# Electroholographic tunable volume grating in the $\mathrm{g}_{44}$ configuration 

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#### Abstract

The $\mathrm{g}_{44}$ grating is an electroholographic transmission grating in which the applied field is perpendicular to both the grating vector and the wave vector of the incident beam. It is argued that in this configuration the incident beam traverses through a periodically rotating index ellipsoid. It is shown that in the $\mathrm{g}_{44}$ configuration the Bragg condition is fulfilled for a specific value of the applied field and for a diffracting beam polarization that is perpendicular to that of the incident beam. Consequently, the $\mathrm{g}_{44}$ grating can be used as an electrically controlled filter. Tunability of 7 nm is demonstrated in a 2 mm thick grating. © 2006 Optical Society of America

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Electroholography (EH) is a wavelength-selective optical switching method based on governing the reconstruction process of volume holograms by means of an electric field. ${ }^{1} \mathrm{EH}$ is based on the voltagecontrolled photorefractive effect at the paraelectric phase where the electro-optic (EO) effect is quadratic. ${ }^{1,2}$ Potassium lithium tantalate niobate (KLTN) doped with copper was found to be a very efficient material system for implementing EH. ${ }^{3}$ EH has originally been implemented as transmission holograms in the $\mathrm{g}_{11}$ configuration, where the electric field is applied across the width of the grating in parallel to both the grating vector and the photorefractive space-charge field. ${ }^{2}$
We present an alternative configuration of EH in which the applied electric field is perpendicular to the grating vector and to the incident beam. In this configuration the effective EO coefficient is $\mathrm{g}_{44}$. It is therefore shown that the $g_{44}$ configuration enables wavelength tunability by the applied voltage.
Consider first the general case of a crystal containing a phase grating given by $\delta(\Delta n) \cos (\mathbf{K} x)$, where $\mathbf{K}$ is the grating vector $(\mathbf{K} \| x,|\mathbf{K}|=2 \pi / \Lambda$, and $\Lambda$ is the grating period). A plane wave with wavelength $\lambda_{o}$ propagating in the $(x, z)$ plane is incident on the crystal at an angle $\phi$. Because of Snell's law of refraction, the direction of the incident beam is shifted as it enters the crystal so that its wave vector within the crystal is $\mathbf{k}_{\text {in }}=\left(-k \sin \theta_{\text {in }}, 0, k \cos \theta_{\text {in }}\right)$, where $\sin \theta_{\text {in }}$ $=\sin \phi / n_{o}, \theta_{\text {in }}$ is the angle of propagation with respect to the $z$ axis, and $k=2 \pi n_{o} / \lambda_{o}$. In general, the Bragg condition is given by

$$
\begin{equation*}
\mathbf{K}=\mathbf{k}_{d}-\mathbf{k}_{\text {in }}, \tag{1}
\end{equation*}
$$

where $\mathbf{k}_{d}$ is the wave vector of the diffracting beam given by $\mathbf{k}_{d}=\left(k \sin \theta_{d}, 0,-k \cos \theta_{d}\right)$, where $\theta_{d}$ is the angle of propagation of the diffracting wave within
the crystal. It can be immediately seen that fulfilling the Bragg condition requires that

$$
\begin{gather*}
\theta_{\mathrm{in}}=\theta_{d},  \tag{2a}\\
2 n_{o} \Lambda \sin \theta_{\mathrm{in}}=\lambda_{o} . \tag{2b}
\end{gather*}
$$

Consider now a transmission electroholographic grating in the $\mathrm{g}_{44}$ configuration implemented in KLTN at the paraelectric phase (Fig. 1). The crystal is positioned so that the crystallographic (100), (010), and (001) axes coincide with the $x, y$, and $z$ axes, respectively. The grating is a space-charge grating with period $\Lambda$ created by a photorefractive process, where the electric field induced by the space charge is given by $\mathbf{E}_{\mathrm{sc}}=\left[E_{\mathrm{sc}} \cos (\mathbf{K} x), 0,0\right]$. In the $\mathrm{g}_{44}$ configuration the applied electric field is $\mathbf{E}_{o}=\left(0, \mathbf{E}_{o}, 0\right)$ so that $\mathbf{E}=\mathbf{E}_{\text {sc }}$ $+\mathbf{E}_{o}=\left[E_{\mathrm{sc}} \cos (\mathbf{K} x), E_{o}, 0\right]$. The matrix of refractive index changes that are induced by the (quadratic) EO effect is given in this case by


Fig. 1. Transmission electrohologram in the $\mathrm{g}_{44}$ configuration for the case of an incident beam ( $I_{\text {in }}$ ) in the TM polarization. The diffracting beam $\left(I_{d}\right)$ is TE polarized, and the direct output beam ( $I_{\text {out }}$ ) is TM polarized.

$$
\Delta \mathbf{n}=-(1 / 2) n_{o}^{3} \varepsilon_{o}^{2} \varepsilon_{r}^{2}\left[\begin{array}{ccc}
\mathrm{g}_{11} E_{\mathrm{sc}}^{2} \cos ^{2}(\mathbf{K} x)+\mathrm{g}_{12} E_{o}^{2} & 2 \mathrm{~g}_{44} E_{o} E_{\mathrm{sc}} \cos (\mathbf{K} x) & 0  \tag{3}\\
2 \mathrm{~g}_{44} E_{o} E_{\mathrm{sc}} \cos (\mathbf{K} x) & \mathrm{g}_{12} E_{\mathrm{sc}}^{2} \cos ^{2}(\mathbf{K} x)+\mathrm{g}_{11} E_{o}^{2} & 0 \\
0 & 0 & \mathrm{~g}_{12} E_{\mathrm{sc}}^{2} \cos ^{2}(\mathbf{K} x)+\mathrm{g}_{12} E_{o}^{2}
\end{array}\right]
$$

where $\mathbf{g}=\left(\mathrm{g}_{i j k l}\right)$ is the fourth-rank quadratic EO tensor. ${ }^{4}$ (For KLTN at the paraelectric phase, $\mathrm{g}_{11}$ $=\mathbf{g}_{i i i i}=0.16 \mathrm{~m}^{4} / \mathrm{C}^{2} ; g_{12}=\mathbf{g}_{i j i j}=-0.02 \mathrm{~m}^{4} / \mathrm{C}^{2}, i \neq j ;$ and $\left.\mathrm{g}_{44}=\mathbf{g}_{i i j j}=0.08 \mathrm{~m}^{4} / \mathrm{C}^{2}, i \neq j.\right)$
As can be seen two diffraction gratings are formed along the $x$ axis: (i) a constant (i.e., independent of $\mathbf{E}_{o}$ ) grating with a $2 \mathbf{K}$ grating vector induced by the diagonal terms, and (ii) an electroholographic grating induced by the off-diagonal term with a grating vector $\mathbf{K}$ and an amplitude $\delta(\Delta n)=n_{o}{ }^{3} \mathrm{~g}_{44} \varepsilon^{2} E_{o} E_{\mathrm{sc}}$.
Assume that the incident wave is TE polarized (i.e., polarized in the $y$ direction), and the angle of incidence $\phi$ is chosen so that Eq. (2b) is fulfilled. In general, upon the application of $\mathbf{E}_{o}$, the $2 \mathbf{K}$ gratings that arise from the diagonal terms in Eq. (3) do not fulfill the Bragg condition and therefore cannot cause diffraction. The $\mathbf{K}$ grating that arises from the offdiagonal term $\Delta n_{12}$ fulfills the Bragg condition and $a$ priori should cause diffraction. This, however, does not happen: $\Delta n_{12}$ is an off-diagonal term that causes the index ellipsoid to rotate in phase with the spatial modulation of the grating. Consequently, the polarization of the TE-polarized incident wave oscillates around the initial TE direction as it traverses through the crystal. Thus, although the Bragg condition (2b) is fulfilled for all values of the applied field, diffraction of a TE-polarized wave is inhibited because in the Bragg direction, which allows the amplitude of the diffracting wave to accumulate, the propagating input beam is not uniformly polarized.
In general, for a given (external) angle of incidence $\phi$, the points of uniform polarization form a straight line inclined to the internal direction of propagation $\theta_{\text {in }}\left(\mathbf{E}_{o}\right)$ at an angle $\psi$. As $E_{o}$ is varied, $\theta_{\text {in }}$ changes and $\psi$ changes accordingly. If the wavefronts that are inclined at $\psi$ to the angle of incidence interfere constructively diffraction will occur. Note however that in this case $\theta_{d}=90^{\circ}-\left(\theta_{\text {in }}+\psi\right)$ so that Eq. (2b) is not fulfilled, and hence diffraction cannot occur for a diffracting wave that is TE polarized.
At the same time it should be noted that TMpolarized components of the incident wave are generated as the wave propagates through the oscillating index ellipsoid. These components can interfere constructively to form a diffracting wave. The condition for diffraction in this case is that the line of uniform polarization of the incident wave is perpendicular to the direction of propagation of a TM-polarized wave, which fulfills the Bragg condition (1) (Fig. 1). The latter is given for this case by $\mathbf{K}=\mathbf{k}_{d}{ }^{(T M)}-\mathbf{k}_{\text {in }}{ }^{(T E)}$, where $\mathbf{k}_{\text {in }}{ }^{(T E)}$ and $\mathbf{k}_{d}{ }^{(T M)}$ are the wave vectors (inside the crystal) of the incident and diffracting waves, respectively. This transforms into the following equations:

$$
\begin{equation*}
0=\frac{2 \pi n_{\mathrm{TE}}}{\lambda_{o}} \cos \theta_{\mathrm{in}}-\frac{2 \pi n_{\mathrm{TM}}}{\lambda_{o}} \cos \theta_{d}, \tag{4a}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2 \pi}{\Lambda}=\frac{2 \pi n_{\mathrm{TE}}}{\lambda_{o}} \sin \theta_{\mathrm{in}}+\frac{2 \pi n_{\mathrm{TM}}}{\lambda_{o}} \sin \theta_{d}, \tag{4b}
\end{equation*}
$$

where $\lambda_{o}$ is the wavelength in vacuum; $n_{\mathrm{TE}}$ $=n_{\mathrm{TE}}\left(\mathbf{E}_{B, \mathrm{TE}}, \mathbf{E}_{\mathrm{sc}}\right)$ and $n_{\mathrm{TM}}=n_{\mathrm{TM}}\left(\mathbf{E}_{B, \mathrm{TE}}, \mathbf{E}_{\mathrm{sc}}\right)$ are the indices of refraction for the TE-polarized wave and TMpolarized wave, respectively, where $\mathbf{E}_{B, \text { TE }}$ is the applied field ( $\mathbf{E}_{o}$ ) for which the Bragg condition is fulfilled. Note that in this case because the incident beam is TE polarized and the diffracting beam is TM polarized $\theta_{\text {in }} \neq \theta_{d}$. In terms of the external angle of incidence $\phi$, the Bragg condition reduces to

$$
\begin{align*}
\lambda_{o}= & \Lambda\{\sin \phi \\
& +\sqrt{\left.\left[n_{\mathrm{TM}}\left(\mathbf{E}_{B, \mathrm{TE}}, \mathbf{E}_{\mathrm{sc}}\right)\right]^{2}-\left[n_{\mathrm{TE}}\left(\mathbf{E}_{B, \mathrm{TE}}, \mathbf{E}_{\mathrm{sc}}\right)\right]^{2}+\sin ^{2} \phi\right\}} . \tag{5}
\end{align*}
$$

Similarly, if the incident wave is TM polarized it will be diffracted with TE polarization at a specific level of the applied field $\mathbf{E}_{B, \mathrm{TM}}$. In this case, the Bragg condition is given by

$$
\begin{align*}
\lambda_{o}= & \Lambda\{\sin \phi \\
& \left.+\sqrt{\left[n_{\mathrm{TE}}\left(\mathbf{E}_{B, \mathrm{TM}}, \mathbf{E}_{\mathrm{sc}}\right)\right]^{2}-\left[n_{\mathrm{TM}}\left(\mathbf{E}_{B, \mathrm{TM}}, \mathbf{E}_{\mathrm{sc}}\right)\right]^{2}+\sin ^{2} \phi}\right\} . \tag{6}
\end{align*}
$$

In general the diffraction efficiency is given by ${ }^{5}$

$$
\begin{equation*}
\eta=\frac{I_{d}}{I_{\mathrm{in}}}=\frac{L^{2} \kappa^{2}}{\cos \theta_{\mathrm{in}} \cos \theta_{d}} \operatorname{sinc}^{2}\left(\frac{L \Omega}{2}\right), \tag{7}
\end{equation*}
$$

where $I_{\text {in }}$ and $I_{d}$ are the intensities of the incident beam and the diffracted beam, respectively, $\theta_{\text {in }}$ and $\theta_{d}$ are the angles of incidence and diffraction, respectively, and $L$ is the thickness of the grating.
The sinc argument $\Omega$ is given by

$$
\begin{equation*}
\Omega=\sqrt{\left(\frac{\nu}{\cos \theta_{d}}\right)^{2}+4\left(\frac{\kappa^{2}}{\cos \theta_{d} \cos \theta_{\mathrm{in}}}\right)} . \tag{8}
\end{equation*}
$$

$\kappa$ is the Kogelnik coupling coefficient given by

$$
\begin{equation*}
\kappa=\frac{\pi \delta(\Delta n)}{\lambda_{o}}, \tag{9}
\end{equation*}
$$

where $\delta(\Delta n)$ is the amplitude of the grating and the Bragg detuning parameter $\nu$ is given by

$$
\begin{equation*}
\nu=\left|\mathbf{K}-\left(\mathbf{k}_{d}-\mathbf{k}_{\text {in }}\right)\right| . \tag{10}
\end{equation*}
$$

In the $\mathrm{g}_{44}$ configuration $\nu$ depends on the applied electric field as well as on the incident beam polarization. In this case, for a given wavelength $\lambda$ and angle of incidence $\phi$, maximum diffraction will be obtained for a specific value of the applied field for which $\nu$ $=0$. Around this value the diffraction efficiency is ex-


Fig. 2. Measurement of the diffraction efficiency as a function of applied electric field for a TE-polarized incident beam.


Fig. 3. Spectrum analyzer measurement of a grating illuminated by a TE-polarized broadband source incident at $\phi=20^{\circ}$. (a) Applied field $E=5.48 \mathrm{kV} / \mathrm{cm}$. (b) Applied field $E=1.21 \mathrm{kV} / \mathrm{cm}$.
pected to be dominated by the sinc ${ }^{2}$ term in Eq. (7). Note that this is substantially different from the $g_{11}$ configuration where the Bragg condition is fulfilled for all values of the applied field so that the diffraction efficiency follows a $\sin ^{2}$ behavior.
We investigated the diffraction off a transmission grating in the $\mathrm{g}_{44}$ configuration in a KLTN crystal at the paraelectric phase. The crystal was grown by using the top seeded solution growth method. ${ }^{6}$ The sample composition was $\mathrm{K}_{0.999} \mathrm{Li}_{0.001} \mathrm{Ta}_{0.61} \mathrm{Nb}_{0.39} \mathrm{O}_{3}$ doped with Cu and V . The sample was cut and polished along the crystallographic (001) axis with dimensions of $5 \mathrm{~mm} \times 4.8 \mathrm{~mm} \times 2 \mathrm{~mm}$. Gold electrodes were deposited on the larger faces. The phase transition temperature was determined by a measurement of the dielectric constant and was found to be $T_{c}$ $=18.3^{\circ} \mathrm{C}$. The diffraction measurements were done at $T=27^{\circ} \mathrm{C}$. A grating with $\Lambda=9 \mu \mathrm{~m}$ was created by interfering two 532 nm beams while applying no external electric field. The diffraction was measured by using a readout beam at $\lambda=1550 \mathrm{~nm}$ polarized in the $y$ direction, (i.e., TE polarized), incident at $\phi=6.3^{\circ}$. The diffraction efficiency versus the applied electric field during readout is presented in Fig. 2. As predicted, the diffraction efficiency is maximized at $\left|\mathbf{E}_{B, \mathrm{TE}}\right|$ $=3.0 \mathrm{kV} / \mathrm{cm}$.
A similar curve was obtained for a TM-polarized beam incident at the same angle $\phi=6.3^{\circ}$. For this
case it was found that the diffraction efficiency is maximized at $\left|\mathbf{E}_{B, \mathrm{TM}}\right|=3.75 \mathrm{kV} / \mathrm{cm}$.

Note that as predicted by the Bragg conditions (5) and (6) for the TE-polarized and TM-polarized incident beams, respectively, it is found that

$$
\begin{aligned}
& {\left[n_{\mathrm{TE}}\left(\mathbf{E}_{B, \mathrm{TE}}\right)\right]^{2}-\left[n_{\mathrm{TM}}\left(\mathbf{E}_{B, \mathrm{TE}}\right)\right]^{2}=\left[n_{\mathrm{TM}}\left(\mathbf{E}_{B, \mathrm{TM}}\right)\right]^{2}} \\
& \quad-\left[n_{\mathrm{TE}}\left(\mathbf{E}_{B, \mathrm{TM}}\right)\right]^{2}=0.01946,
\end{aligned}
$$

confirming further the interpretation given above.
The $\mathrm{g}_{44}$ configuration enables electrically controlled Bragg detuning in a transmission hologram. Namely, for a given grating, for each value of the applied field, the diffraction efficiency is maximized at a different wavelength. This feature is demonstrated in Fig. 3. The experiment was conducted in a KLTN crystal sized $2 \mathrm{~mm} \times 2 \mathrm{~mm} \times 2 \mathrm{~mm}$. A volume spacecharge grating was created with $\Lambda=2.25 \mu \mathrm{~m}$. The light source during reconstruction was an erbiumdoped fiber amplifer with continuum spectrum between 1520 and 1570 nm incident on the crystal at $\phi=20^{\circ}$. The diffracting beam was measured with a spectrum analyzer. The results for two levels of the applied field are presented in Fig. 3. The FWHM bandwidth was found to be 1.05 nm , which is the expected value for these grating dimensions. The tunable spectral range was found to be $\Delta \lambda_{\max }=6.95 \mathrm{~nm}$ and is spanned between 1526.95 nm at $1.21 \mathrm{kV} / \mathrm{cm}$ and 1533.9 nm at $5.5 \mathrm{kV} / \mathrm{cm}$, where the induced polarization reaches the saturation level. The reduced bandwidth of a single grating can be improved by increasing the size of the crystal and the angle of incidence.

In summary, the $\mathrm{g}_{44}$ configuration of electroholographic transmission gratings enables the construction of tunable electrically controlled filters. Furthermore, the fact that the applied field is perpendicular to the plane of the incident beam, the diffracting beam, and the grating vector enables the $g_{44}$ grating to be operated at moderate voltages.

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