Wave propagation in nonlinear lattices is fundamental to various branches of physics. The transport dynamics of incoherent (random-phase) waves in such structures is of particular interest, as most natural sources emit waves with only partial coherence. The distinction between coherent and incoherent waves in periodic structures becomes especially significant when the correlation distance is of the order of the lattice spacing or smaller. An incoherent field can be decomposed into a set of coherent modes with stochastic coefficients. In linear systems, the evolution of these modal constituents is uncoupled; hence the dynamics of a weakly correlated (incoherent) wave packet in a linear lattice can be analyzed through superposition. On the other hand, in nonlinear lattices the modal constituents are coupled through nonlinearity; hence superposition is inapplicable. Consequently, weakly correlated waves in nonlinear periodic systems yield a wealth of new phenomena, arising from the interplay between interference effects (partial reflections in the lattice), nonlinearity, and the statistical (coherence) properties of the waves. These ideas are augmented by the recently discovered random-phase lattice solitons (RPLSs), which introduced the first study of incoherent wave dynamics in nonlinear lattices. In Refs. 3–5, a significant part of the modes originated from the first Brillouin zone, residing in the semi-infinite gap above the first band. These modes were localized through total internal reflection from the defect in the lattice induced by the soliton intensity. Here, we demonstrate incoherent gap solitons for which all modal constituents reside within the photonic bandgaps between bands. All the modal constituents of such a gap RPLS are localized through Bragg reflection.

A gap soliton (GS) is a self-localized wave packet in a nonlinear periodic system whose propagation constant resides in the gap between two bands. Gap solitons were first found as temporal pulses in fiber gratings, and later as self-trapped beams in waveguide arrays. The recent progress with nonlinear photonic lattices has led to the first observation of dark and bright spatial GSs, which were followed by observations of 2D spatial GSs and of GSs originating from higher bands in 1D and 2D lattices. All of these spatial GSs were constructed from spatially coherent light. Building on the knowledge accumulated on incoherent solitons, our group has recently predicted and observed random-phase solitons in photonic lattices. Subsequent theoretical studies predicted the existence of gap RPLSs in 1D waveguide arrays: gap solitons constructed from partially spatially incoherent light.

Here we present what is to our knowledge the first experimental observation of gap RPLSs. The modal constituents of such solitons reside in the gaps of the unperturbed lattice, having no contribution from modes whose propagation constant lies in the semi-infinite gap above the first band and are localized by total internal reflection. We observe the self-trapped structure of these gap RPLSs as well as their multi-hump power spectra.

A spatially incoherent field \( \mathcal{E}(x,y,z,t) \) can be described as an incoherent superposition of coherent modes \( \mathcal{E} = \sum_m c_m(t) \psi_m(x,y,z) \), where \( c_m(t) \) are randomly fluctuating coefficients with \( \langle c_m c_n \rangle = d_m \delta_{mn} \), where brackets denote the average over the response time of the nonlinearity, and \( d_m \) is the weight of the \( m \)th mode. The concept of gap RPLS is general. However, to describe our experiments, we analyze incoherent beams propagating in optically induced waveguide arrays in photorefractives. The paraxial dynamics of such an incoherent beam is described by the set of dimensionless equations:

\[
\frac{\partial \Psi_m}{\partial z} + \left( \frac{\partial^2 \Psi_m}{\partial x^2} + \frac{\partial^2 \Psi_m}{\partial y^2} \right) - \left[ \frac{\sigma}{1 + V(x,y) + I(x,y,z)} \right] \Psi_m = 0, \tag{1}
\]

where \( I = \langle |\mathcal{E}|^2 \rangle = \sum_m d_m |\psi(x,y,z)|^2 \) is the time-averaged intensity. Here we use a self-defocusing nonlinearity \( \sigma = -1 \) and \( V = V_0 (\cos(2\pi x/d) + \cos(2\pi y/d))^2 \), where \( V_0 \) is the maximal lattice depth and \( d \) is the
lattice spacing. The lattice in this case is the “backbone lattice,” with its maximal points connected by a grid of equipotential lines. The bandgap structure of our lattice is sketched in Fig. 1(a). For typical experimental parameters, there is only one gap (between the first and the second bands) in a backbone lattice of a square symmetry. We therefore expect that the propagation constants of the modal constituents of our gap RPLSs will reside in that gap, with the modes arising from the anomalous diffraction regions of the first band. We calculate the modes of a representative gap RPLS by employing the self-consistency method. The soliton in this example is composed of five localized modes. The time-averaged intensity of the beam induces a defect in the lattice, and the soliton forms when its modal constituents populate, with the proper weights, the bound states of the defect that they jointly induce. The intensity structure of the soliton and its (Fourier) power spectrum are shown in Fig. 1. The stability of the soliton is checked numerically by adding small-amplitude initial noise to the modes and propagating the soliton for a number of diffraction lengths. Figure 1(b) depicts linear diffraction of the soliton in the lattice.

Our experiments are carried out in optically induced nonlinear lattices formed in a photorefractive SBN:75 crystal [Fig. 2(a)]. The response time of the nonlinearity is much longer than the characteristic time of random field fluctuations; hence the nonlinearity responds to the time-averaged intensity. The bias field is 1700 V/cm, corresponding to a 0.001 ratio of the lattice depth to the linear index of refraction. Our probe (soliton-forming) beam is extraordinarily polarized and is made spatially incoherent by passing a laser beam through a rotating diffuser and then focusing the beam onto the input face of the 2D photonic lattice [Fig. 2(b)]. We control the degree of spatial coherence and the shape of the power spectrum of the input probe beam by means of a spatial filter in the Fourier plane of the 4f system. The lattice period is \(~11\) µm, and the probe beam is 20 µm FWHM, covering an area of \(~10\) channel waveguides. We image the input and output planes of the lattice onto a CCD camera, and in parallel view the power spectrum of the beam exiting the lattice.

Two experimental examples of gap RPLSs are shown in Figs. 3 and 4. The input beams have similar widths, yet their structures in \(k\) space differ; hence under proper nonlinear conditions, they evolve to different gap RPLSs. The input beams are engineered with the central region of their power spectra removed, so they do not excite first-band Bloch modes from the normal diffraction region. We find (numerically and experimentally) that without such “spectral engineering” the random-phase beams do not evolve into gap RPLSs within a reasonable propagation distance (\(~1\) cm).

Figure 3 demonstrates the evolution of an input beam whose power spectrum consists of four humps, each centered on an \(M\)-symmetry point of the square lattice, into a gap RPLS. Such a beam excites mostly anomalously diffracting Bloch modes. When the input probe beam is at low intensity (peak intensity 50

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Fig. 1. (Color online) (a) Backbone lattice band structure. The three lowest bands are presented top to bottom. Note that there is no gap between the second and the third bands. (b) Linear lattice diffraction of the calculated example of gap RPLS at low intensity. (c) Intensity and (d) power spectrum of the gap RPLS example.

Fig. 2. (Color online) (a) Optical induction technique used to obtain a nonlinear photonic lattice. (b) The setup for producing a partially spatially incoherent beam with an engineered power spectrum.

Fig. 3. (Color online) Experimental observation of a gap RPLS with a four-hump power spectrum. (a) Intensity and (b) power spectrum of the input beam. (c) Diffraction of the input beam after 5 mm free space propagation. (d) Low-intensity output beam after 5 mm propagation in the lattice. (e) Intensity and (f) power spectrum of the high intensity gap RPLS exiting the lattice.
times smaller than $V_0$ the beam broadens to an output of $\sim 50 \mu$m FWHM after 5 mm propagation. When the intensity of the probe beam is sufficiently high (peak intensity $\sim V_0/4$), the beam self-traps and forms a gap RPLS, with a width of $\sim 18 \mu$m FWHM. Figure 4 shows the evolution of another incoherent input beam into a gap RPLS. In this second example, the input beam has an annular power spectrum with a square hole in the middle. At low intensity, this beam broadens to $\sim 80 \mu$m after 5 mm propagation in the lattice. When the beam intensity is increased to $\sim V_0/4$, it evolves into a gap RPLS of a width $\sim 30 \mu$m FWHM, and its power spectrum reshapes to have most of its power located in the areas of anomalous diffraction. Such a soliton is different in shape, width, and power spectrum from the gap RPLS of Fig. 3.

In conclusion, we have presented what is to our knowledge the first experimental observation of gap random-phase lattice solitons. The modal constituents of such solitons reside in the gaps between adjacent bands of the lattice, having no contribution from modes whose propagation constant lies in the semi-infinite gap, being self-trapped only by virtue of Bragg reflections. We have observed the self-trapped intensity structure of two different gap RPLSs, as well as their multihump power spectra, and verified the nonlinear nature of the self-trapping process by observing significant broadening of these beams at lower intensities. We have shown that, by engineering the power spectrum of the input beam, one can control the width of the random-phase lattice soliton. Random-phase gap solitons are generic entities; hence such weakly correlated self-trapped waves should also appear in other nonlinear periodic systems. One possibility is with incoherent temporal gap solitons in fiber gratings, which seem feasible following the recent progress with incoherent temporal solitons$^{21}$ and resonance solitons in photo-induced gaps.$^{22}$ Another possibility is with random-phase matter wave solitons, which were predicted to occur at finite temperatures.$^{23}$

Following the observation of matter–wave gap solitons,$^{24}$ it is natural to envision matter–wave gap RPLSs. We anticipate the observation of these gap random-phase lattice solitons in these other fields in the near future.

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References