Grating-Mediated Waveguiding

Oren Cohen,[1] Barak Freedman,[1] Jason W. Fleischer,[1,2] Mordechai Segev,[1] and Demetrios N. Christodoulides[2]

[1]*Department of Physics and Solid State Institute, Technion, Haifa 32000, Israel*

[2]*School of Optics/CREOL, University of Central Florida, Orlando, Florida 32816-2700, USA*

(Received 7 September 2003; revised manuscript received 9 June 2004; published 2 September 2004)

We propose a method of optical waveguiding, which relies on Bragg diffractions from a 1D grating that gives rise to waveguiding in the direction normal to the grating wave vector. The waveguide structure consists of a shallow 1D grating that has a bell- or trough-shaped amplitude in the confinement direction. Finally, we provide an experimental proof of the concept for this mechanism.

DOI: 10.1103/PhysRevLett.93.103902 PACS numbers: 42.79.Gn, 42.40.Eq

Optical waveguides are widely used in modern opto-electronic systems. Thus far, three physical mechanisms for optical waveguiding have been proposed and demonstrated. The most commonly used waveguiding scheme relies on total internal reflection (TIR) [1,2], in which light is propagating in a core region that has a refractive index higher than that of the cladding. The second method exploits the process of "Bragg reflection" and is sometimes called "photonic band gap waveguiding" [3–5]. In this scheme, light is confined into a core region that is surrounded by stacks of alternating dielectric layers. The propagation constants of the guided modes reside in the forbidden gaps of the cladding's transmission spectrum and, consequently, light is trapped due to Bragg reflections from the cladding regions. The third type of optical waveguiding is based on the CROW (coupled resonator optical waveguide) system [6], in which waveguiding is achieved through weak coupling between adjacent high-Q optical microresonators.

Here we propose a method for optical waveguiding: grating-mediated waveguiding (GMW). GMW is driven by a shallow 1D grating with a bell- or trough-shaped amplitude that is slowly varying in the direction normal to the grating wave vector. The waveguiding in this system occurs in the direction normal to the grating wave vector, when two Bragg-matched beams are incident upon the grating in an angularly symmetric fashion. These beams are simultaneously Bragg reflected from the grating and are jointly guided in the direction normal to the grating wave vector. As explained below, GMW differs from the other three generic waveguiding methods. This technique can be implemented in wave systems beyond optics as well, such as matter waves in Bose-Einstein condensates and density waves in acoustics.

For demonstration purposes, let the refractive index profile associated with such a planar grating-mediated waveguide be given by:

$$n(x, y) = n_0[1 + \varepsilon A(y)\cos(2\pi x/d)]. \quad (1)$$

This structure consists of a shallow grating in x with periodicity d and an amplitude $\varepsilon A(y)$. In Eq. (1), $A(y)$ is a normalized profile $[0 \leq A(y) \leq 1]$, and the small parameter ε $(0 < \varepsilon \ll 1)$ indicates the peak amplitude of the grating involved. The structure is uniform in the z direction, which is also the propagation direction of the waveguide. Here we focus on two generic families of GMW, corresponding to $A(y)$ having a bell shape [Fig. 1(a)] or a trough shape [Fig. 1(b)]. In both cases, the average index in every "layer" (cross section) in the y direction is equal to n_0, i.e., $\bar{n}(y) = \int_0^d n(x, y)dx = n_0$.

We begin by giving a qualitative explanation of the basic mechanism underlying GMW. For this purpose, we assume that the width of the grating amplitude is several times larger than the grating period and, of course, much larger than the wavelength. Within this limit, the guided modes of these systems can be described by the modes of the 1D grating, with their amplitudes slowly modulated (on a wavelength scale) in the y direction through the change in the grating amplitude. Figure 2(a) shows three gratings with different amplitudes, corresponding to three different y planes (layers) of this structure. The dispersion curves in these layers, namely, the propagation constant β vs the transverse momentum k_x, are shown in Fig. 2(b). The dispersion curves coincide everywhere except near the edges of the Brillouin zones [see magnified section of Fig. 2(b)], where a large amplitude ε yields a larger gap (outermost curves) and $\varepsilon \rightarrow 0$ leads to a diminishingly small gap. The modes of shallow gratings are approximately plane waves $\exp(\pm ik_x x)$ or, equivalently, standing waves $\cos(k_x x)$ and $\sin(k_x x)$ [7]. Note that near the Brillouin zone edge, that is near $k_x = \pm\pi/d$, the grating removes the degeneracy between the cos and sin modes. In the vicinity of this region, the wave with the $\cos(\pi x/d)$ dependence is more concentrated in the higher index regions, whereas

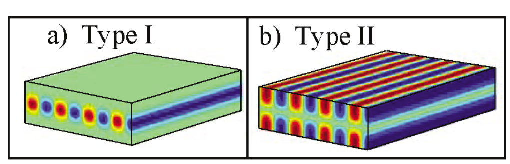

FIG. 1 (color online). Structure of (a) type I (bell-shaped) and (b) type II (trough-shaped) grating-mediated waveguides.

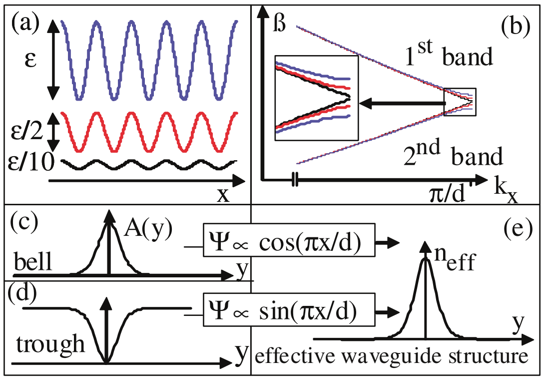

FIG. 2 (color online). (a) Index grating with three different amplitudes, corresponding to three different y planes (layers) in grating-mediated waveguides. (b) The dispersion curves (propagation constant β vs the transverse momentum k_x) near the edge of the first Brillouin zone for the gratings shown in (a). The inset shows the dispersion curves at the edge of the first Brillouin zone. (c),(d) Typical grating index amplitudes of (c) type I and (d) type II grating-mediated waveguides. (e) The effective waveguide structure in y; type I beams need a $\cos(\pi x/d)$ dependence and type II beams need a $\sin(\pi x/d)$ dependence to experience Bragg-mediated waveguiding.

the $\sin(\pi x/d)$ wave is more concentrated in the lower index regions. Hence, the propagation constant (or, equivalently, the effective refractive index $n_{\text{eff}} = \beta/k_0$) of the cosine wave is shifted upward, while the propagation constant of the sine wave is shifted downward, as shown in Fig. 2(b). For shallow gratings, this shift is proportional to the grating amplitude [7].

The first generic type of GMW, the type I waveguide, has a bell-shaped amplitude in y [Fig. 2(c)]. The change in propagation constant (the effective index) for the $\cos(\pi x/d)$ grating mode, due to the grating, is proportional to the grating amplitude $\varepsilon A(y)$. Hence, beams with a $\cos(\pi x/d)$ dependence experience effective waveguiding in y; that is, the effective index for such beams has a waveguide structure, as shown in Fig. 2(e). GMW of the second generic kind, type II, relies on a trough-shaped grating amplitude $A(y)$ [Fig. 2(d)]. Here the change in propagation constant β (the effective index) for beams having a $\sin(\pi x/d)$ dependence is proportional to $-\varepsilon A(y)$, which (again) results in an effective waveguide structure for these sin beams [Fig. 2(e)].

It is instructive to highlight the difference between GMW and waveguides that rely either on Bragg reflection or on TIR. At first sight, one might mistakenly think that because the structures of a GMW are periodic, then the waveguiding effect is identical to that of Bragg reflection waveguides [3–5]. However, in Bragg reflection waveguides, the waves are Bragg reflected from the cladding regions back into the core region; i.e., the reflections are in the plane defined by the propagation and confinement directions. In contrast, for GMW, the grating is perpendicular to the direction of confinement, and the confined waves are Bragg scattered in the direction normal to the confinement direction. Likewise, grating-mediated waveguides are also different from TIR waveguides: in TIR structures, the light is confined in regions of higher refractive index [2], whereas for GMW all the waveguide layers have the same average refractive index n_0. This last argument becomes more apparent in type II grating-mediated waveguides, for which the grating amplitude has a trough shape. In this case, waveguiding is achieved even though light is not concentrated in the regions of the peaks of the refractive index.

We find the guided modes of the GMW through coupled mode theory. Since the index change is small, we neglect any vectorial effects and solve for the guided modes $\Psi_\beta(x, y)$ and propagation constants β of the scalar Helmholtz equation:

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + (k_0^2 n^2 - \beta^2)\Psi = 0, \tag{2}$$

where $n(x, y)$ is given by Eq. (1) and $k_0 = \omega/c$. Following the above discussion, we seek modes of the form [8]

$$\Psi(x, y) = \Phi(y)\begin{bmatrix} \cos(\pi x/d) \\ \sin(\pi x/d) \end{bmatrix}.$$

Inserting this ansatz and Eq. (1) into Eq. (2) and neglecting the ε^2 term yields

$$\begin{bmatrix} \cos(\pi x/d) \\ \sin(\pi x/d) \end{bmatrix}\{\Phi'' + \Phi[k_0^2 n_0^2 - (\pi/d)^2 - \beta^2 + 2\varepsilon A(y) k_0^2 n_0^2 \cos(2\pi x/d)]\} = 0. \tag{3}$$

Utilizing trigonometric identities and neglecting asynchronous terms of spatial frequency $3\pi/d$ yields

$$\Phi'' + \Phi[k_0^2 n_0^2 - (\pi/d)^2 - \beta^2 \pm \varepsilon k_0^2 n_0^2 A(y)] = 0, \tag{4}$$

where the plus (minus) corresponds to the cos (sin) x dependence of $\Psi(x, y)$.

Consider first Eq. (4) with the plus sign. As a concrete example of a type I waveguide, we analyze the case where $A(y) = \text{sech}^2(y/y_0)$, with $y_0 = \sqrt{2/\varepsilon k_0^2 n_0^2}$. The first guided (bound) solution to Eq. (4) is $\Phi_1 = \text{sech}(y/y_0)$, with propagation constant $\beta_1 = \sqrt{k_0^2 n_0^2 (1 + \varepsilon/2) - (\pi/d)^2}$. The second guided mode is $\Phi_2 = \tanh(y/y_0)$, with $\beta_2 = \sqrt{k_0^2 n_0^2 - (\pi/d)^2}$. We solved Eq. (2) numerically with the following parameters: $\varepsilon = 10^{-4}$, $d = 5\ \mu\text{m}$, $k_0 = 2\pi\ \mu\text{m}^{-1}$, and $n_0 = 2$ (resulting with $y_0 \sim 11.25\ \mu\text{m}$) and found that the numerical solutions almost perfectly match the analytical solutions. The analytic approximation is excellent for shallow gratings with envelopes that are several times wider than the grating period. We emphasize, however, that the idea of grating-mediated waveguides works well beyond the regime described by coupled mode theory.

To further study GMW, we simulate the propagation of various beam configurations when launched into this system. Our simulation is carried out using a standard beam-propagation method and is based on the paraxial version [2] of the Helmholtz equation. The parameters used are those mentioned in the previous section. Figures 3(a)–3(c) show a typical evolution of such beams when they are not Bragg matched with the index grating. Specifically, these figures show the evolution of a $\mathrm{sech}(y/y_0)$ beam, launched on axis. The beam broadens from 20 μm at $z = 0$ to 150 μm at $z = 2$ cm. In a homogenous medium with $n = n_0$, the same beam would have diffracted (broadened) to 167 μm. On the other hand, Figs. 3(d)–3(f) display the evolution of a beam with the same shape in y but with a $\cos(\pi x/d)$ modulation in x. Such a beam displays stationary evolution, maintaining its narrow 20 μm width in y, thereby demonstrating grating-mediated waveguiding. However, as discussed above, type I GMW necessitates matching between the phase front of the propagating field and the grating. This phase selectivity is demonstrated in Figs. 3(g)–3(i), which show the evolution of a single $\mathrm{sech}(y/y_0)$ beam incident at the Bragg angle, which can also be viewed as a superposition of $\cos(\pi x/d)$ and $\sin(\pi x/d)$ beams. The sine component radiates [Fig. 3(h)], whereas the cosine component stays guided within the structure.

Next we demonstrate type II GMW, for which the grating envelope in y has a trough shape. We solve Eq. (4) with the minus sign and set $A(y) = 1 - \mathrm{sech}^2(y/y_0)$. The only difference in Eq. (4) in this example from the previous one is the constant term $\varepsilon k_0^2 n_0^2$. Hence, the envelopes of the guided modes are identical to those of the previous example, $\Phi_1 = \mathrm{sech}(y/y_0)$ and $\Phi_2 = \tanh(y/y_0)$, but with different propagation constants: $\beta_1 = \sqrt{k_0^2 n_0^2(1 - \varepsilon/2) - (\pi/d)^2}$ and $\beta_2 = \sqrt{k_0^2 n_0^2(1 - \varepsilon) - (\pi/d)^2}$, respectively. Figure 4 shows the propagation in such a waveguide with the same parameters $\{\varepsilon, d, k_0, n_0\}$ as in Fig. 3. Figures 4(a)–4(c) show the propagation of an on-axis $\mathrm{sech}(y/y_0)$ beam with a uniform x structure, which broadens from 20 μm at $z = 0$ to 190 μm at $z = 2$ cm. Figures 4(d)–4(f) display the stationary evolution of the $\mathrm{sech}(y/y_0)\sin(\pi x/d)$ beam. On the other hand, a beam that is $\mathrm{sech}(y/y_0)$ in y but has a $\cos(\pi x/d)$ shape in x broadens considerably [Figs. 4(g)–4(i)]. We emphasize that waveguiding occurs in this trough-shaped type II GMW in spite of the depression in the grating amplitude. Waveguiding, however, is achieved only at the proper angles of incidence, with the proper relative phase between the waves, so that the interference maxima coincide with the valleys in the index grating.

An experimental proof of concept for the type II (trough-shaped) GMW is shown in Fig. 5. We generate the waveguide in a 5 mm long photorefractive SBN:60 crystal through optical induction. An index grating is induced in the crystal by interfering two plane waves, while the trough is created by illuminating the central region with a narrow stripe beam that is incoherent with the interfering waves [Fig. 5(a)]. The total trough-shaped GMW structure $\Delta n \propto I_1 \cos(2\pi x/d + \pi/2)/[I_1 + I_2 \exp(-y^2/y_0^2)]$, forms through photorefractive diffusion effects [2]. Here d is the periodicity of the grating intensity interference pattern, y_0 is the width of the stripe, and I_1, I_2 are the peak intensities of the grating and stripe beams, respectively. The beams inducing the waveguide are ordinarily polarized; hence, they experience approximately linear propagation, as if they propagate in a homogenous linear medium. Next we launch the probe beams into the trough structure. The probe beams are very weak and extraordinarily polarized; hence, they "feel" the waveguide structure. To generate a probe beam with a proper guided mode profile, we interfere two beams that are wide in x, narrow in y, propagate at $\pm\theta_{\mathrm{Bragg}}$ with respect to the z axis, and posses the "correct" π phase shift relative to the grating index. Figures 5(b) and 5(c) show the intensity pattern at the

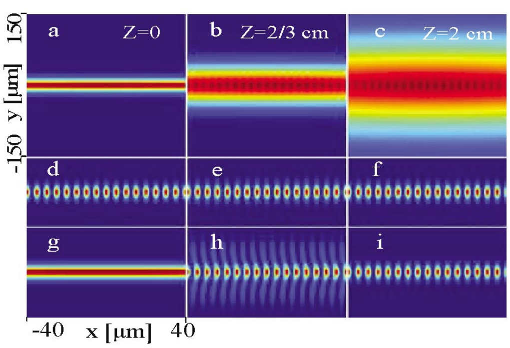

FIG. 3 (color online). Propagation of different beams in a type I (bell-shaped) GMW, each with a y profile of the first guided mode. Shown are the intensities at the propagation planes $z = 0$, 2/3, and 2 cm for (a)–(c) a beam uniform in x, (d)–(f) a beam with $\cos(\pi x/d)$ dependence, and (g)–(i) a beam with $\exp(i\pi x/d)$ dependence. Note that, for the latter case, the sine component radiates while the cosine component is trapped.

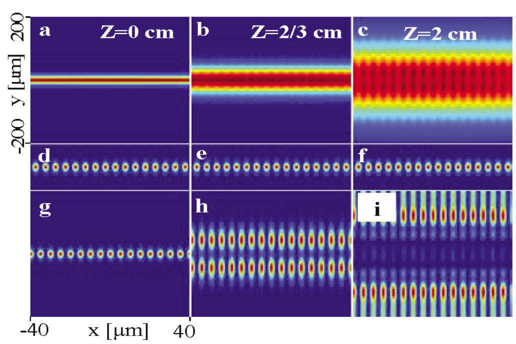

FIG. 4 (color online). Propagation of different beams in a type II (trough-shaped) grating-mediated waveguide, each with a y profile of the first guided mode. Shown are the intensities at the propagation planes $z = 0$, 2/3, and 2 cm for (a)–(c) a beam uniform in x, (d)–(f) a beam with $\sin(\pi x/d)$ dependence, and (g)–(i) a beam with $\cos(\pi x/d)$.

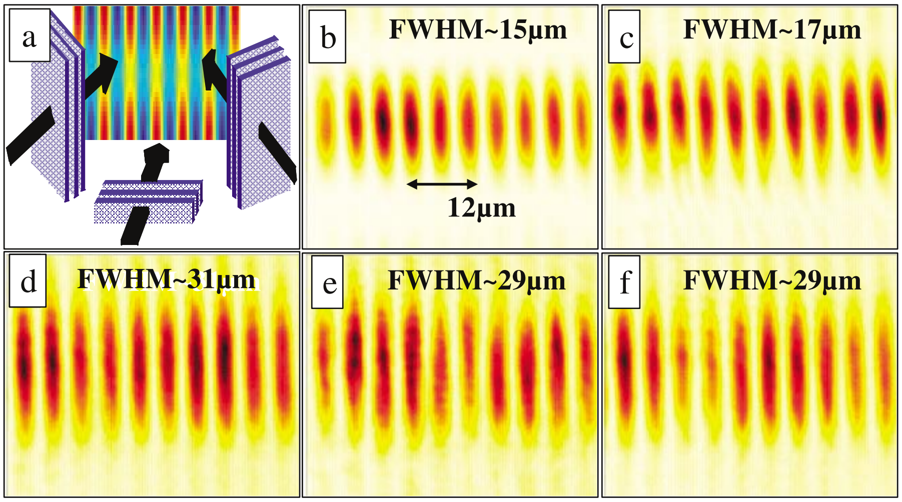

FIG. 5 (color online). Experimental scheme and results of a type II grating-mediated waveguide. (a) Schematic of the optically induced technique we use to obtain a type II GMW. The photographs are of a probe beam at (b) the input and (c) output of the waveguide providing experimental proof of concept for GMW. (d)–(f) Diffraction in unguided conditions: (d) a homogeneous medium, (e) when the beams are not Bragg matched with the index grating, (f) when the beams are Bragg matched with the grating but have the "wrong" phase relative to the grating. In (b)–(f), the intensity in each figure is normalized to its own peak intensity.

input and output faces of the crystal, respectively, demonstrating the confinement of the beam in the GMW structure. For comparison, Figs. 5(d)–5(f) show diffraction in three different conditions: propagation in a homogeneous medium [Fig. 5(d)], when the beams are not Bragg matched with the index grating [Fig. 5(e)], and when the beams are Bragg matched with the grating but have the "wrong" phase relative to the grating [Fig. 5(f)].

Let us now briefly discuss other interesting choices for the grating amplitude $A(y)$ which arise from combinations of the two generic (bell and trough) cases. For example, the structure with envelope $A(y) = \mathrm{sech}^2(y/y_0) - \tanh^2(y/y_0)$ also acts as a GMW in the vicinity of $y = 0$. Another interesting example is $A(y) = \mathrm{sech}^2(y/y_0) - \tanh^2(y/y_0)\exp[-(y/4y_0)^2]$, which guides both the cosine and the sine beam components but at different layers (in the vicinities of $y = 0$ and $y = \pm 2y_0$, respectively). A third example is the structure with a periodic grating amplitude $A(y) = \cos(y/y_0)$, which results in an array of grating-mediated waveguides. Here beams with different x dependencies experience different "effective structures" in y. Specifically, beams with $\cos(\pi x/d)$ exhibit waveguiding in layers $y = 2\pi y_0 m$, whereas beams with $\sin(\pi x/d)$ are guided at $y = \pi y_0(1 + 2m)$ [where $(m = 0, \pm 1, \pm 2, \ldots)$]. At the same time, beams lacking a spatial frequency of π/d do not feel any waveguide array but behave approximately as they would in a homogenous medium.

While we considered here the optical domain, a GMW mechanism is clearly applicable to any (2 + 1)D or (3 + 1)D wave equation [e.g. the Helmholtz equation (2) or the Schrödinger equation]. For example, a periodic density variation with a transverse profile (established, say, by a standing wave of suitable frequency) can waveguide acoustic or ultrasonic waves. The only requirement is the ability to create (or induce) a sinusoidal grating and propagate in the structure a distance greater than the grating period by orders of magnitude. This can be achieved with sound waves [9] as well as with matter waves [10]. Indeed, any coherent wave propagating in a potential that is periodic in one dimension, with a slowly varying amplitude in one or two transverse dimensions, will experience grating-mediated waveguiding.

In conclusion, we have proposed and provided an experimental proof of concept for a new method of optical waveguiding that is driven by a shallow 1D grating with a bell- or trough-shaped amplitude, which is slowly varying in the direction normal to the grating wave vector.

This work was supported by the Israel–USA Binational Science Foundation. O. C. gratefully acknowledges support by the Israeli Ministry of Science, Israel.

[1] D. Collodon and J. Babinet, C.R. Hebd. Seances Acad. Sci. **15**, 800 (1842).
[2] A. Yariv, *Optical Electronics in Modern Communications* (Oxford, New York, 1997).
[3] P. Yeh and A. Yariv, Opt. Commun. **19**, 427 (1976).
[4] M. Ibanescu, Y. Fink, S. Fan, E. L. Thomas, and J. D. Joannopoulos, Science **289**, 415 (2000).
[5] P. St. J. Russell, Science **299**, 358 (2003).
[6] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).
[7] N.W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders, Philadelphia, PA, 1976).
[8] In general, the grating modes at the edge of the first Brillouin zone that belong to the odd bands can be written as an infinite sum $\sum_{m=0}^{\infty} a_m \cos[(2m+1)\pi x/d]$. Here the grating is shallow; hence, we neglect the coupling to higher bands and approximate the grating mode of the first band as $\cos(\pi x/d)$ [7]. Similarly, we approximate the grating mode of the second band at the edge of the Brillouin zone as $\sin(\pi x/d)$.
[9] R. J. D. Miller *et al.*, J. Phys. Chem. **88**, 3021 (1984).
[10] M. Greiner *et al.*, Nature (London) **419**, 51 (2002).