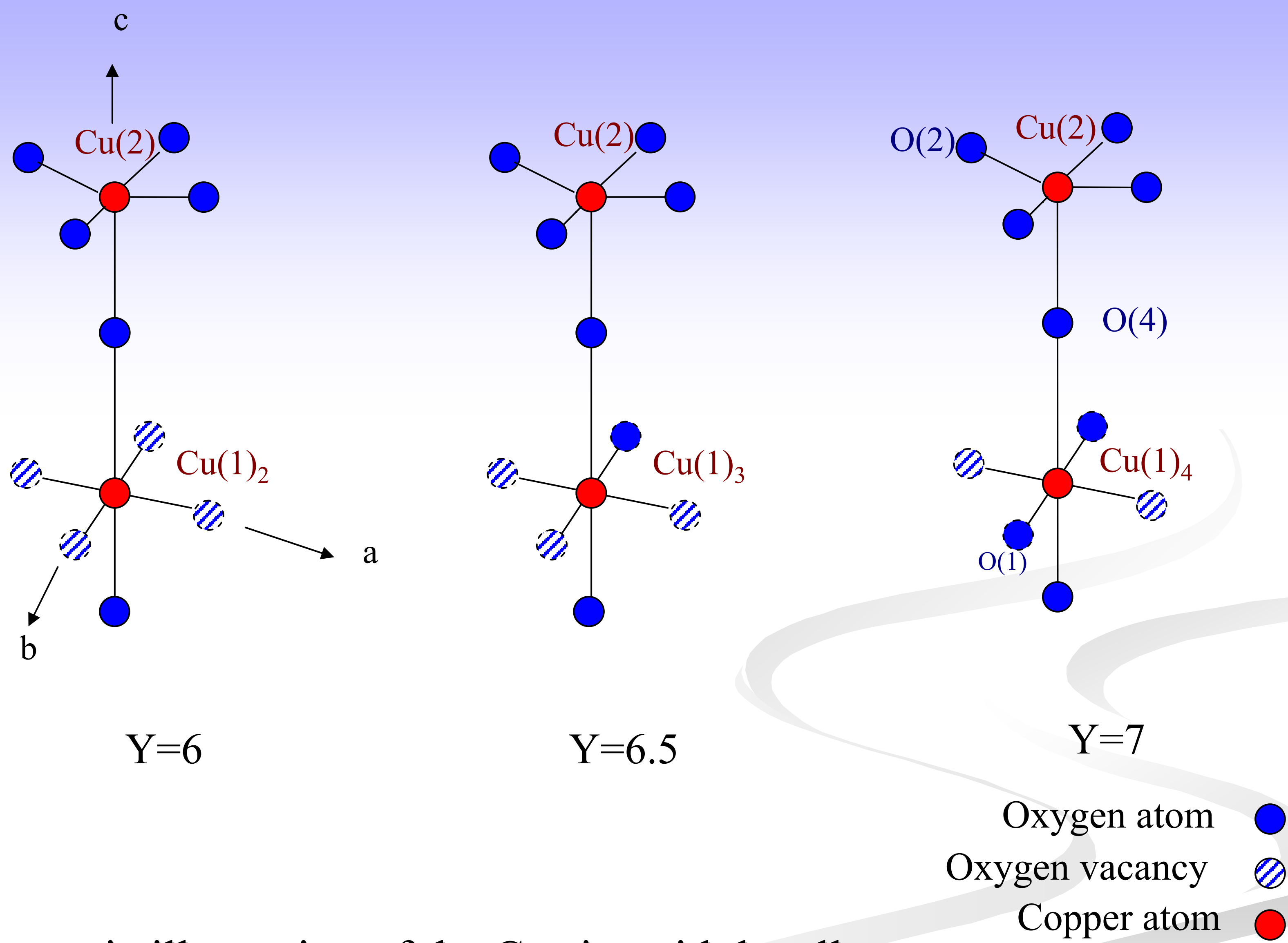


# NQR Raw data



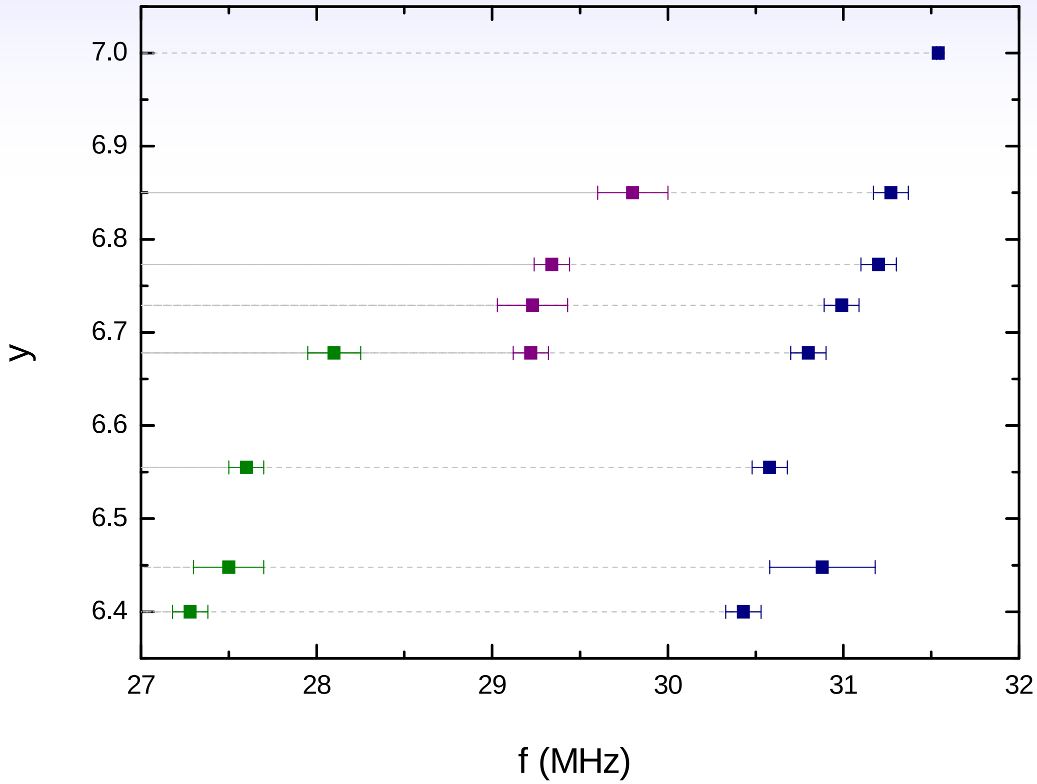

One can see the importance of enrichment, without it lines would overlap.




Schematic illustration of the Cu site with locally different oxygen coordinations

$Y_1Ba_2Cu_3O_y$

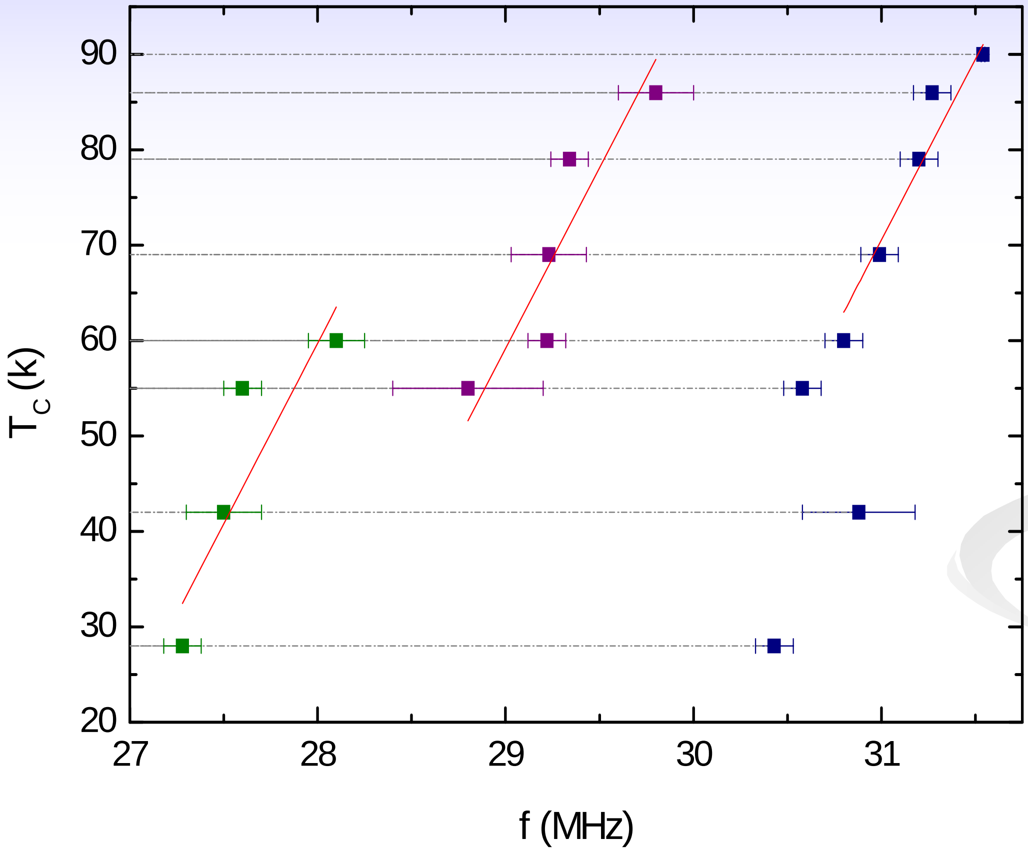
# NQR Results




3 different Cu(2) lines, for the 3 different ionic environments:

- 31MHz: Cu(2) with full chain.
- 29MHz: Cu(2) with chain half full, Cu(1) with corrdination 3.
- 27.5MHz: conducting Cu(2) with empty chains.

# NQR Results




When plotting $T_C$ vs. the NQR frequency → three lines with a common slope

$dT_C/df$=38±6 (K/MHz)

This is the first Uemura plot obtained not through penetration depth measurements. It allows the separation of $J$ ($m^*$) from $n_s$.

Following Haase et al: J. Haase et al, Phys. Rev. B **69**, 094504 (2004).

- **3 main contributions to the EFG of the Cu(2) nucleus:**

I. The 3d hole concentration on the copper ion.

II. There is a virtual hopping of electrons from neighboring oxygen ions to unoccupied 4d orbitals of Cu.

III. The electric field of distant ions.

→ NQR frequency

$$f \propto A \cdot n_d - B \cdot \beta^2 (8 - 4n_p) + C$$

$n_d$ - is the number of Cu holes

$(8\text{-}4n_p)$ - is the number of electrons in the neighboring oxygen

$\beta^2$ - is the probability to jump to the Cu 4d orbital

$C$ - is related to the contribution of distant ions.

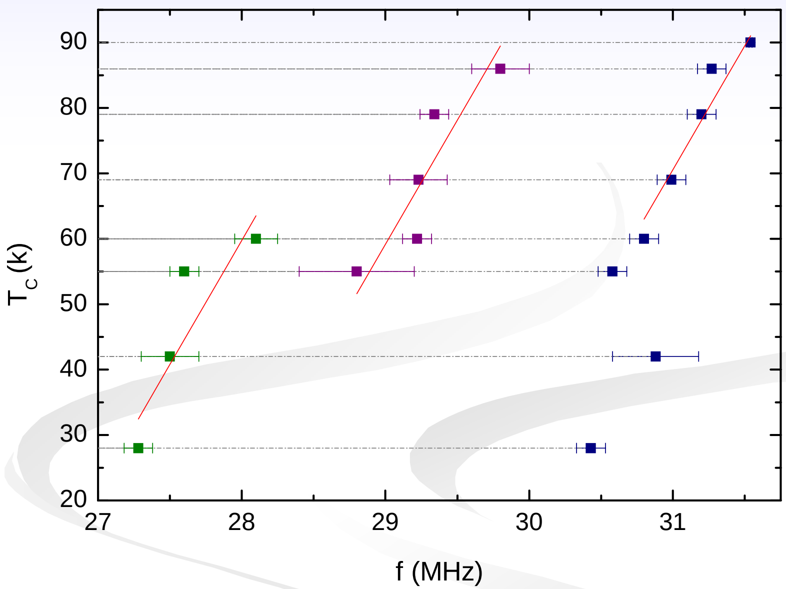
- **It is widely accepted that doping increases the hole content only for the oxygen in the planes ($n_d$ is a constant)**

# Conclusions of the first part

According to Haase et al: $\Delta f \propto$ number of holes in the plane $\propto n_s$

From our results, for each ionic environment:

$$\Delta f \propto T_C$$




Therefore:

$$T_C \propto n_S$$

From the Uemura relation:

$$T_C = J_f \cdot n_S$$

**Our data for the YBCO family is consistent with a $J_f$ independent of the oxygen doping.**

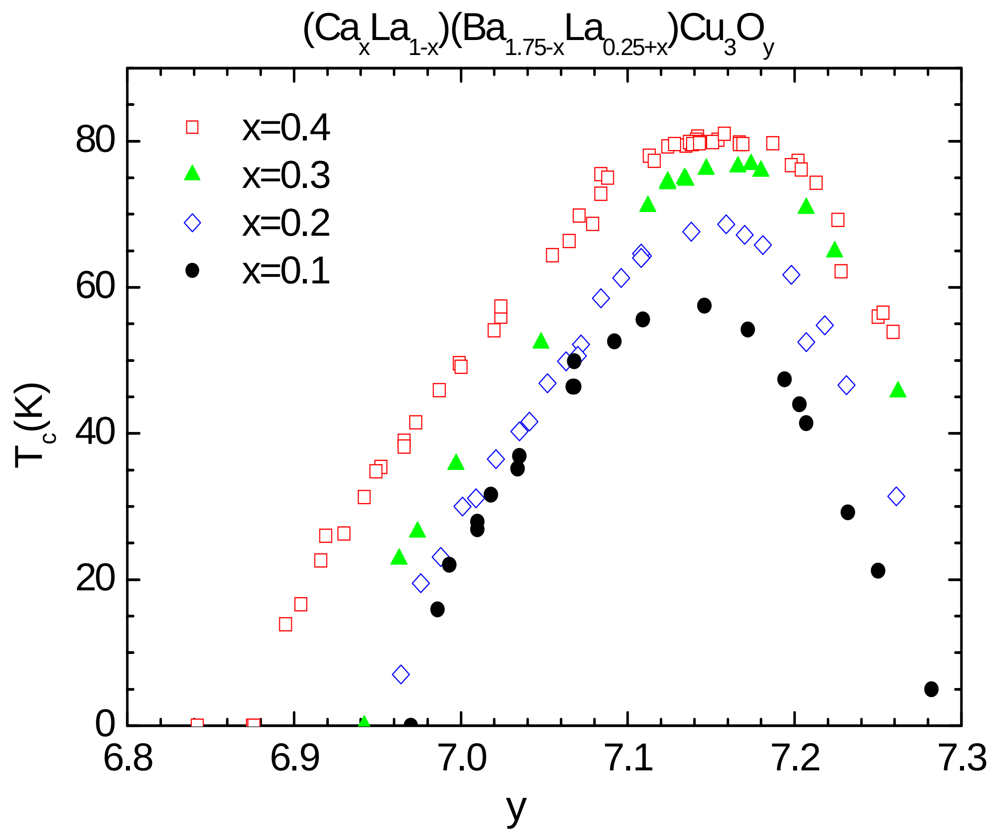
# Motivation

Is $T_C^{max}$ correlated with the Néel temperature $T_N$ of the parent antiferromagnetic compound?

μSR measurements on the CLBLCO family

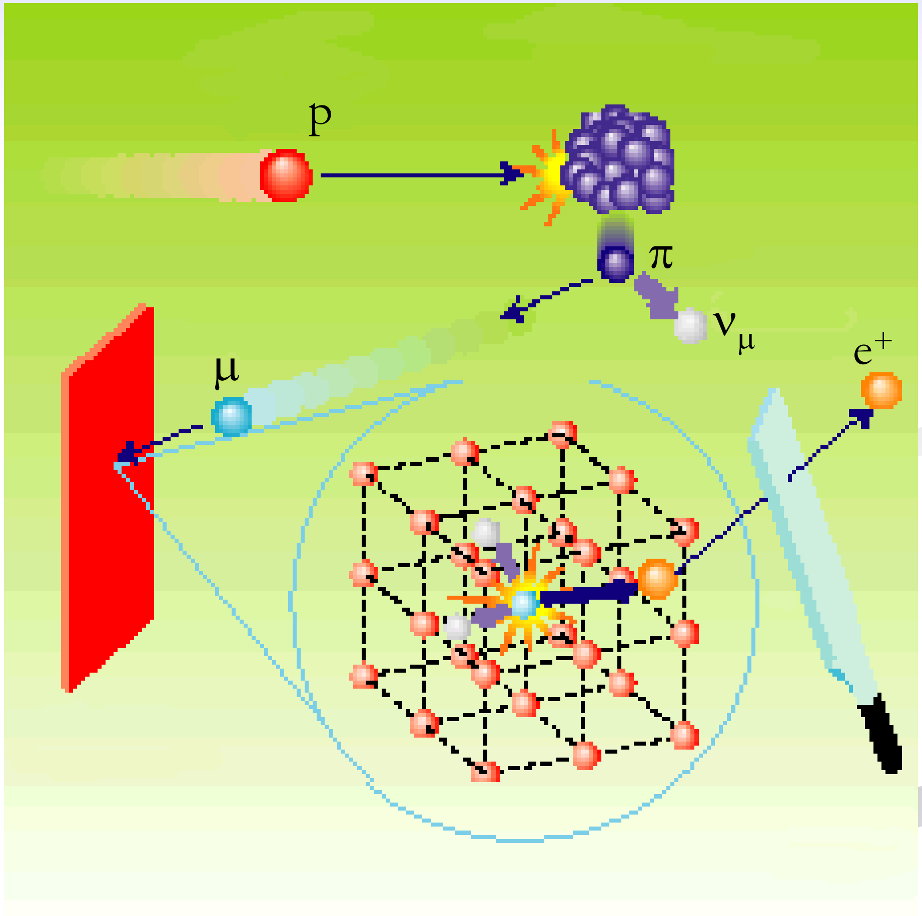
# The CLBLCO family

- Tetragonal - no ordered Cu-O chains.
- Stable throughout all parabolic $T_C$ curves.
- allows $T_C$ to be kept constant and other parameters to be varied, with minimal structural changes.



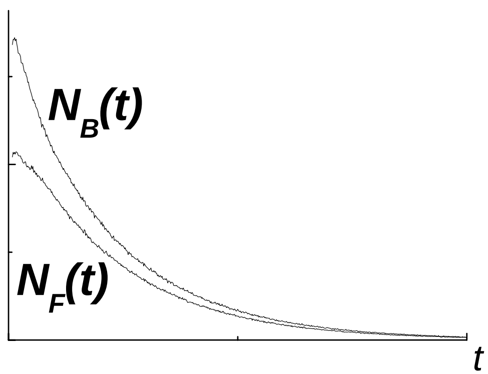
# μSR

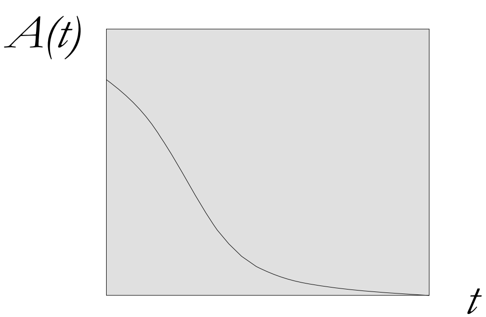
- 100% spin polarized muons
- Muon life time : 2.2μsec
- Positron emitted preferentially in the muon spin direction

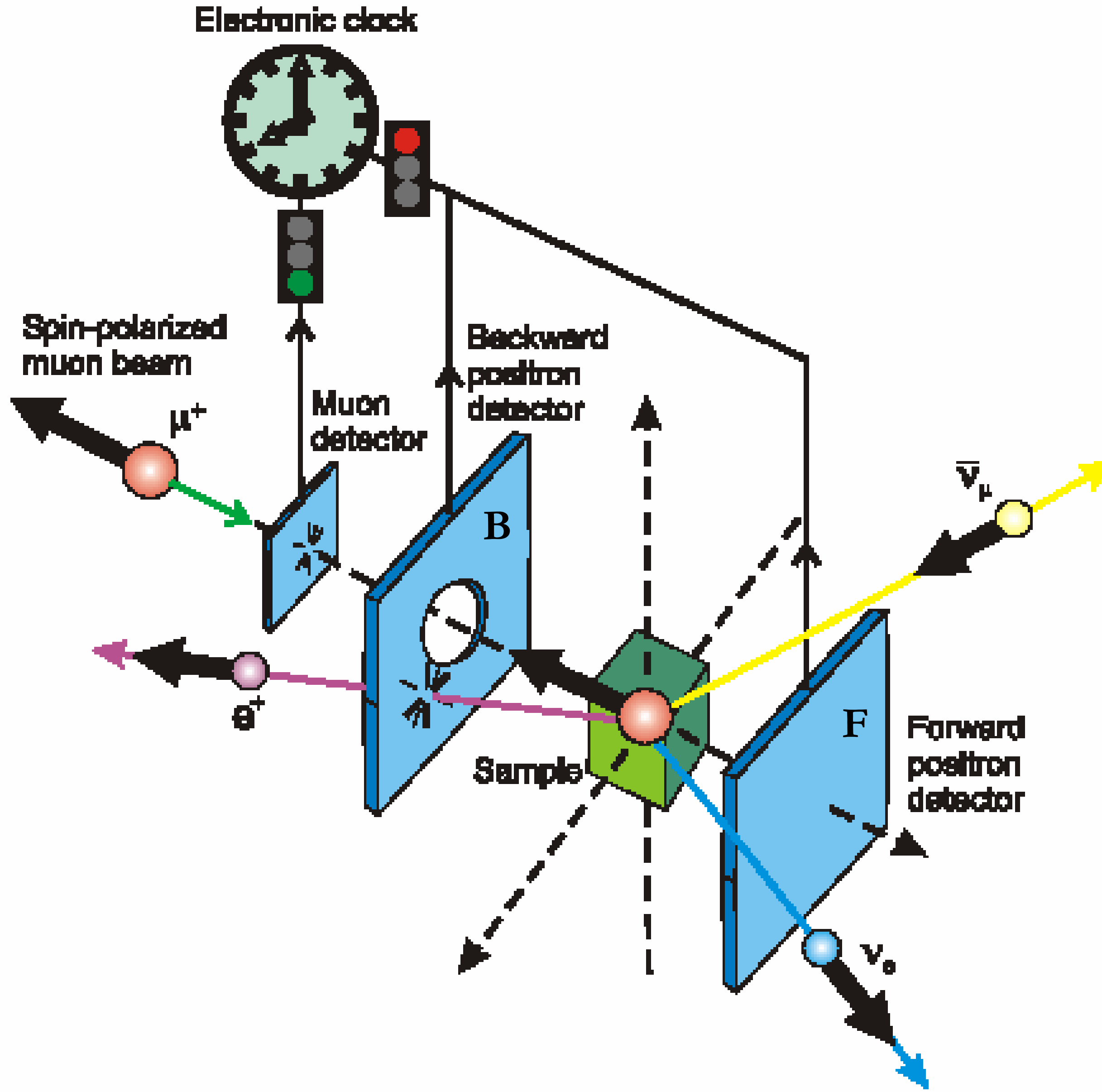


# μSR

$$A(t) = \frac{N_B(t) - N_F(t)}{N_B(t) + N_F(t)}$$



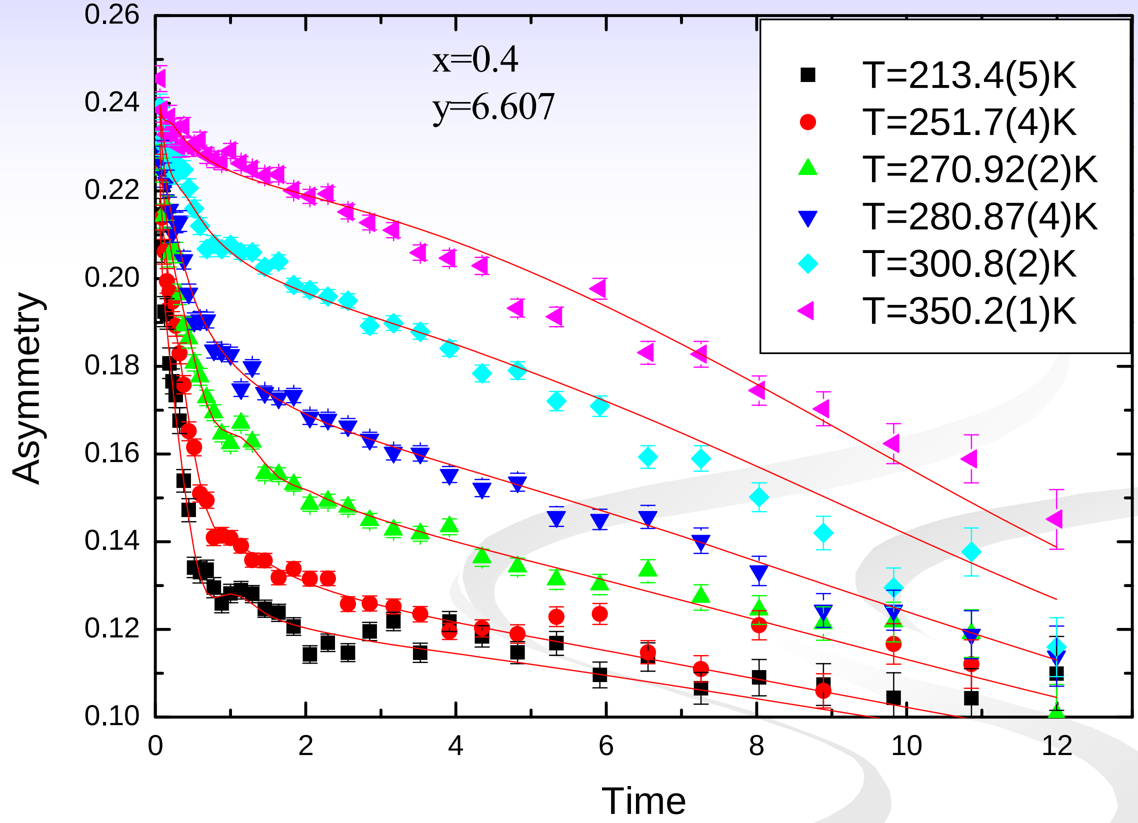




Schematic of a zero field (ZF) μSR setup.

# μSR raw data for CLBLCO

There are 2 contributions to *Az:*




$A_n$ – the amplitude of the nuclear frozen moments.

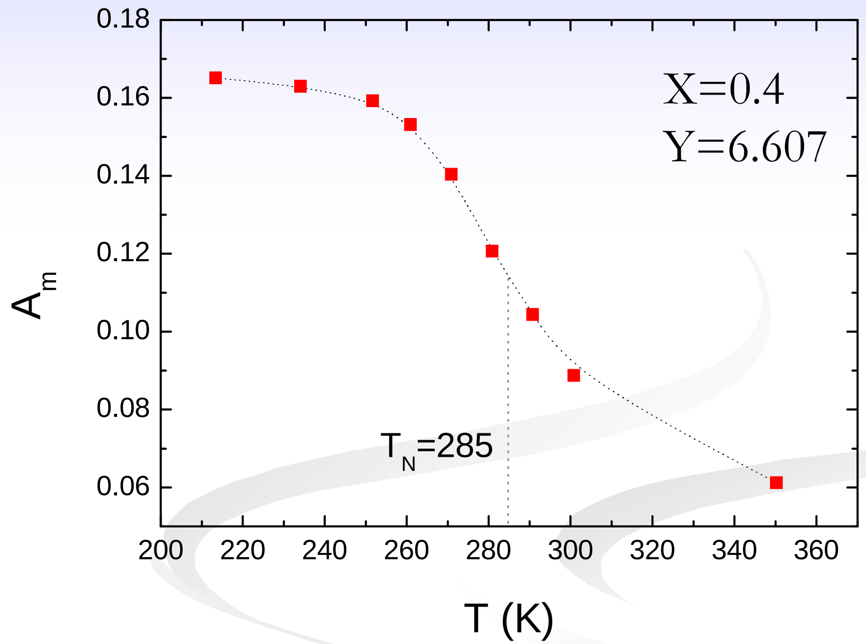
$A_m$ – the amplitude of the magnetic phase.

$A_{LR}$ – the amplitude of long range magnetic order.

$$A_Z = A_n \exp\left(\frac{-\Delta^2 t^2}{2}\right) + A_m \exp\left(-\sqrt{\lambda t}\right) + A_{LR} \exp\left(\frac{-t}{T_1}\right)\cos(\omega t)$$

# Measuring $T_g/T_N$

- As the temperature decreases, $A_m$ the electronic magnetic part increases, at the expense of $A_n$.

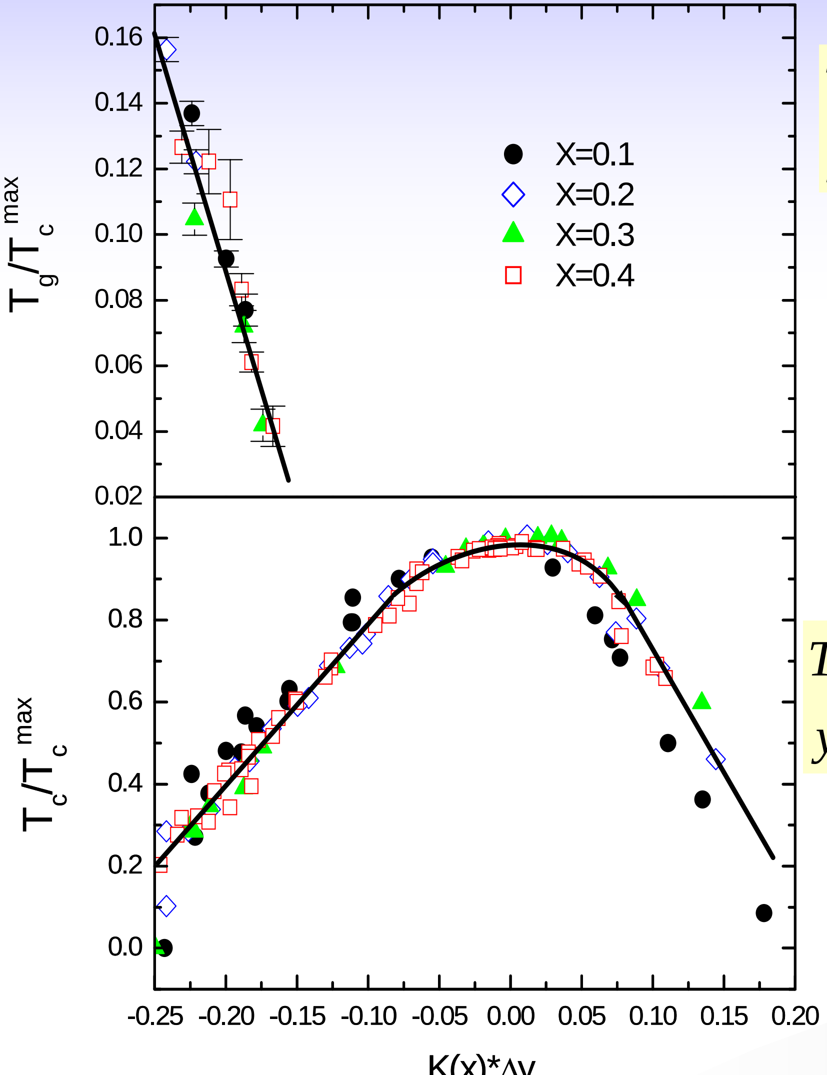



- the transition temperature is the temperature at which:

$A_m = A_m^{max}/2$

$$A_Z = A_n \exp\left(\frac{-\Delta^2 t^2}{2}\right) + A_m \exp\left(-\sqrt{\lambda t}\right) + A_{LR} \exp\left(\frac{-t}{T_1}\right)\cos(\omega t)$$

# Scaling relation in the superconducting phase for CLBLCO

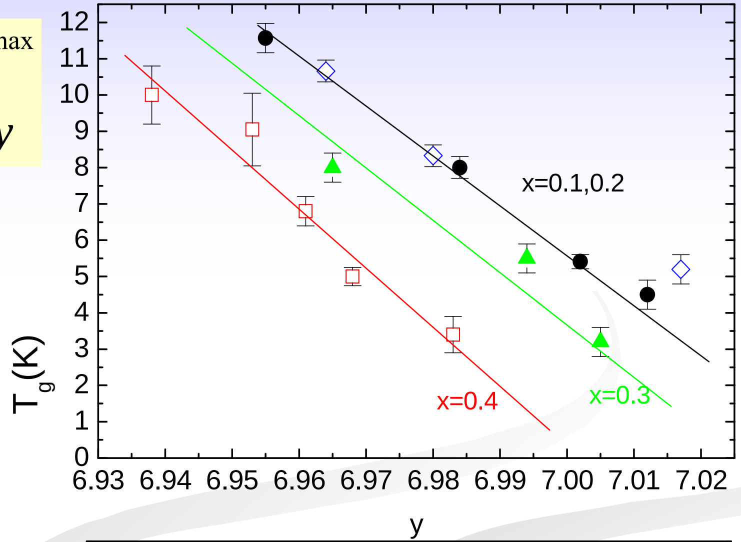


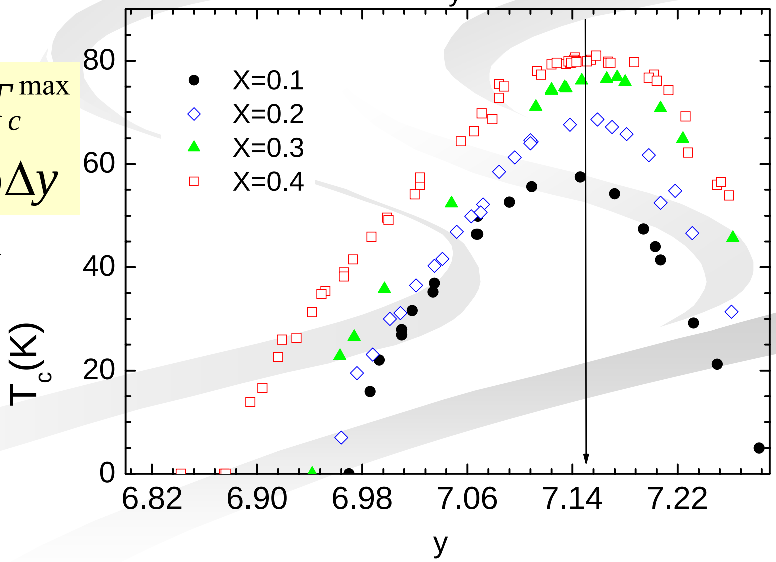

$T_g \Rightarrow T_g / T_c^{max}$

$y \Rightarrow K(x)\Delta y$

$T_c \Rightarrow T_c / T_c^{max}$

$y \Rightarrow K(x)\Delta y$




A. Kanigel et al, Phys. Rev. Lett **88**,137003 (2002).

# Interpretation

If

$$T_c = J_f^s n_s(\Delta p_m)$$

$$T_g = J_f^s f(\Delta p_m)$$

where

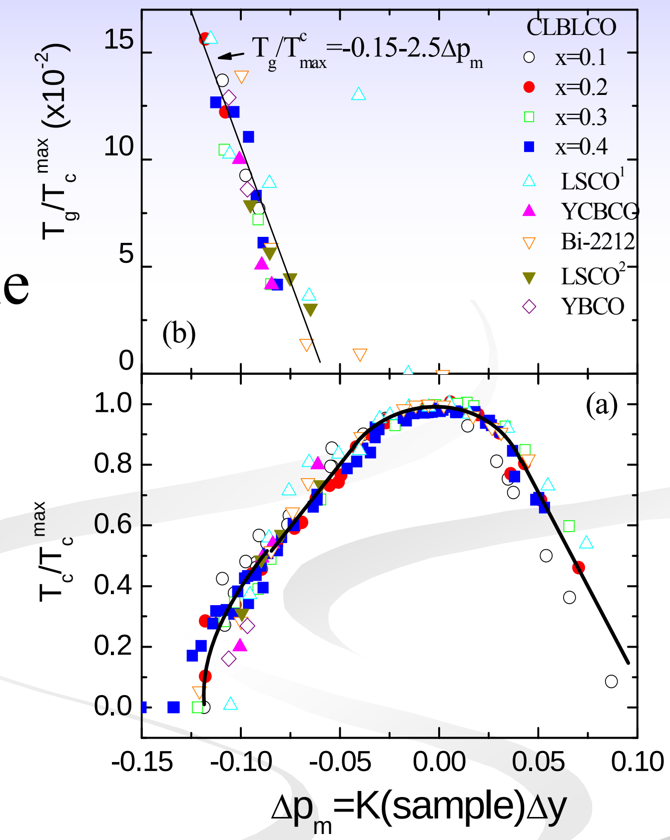
- $J^s{}_f$ is a family dependent magnetic energy scale
- $f$ is a function of doping only

then

$$T_c / T_c^{\max} = n_s(\Delta p_m) / n_s(0)$$

$$T_g / T_c^{\max} = f(\Delta p_m) / n_s(0)$$

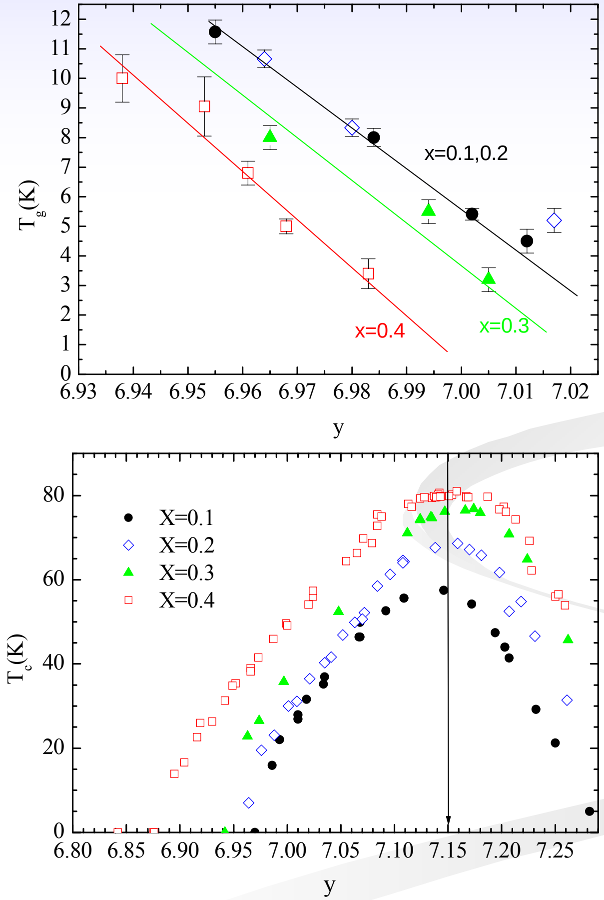
with no family index.



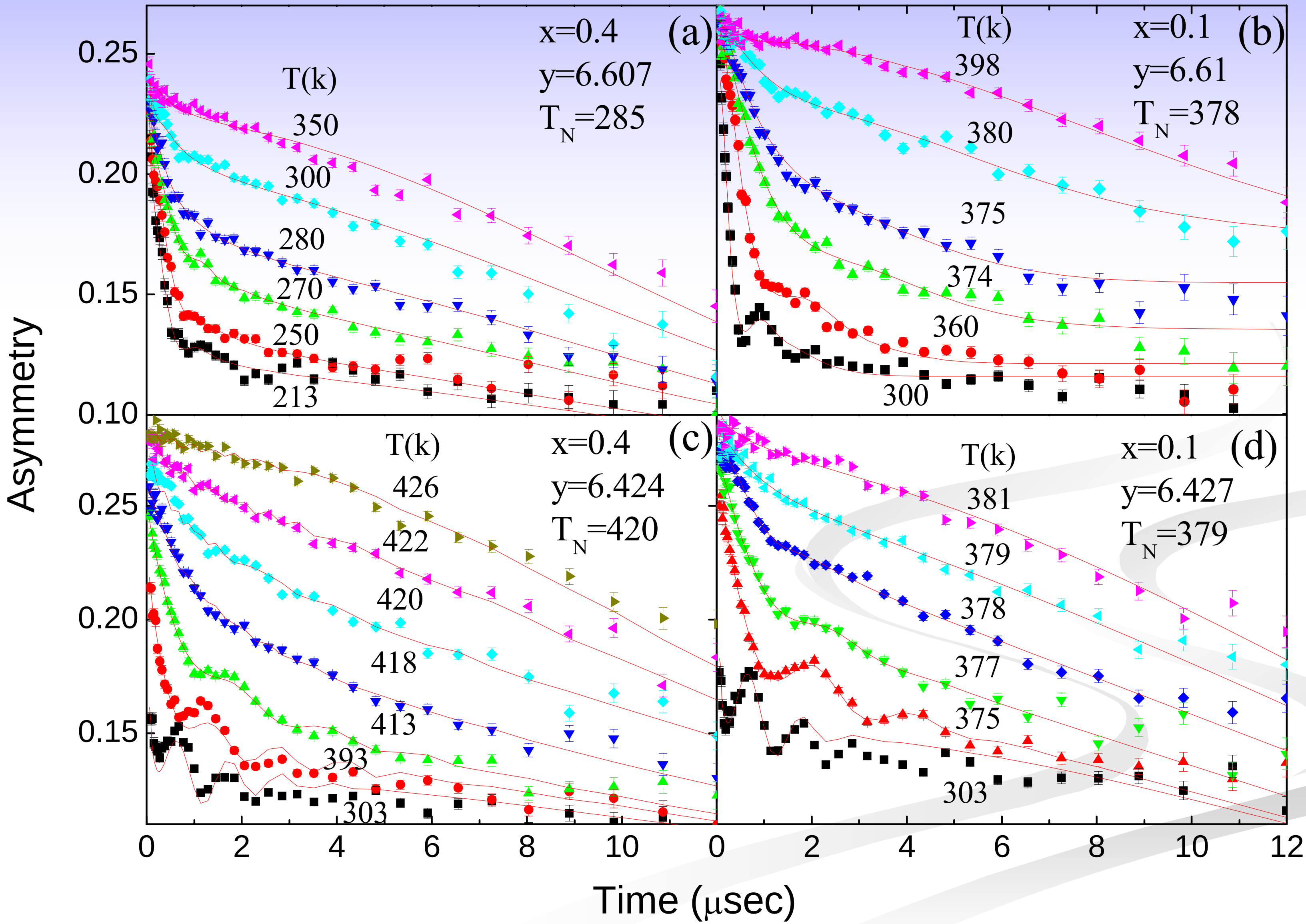

There is a common energy scale for magnetism and superconductivity

# Problem

$T_g$ of high $T_c^{max}$ family is lower than $T_g$ of low $T_c^{max}$ family.

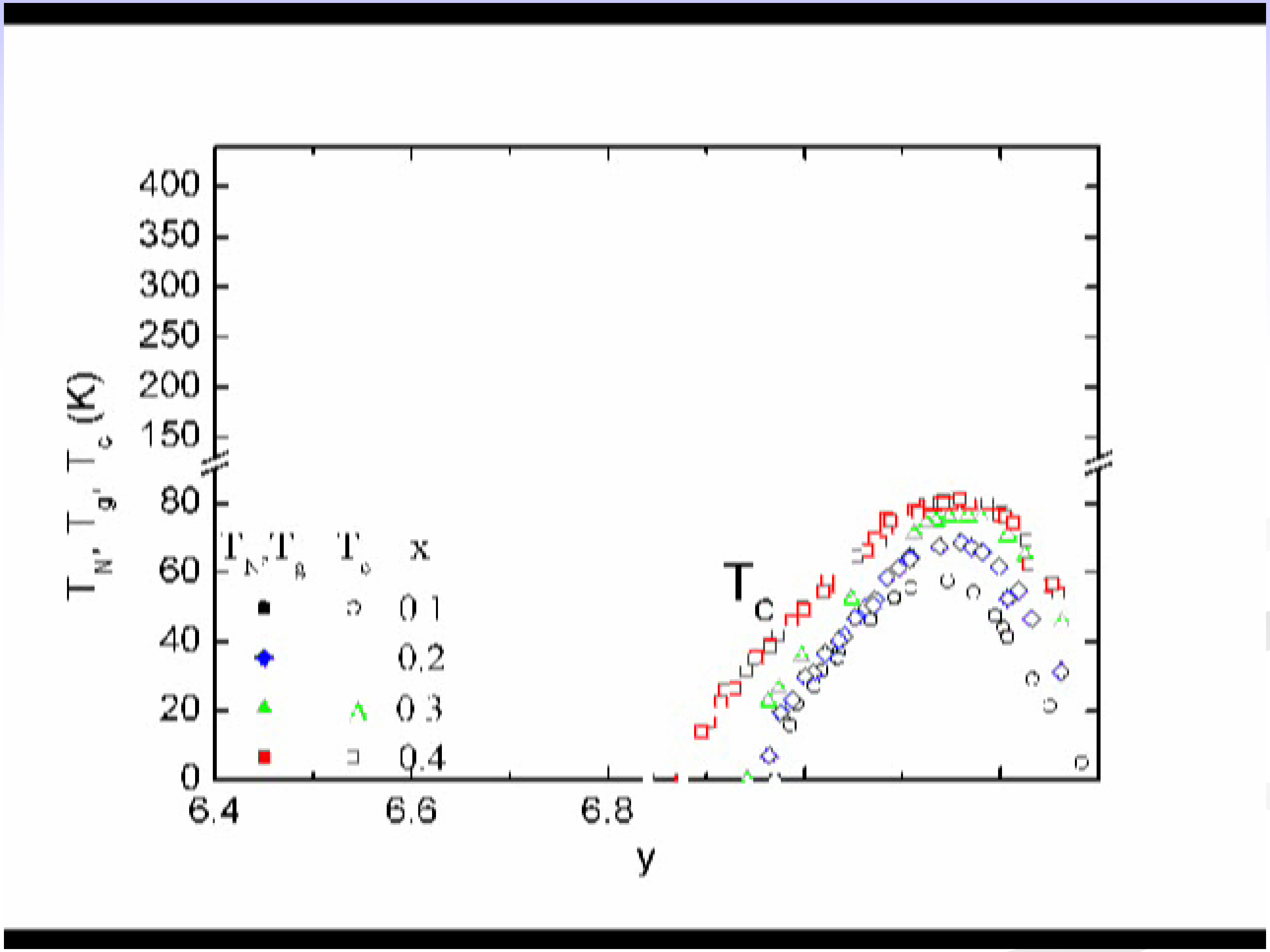



Without the *y* scaling one might reach the opposite conclusion.



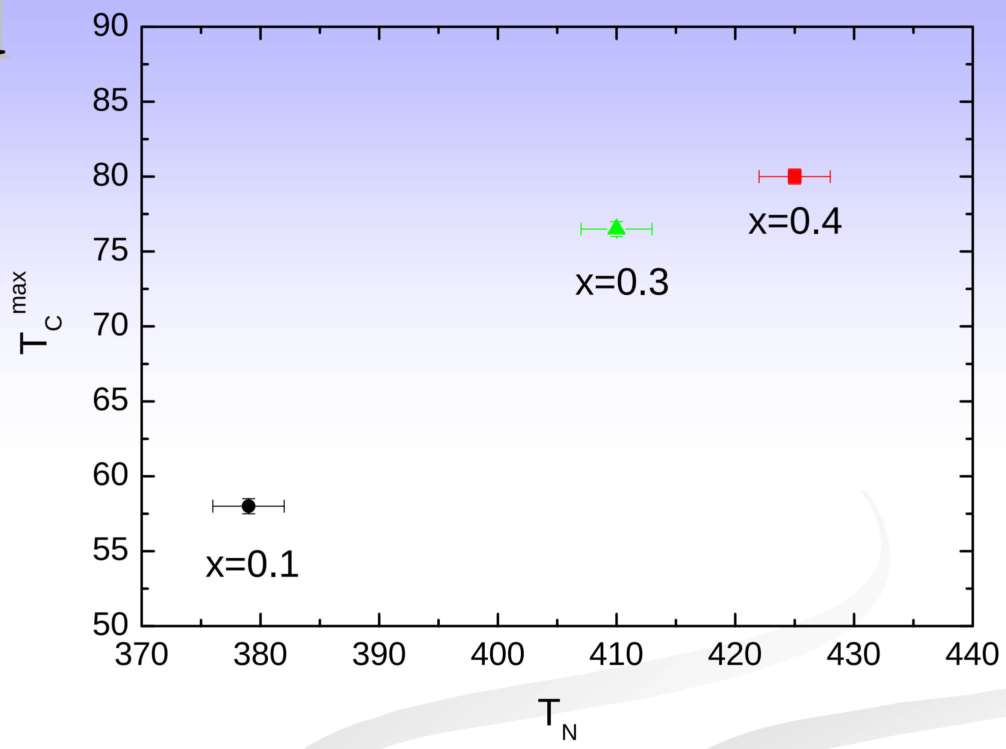
Asymmetry
Time (μsec)
x=0.4
y=6.607
T_N=285
(a)
T(k)
350
300
280
270
250
213
T(k)
x=0.1
y=6.61
T_N=378
(b)
398
380
375
374
360
300
T(k)
x=0.4
y=6.424
T_N=420
(c)
426
422
420
418
413
393
303
T(k)
x=0.1
y=6.427
T_N=379
(d)
381
379
378
377
375
303
0.25
0.20
0.15
0.10
0 2 4 6 8 10 12

# μSR results: Phase diagram of CLBLCO




**The family with higher $T_C^{max}$ has a higher $T_N$ at very low doping**

# Problem solved




**$T_C^{max}$ is a monotonic function of $T_N$ in a system with minimal structural changes.**

- However, while $T_c^{max}$ changes by 30%, $T_N$ changes by 10%.
- The scaling relation no longer holds in the non superconducting samples.

# Theory

B.Keimer et.al. Phys. Rev. B 45, 7430 (1989)

The spin Hamiltonian for a weakly anisotropic, nearly 2D Heisenberg antiferromagnet:

$$H = J\left(\sum_{i,\delta\|} S_i \cdot S_{i+\delta\|} + \alpha_{xy}\sum_{i,\delta\|} S_i^z \cdot S_{i+\delta\|}^z + \sum_{i,\delta_{\perp j}} \alpha_{\perp j} S \cdot_i S_{i+\delta_{\perp j}}\right)$$

$\alpha_{xy}, \alpha_{\perp} << 1$ $\quad \delta\|, \delta_{\perp j}$ - the nearest neighbor spacings

$$T_N \approx 2J \frac{M_0 \pi}{\ln\left\{\dfrac{4\alpha_{eff}}{M_0 \pi^2 \ln(4\alpha_{eff}/\pi)}\right\}}$$

$$\alpha_{eff} = z\|\alpha_{xy} + z_{\perp}\alpha_{\perp}$$

z – coordination number

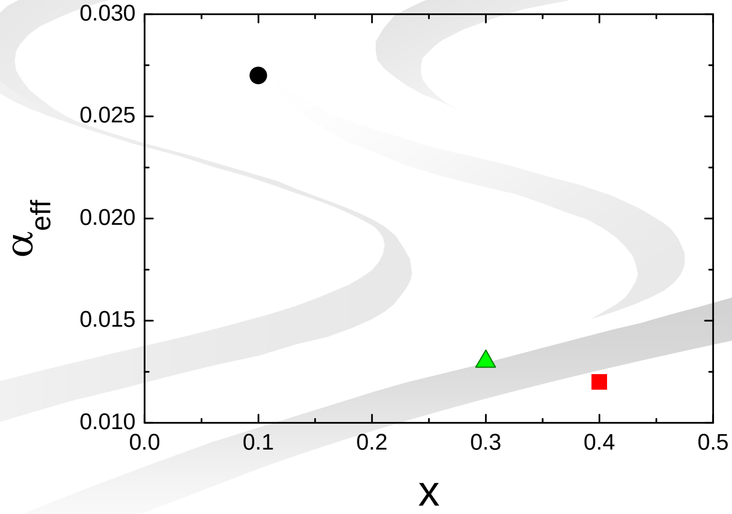
$M_0$ - sublattice magnetization

***$T_N$* is determined by *J*** while various anisotropies (α's) enter logarithmically.

- In order for the energy scale $J_f^s$, which is common to $T_g$ and $T_c$ in the superconducting region, to be exactly $J_f$ of the AFM, one needs the alpha's to changes between different families.

We can predict how these alphas must change.

- We write: $J_f^s = CJ_f$ so that $T_c^{\max} = CJ_f n_s$
- Calibrate C using YBCO ($n_s(0)$=0.08, $T_C^{max}$ =90, $J$=100meV) ➡ $C$=0.8.
- Assume $J_f = \dfrac{T_c^{\max}}{Cn(0)}$ for all CLBLCO families.
- Use the Eq. for $T_N$ to obtain the α's.
- We obtain: ➡




This is a prediction for future experiments (muon rotation frequency vs. T )

# Conclusions of the second part

- $T_c^{max}$ is a monotonic function of $T_N$.
- However, the magnetic energy scale $J^s_f$ that enters in

$$T_c = J_f^s n_s$$

is not necessarily the AFM *J*. The anisotropies α's might be important for *Tc*.

$J_f^s$ could still be the AFM *J* if the α's change.

# Summary

- For the YBCO family we confirmed that $J^s{}_f$ is a constant for all doping values in the superconducting dome.
- $T_C{}^{max}$ is a monotonic function of $T_N$. However, $T_N$ *a priori* has a magnetic energy scale of its own.
- $T_N$, $T_g$, and $T_c$ could all be proportional to the AFM $J$ if the $\alpha$'s varied between families.

# Acknowledgments

Galina Bazalitsky

Leonid Iomin

Larisa Patalgen

Arkadi Knizhnik

Oren Shafir, Eva Sagi, Oshri Peleg, Meni Shay, Ariel Maniv, Lior Shkedy,
**Shahar Levy, Amit Kanigel**