Incoherent modulation instability in a nonlinear photonic lattice

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Abstract: We study modulation instability (MI) of random-phase waves in nonlinear photonic lattices. We find that an incoherent superposition of extended nonlinear eigenstates of the system, that is, an incoherent extended stationary beam, may become unstable due to nonlinearity. The instability process depends on the nonlinearity, on the structure of the diffraction curves of the lattice, as well as on the properties of the beam, whose spectrum can be comprised of Bloch modes from different bands, and from different regions of diffraction (normal/anomalous). This interplay among diffraction, incoherence, and nonlinearity leads to a variety of phenomena, including the possibility of tailoring the diffraction curve of the lattice, or the coherence properties of the beam, to enhance or suppress the instability. We present several examples of such phenomena, including a case where increasing the lattice depth flattens the diffraction curve thereby enhancing the instability, ”locking” the most unstable mode to the edge of the 1st Brillouin zone for large nonlinearity, and incoherent MI in self-defocusing media.

OCIS codes: (190.5940) Self-action effects, (190.5530) Pulse propagation and solitons, (030.6600) Statistical optics.

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1. Introduction

The nonlinear phenomenon of modulation instability (MI) occurs in diverse physical systems. In nonlinear optics, it usually refers to a process where small perturbations upon a uniform intensity beam grow exponentially due to the interplay between nonlinearity and dispersion/diffraction [1], thereby breaking the symmetry of the uniform beam. The process of MI has been studied for decades with coherent optical waves. A new direction of research on the MI phenomenon was initiated with the theoretical prediction [2] and experimental observation [3] of MI with incoherent light in noninstantaneous nonlinear media. The phenomenon of incoherent MI draws interest because the three-fold interplay among coherence, diffraction, and nonlinearity strongly affects the process of the instability [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Following its discovery [2, 3], incoherent MI has been studied in the context of induced instability [4], and clustering [5]. MI was demonstrated to occur with spatially and temporally incoherent white-light [6, 7]. It has been shown that the MI process may depend on the shape of the power spectrum [8]. Besides noninstantaneous local nonlinear media [2, 3, 4, 5, 6, 7, 8], incoherent MI occurs in nonlocal nonlinear media such as liquid crystals [10], in media with integrating nonlinearity [11], and under certain conditions, it is possible even in nonlinear media with instantaneous temporal response [9]. However, to the best of our knowledge, the process of incoherent MI has not yet been addressed in nonlinear photonic lattices.

The physics of coherent optical beams in nonlinear photonic lattices gives rise to lattice solitons (initially referred to as ”discrete” solitons, following the discrete model used in their prediction). Optical lattice solitons were predicted in 1988 [12], and observed ten years later [13]. The intriguing features of nonlinear dynamics in these systems have stimulated considerable activity in this field (for recent reviews, e.g., see Refs. [14, 15]). The nonlinear phenomenon of MI, which is closely related to solitons, has been predicted within the context of the discrete nonlinear Schrödinger equation (discrete NLSE), first at the base of the Brillouin zone [14], and later within the entire band [16]. The interplay between the spatial and temporal effects on the MI process has been studied in Ref. [17]. The first experimental observation of MI in discrete optical systems was reported in [18]. Further experiments demonstrated discrete MI in periodically poled lithium niobate waveguide arrays [19], in self-defocusing media [20], and in a layered Kerr medium (nonlinearity is periodic in the evolution variable) [21].

Our current study of incoherent lattice MI combines the phenomenon of coherent MI in (”discrete”) periodic nonlinear systems [14, 16, 17, 18, 19, 20, 21], and that of incoherent MI in homogeneous nonlinear systems [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. This analysis draws upon recent studies of partially coherent wave dynamics in noninstantaneous nonlinear photonic lattices [22, 23, 24, 25, 26, 27, 28, 29], including the prediction [22] and observation [23] of random-phase lattice solitons, and the possibility of Brillouin-zone (BZ) spectroscopy [24] with incoherent light (see Ref. [25] for a review of the topic).

Here we study modulation instability of incoherent extended stationary beams in nonlinear photonic lattices. The intensity and coherence structure of these extended states posses the periodicity of the lattice. In contrast to coherent MI in a lattice, these incoherent beams may excite an ensemble of Bloch modes from different bands and from different regions of diffraction (normal/anomalous). The instability process depends on the properties of the excitation, the nonlinearity, and the structure of diffraction (spatial dispersion) curves. This leads to a variety of phenomena, and includes the possibility of tailoring diffraction curves and/or excitation (e.g., degree of coherence) to enhance or suppress the instability. We present the features of incoherent MI in several examples including (i) a case where the increase of the lattice depth flattens the diffraction curve, thereby enhancing the instability, (ii) MI of an incoherent extended beam whose constituents arise from multiple bands, (iii) locking of the most unstable mode to the edge of the 1st Brillouin zone for large nonlinearity, (iv) suppression of MI due to incoherence, and (v) incoherent MI in self-defocusing media.

It is important to emphasize that, even though this study focuses on instability of incoherent beams, some of our conclusions (e.g., Sec. 5) hold generally for *both* coherent and incoherent MI in continuous nonlinear periodic structures.

2. The physical system and the corresponding model

The physical system is identical to the one considered in Refs. [22, 23, 25]: We study propagation of a spatially incoherent, quasimonochromatic, and linearly polarized light beam in a nonlinear photonic lattice with a noninstantaneous nonlinearity. The electric field $E(x,z,t)$ of such a beam randomly fluctuates on time-scales much shorter than the response time of the nonlinearity τ_m [2, 3], which implies that nonlinearity cannot follow fast fluctuations of the field, but responds to the time-averaged intensity. The state of the system can be described with a mutual coherence function $B(x_1,x_2,z) = \langle E^*(x_2,z,t)E(x_1,z,t)\rangle$, which in the paraxial approximation, obeys [30]

$$i\frac{\partial B}{\partial z} + \frac{1}{2k}\left[\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2}\right]B + \frac{k}{n_0}[V(x_1,z) - V(x_2,z)]B = 0. \quad (1)$$

The potential $V(x,z) = p(x) + \delta n[I(x,z)]$ contains both the periodic $p(x) = p(x+D)$, and the nonlinear term $\delta n[I(x,z)]$; $I(x,z) = B(x,x,z)$ denotes the time-averaged intensity, n_0 is the linear part of the index of refraction, and $k = 2\pi n_0/\lambda$, where λ denotes the vacuum wavelength.

Instead of the mutual coherence function approach, equations of motion can be written in a fully equivalent modal form [31, 32], which is suitable for our numerical calculations. Within the modal theory, the electric field is written through a superposition of coherent waves (modes) with randomly varying coefficients: $E(x,z,t) = \sum_m c_m(t)\psi_m(x,z)$ [31], while the statistics follows from $\langle c_m(t)c^*_{m'}(t)\rangle = d_m\delta_{mm'}$; coherent waves $\psi_m(x,z)$ are connected to the mutual coherence function via

$$B(x_1,x_2,z) = \sum_m d_m \psi_m(x_1,z)\psi^*_m(x_2,z). \quad (2)$$

The functions $\psi_m(x,z)$ form an orthonormal set, and d_m denotes the power within the mth wave [32]. The evolution of coherent waves ψ_m follows a set of coupled nonlinear equations (e.g., see [25]),

$$i\frac{\partial \psi_m}{\partial z} + \frac{1}{2k}\frac{\partial^2 \psi_m}{\partial x^2} + \frac{V(x,z)k}{n_0}\psi_m(x,z) = 0. \quad (3)$$

where $V(x,z) = p(x) + \delta n[\sum_m d_m|\psi_m(x,z)|^2]$ again contains both the linear periodic and the nonlinear term.

3. Extended stationary states of the nonlinear system

The state of the system is given by the mutual coherence function, or equivalently a set of modes and their weights. Thus, the *trajectory* in the phase space of this nonlinear dynamical system is given by a set of functions $B(x_1,x_2,z)$ for continuous set of values $z \geq 0$ (which plays the role of time in our dynamical system). The concept of stability of this trajectory is well defined irrespective of whether this trajectory is stationary $\partial B/\partial z = 0$, or not. The stability describes the behavior of trajectories that are initially (at $z = 0$) nearby $B(x_1,x_2,z=0)$, that is, the trajectories from initial conditions $B'(x_1,x_2,z=0) = B(x_1,x_2,0) + \delta B(x_1,x_2,0)$, where δB denotes a small perturbation at $z = 0$, $|\delta B| << |B|$. The trajectory $B(x_1,x_2,z)$ is stable if the perturbation δB remains small during the evolution. Conversely, its exponential increase indicates

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instability. Even though stability is a well-defined concept for various types of trajectories, the stability of stationary states is much easier to analyze theoretically (analytically/numerically) and experimentally, and it provides important insight into the dynamics of the system.

The concept of MI in nonlinear optical systems usually refers to a stability analysis of certain steady states, which are in most cases *uniform-intensity waves* (e.g., plane waves are steady states in homogeneous nonlinear media). The first studies of MI with coherent waves in nonlinear waveguide arrays were performed within "discrete" model(s) [12], e.g., a discrete NLSE. Within these models a "discrete" plane wave is a steady state [18], and the linear stability analysis leads to a closed form expression for a gain curve [18]. However, here we investigate MI in a nonlinear photonic lattice by using a continuous model, and uniform intensity waves are (in general) *not* steady states within this model (i.e., an initial condition with uniform intensity evolves in a nontrivial fashion). For this reason, we first describe the extended, stationary, incoherent states in our system.

Extended stationary states of the system self-consistently obey the equations of motion, with appropriate boundary conditions. We seek solutions with intensity structure displaying the lattice periodicity $I_0(x) = I_0(x+D)$. Such solutions can be constructed in the form

$$B_0(x_1,x_2) = \sum_{\kappa} c_\kappa \phi_\kappa^N(x_1)\phi_\kappa^{N*}(x_2), \tag{4}$$

where $\phi_\kappa^N(x)$ are the Floquet-Bloch (FB) waves of the periodic potential $P(x) = p(x) + \delta n[I_0(x)]$, $P(x) = P(x+D)$; here, κ denotes the Bloch-wave vector. The nonlinear FB waves $\phi_\kappa^N(x) = u_\kappa^N(x)\exp(i\kappa x)$ are eigenfunctions of the operator $\hat{N}(x)$,

$$\hat{N}(x)\phi_\kappa^N(x) = \beta_\kappa^N \phi_\kappa^N, \tag{5}$$

where

$$\hat{N}(x) = \frac{1}{2k}\frac{\partial^2}{\partial x^2} + \frac{k}{n_0}[p(x) + \delta n(\sum_\kappa c_\kappa |\phi_\kappa^N(x)|^2)]; \tag{6}$$

the boundary conditions are $u_\kappa^N(x) = u_\kappa^N(x+D)$. For the sake of clarity, the FB modes and eigenvalues corresponding to the *linear* periodic potential $p(x)$ will be denoted by $\phi_\kappa^L(x)$ and β_κ^L, respectively. For a given set of parameters, the solution B_0 (provided it exists) may be found numerically by iteratively solving a set of modal equations (5); incoherent solitons are found in an identical fashion, with different boundary conditions [22]. We should state that our extended stationary states are, in fact, composite nonlinear Bloch waves [33], and in some cases multi-band nonlinear Bloch waves.

It should be emphasized that by changing any parameter of the system, e.g., the strength of the nonlinearity, we change the stationary state solution $B_0(x_1,x_2)$, and effectively, for each value of the parameter(s), we analyze the stability of a different solution $B_0(x_1,x_2)$. Nevertheless, in order to characterize lattice MI, it is instructive to study the stability in dependence of certain parameters, e.g., the depth of the lattice or nonlinearity, as we discuss below.

4. Linear stability analysis

In order to analyze the stability of B_0, we slightly perturb it at $z=0$, and observe the evolution of the mutual coherence: $B(x_1,x_2,z) = B_0(x_1,x_2) + B_1(x_1,x_2,z)$, where B_1 denotes a small perturbation $|B_1| << |B_0|$. The linearized evolution equation for B_1 reads [2]

$$i\frac{\partial B_1(x_1,x_2,z)}{\partial z} + [\hat{N}(x_1) - \hat{N}(x_2)]B_1(x_1,x_2,z) + \frac{k\gamma}{n_0}[B_1(x_1,x_1,z) - B_1(x_2,x_2,z)]B_0(x_1,x_2) = 0; \tag{7}$$

we have assumed the nonlinearity is of the Kerr-type $\delta n(I) = \gamma I$, thus, $\hat{N}(x_j) = 1/(2k)\partial^2/\partial x_j^2 + k/n_0[p(x_j) + \gamma B_0(x_j,x_j)]$, $j = 1,2$. The perturbations B_1 can be generally expanded in a set of eigenmodes, $B_1(x_1,x_2,z) = \sum_\alpha f_\alpha[B_\alpha(x_1,x_2)e^{g(\alpha)z} + B^*_\alpha(x_2,x_1)e^{g(\alpha)^*z}]$, where $B_\alpha(x_1,x_2)$ denotes the structure of the eigenmode, while $g(\alpha)$ is the corresponding eigenvalue. Clearly, if $\Re\{g(\alpha)\} > 0$ the mode (and therefore the whole beam) is unstable. The eigenmodes $B_\alpha(x_1,x_2)$ and their eigenvalues $g(\alpha)$ can be found from the equation

$$[\hat{N}(x_1) - \hat{N}(x_2)]B_\alpha + \frac{k\gamma}{n_0}[B_\alpha(x_1,x_1) - B_\alpha(x_2,x_2)]B_0(x_1,x_2) = -igB_\alpha(x_1,x_2). \qquad (8)$$

In order to proceed with the stability analysis in the fashion of Refs. [2, 6], which consider incoherent MI in a homogeneous medium, we would have to guess the diagonal structure of the growing modes $I_\alpha(x) = B_\alpha(x,x)$. For the sake of clarity it should be noted that the diagonal structure of B_α is *not* intensity (it can assume complex values); the intensity of the perturbation corresponding to the eigenmode B_α is $B_\alpha(x,x)e^{g(\alpha)z} + c.c.$, which is a real quantity. In a homogeneous medium, the diagonal structure of the growing eigenmodes are plane waves, $I_\alpha \propto \exp(i\alpha x)$ [2, 6]. Intuition therefore suggests that $I_\alpha(x)$ should be related to FB waves, which we corroborate in the next section.

Here we obtain an integral equation for the gain from Eq. (8) by using the fact that every mode B_α can be represented in the FB basis $\phi^N_\kappa(x)$: $B_\alpha(x_1,x_2) = \sum_{\alpha_1\alpha_2} H^\alpha_{\alpha_1\alpha_2}\phi^{N*}_{\alpha_2}(x_2)\phi^N_{\alpha_1}(x_1)$ (the summation goes over FB modes in the extended Brillouin zone scheme). After a straightforward calculation equivalent to the one presented in Refs. [2, 6], Eq. (8) may be cast into an integral form:

$$I_\alpha(x') = \sum_{\alpha_1\alpha_2} \frac{k\gamma}{n_0} \frac{-\phi^{N*}_{\alpha_2}(x')\phi^N_{\alpha_1}(x')}{ig + \beta(\alpha_1) - \beta(\alpha_2)} \int I_\alpha(x)\phi^N_{\alpha_2}(x)\phi^{N*}_{\alpha_1}(x)dx[c_{\alpha_2} - c_{\alpha_1}]. \qquad (9)$$

Equation (9) can be solved (numerically) in an iterative fashion. First, we assume $I_\alpha(x) = \phi^L_\alpha$, and find $g(\alpha)$ by using $x' = 0$. From this value of $g(\alpha)$, and the r.h.s. of Eq. (9), we obtain the next iterate for $I_\alpha(x)$, and repeat this procedure until convergence, which yields the gain curve $g(\alpha)$.

We conclude this section by emphasizing that we can use straightforward numerical evolution with initial conditions $B_0 + B_1$, where B_1 denotes small initial noise, and characterize the instability. The evolution is performed with a standard split-step Fourier method applied on evolution Eq. (3). Alternatively, we can obtain the gain curve from Eq. (9). In the following sections we utilize both procedures to study incoherent MI and its features in photonic lattices.

5. Inducing MI by flattening the diffraction curve

In this section we demonstrate how a change in the lattice parameters (the lattice depth) can affect MI through the change of the diffraction curves. Although we consider incoherent beams, the results of this section can be applied also to the case of coherent MI. Let us consider extended steady states B_0 with a broad and symmetric c_κ-spectrum ($c_\kappa = c_{-\kappa}$) shown in Fig. 1(a). The c_κ-spectrum is the FB power spectrum [22] expressed in terms of the FB eigenmodes ϕ^N_κ (for $p(x) = 0$, it is in fact a Fourier power spectrum). We investigate the stability of the solution B_0 in dependence of the lattice depth p_0, $p(x) = p_0\cos(\pi x/D)^2$, while keeping the c_κ-spectrum and all other parameters fixed: $n_0 = 2.3$, $\lambda = 488$nm, $D = 10\ \mu$m, $\gamma I_0 = 1.97\times10^{-5}$.

First we analyze the stability of B_0 by direct numerical evolution with initial condition $B_0 + B_1$, $|B_1| << |B_0|$. When the lattice depth is zero, the stationary state B_0 is stable. However, by increasing the lattice depth, above a certain threshold value, B_0 becomes unstable. The intensity structure of B_0 for $p_0 = 3.4\times10^{-4}$ is shown in Fig. 1(b) (red dashed line); black solid line

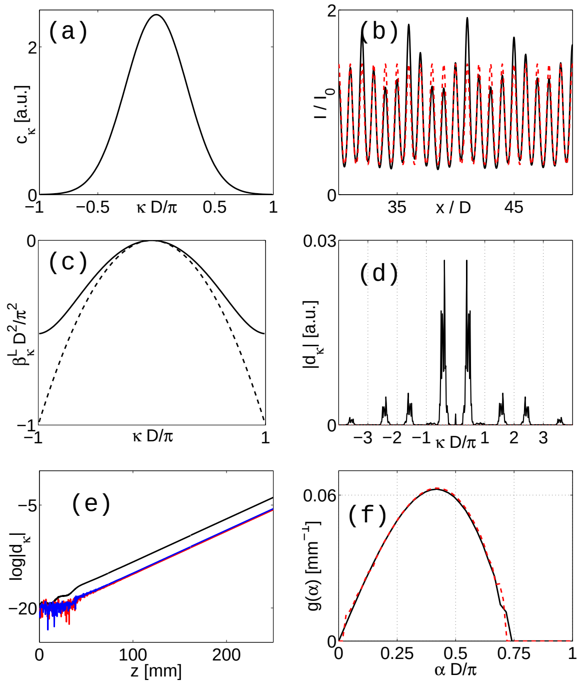

Fig. 1. Features of incoherent lattice MI. (a) The c_κ-spectrum of a stationary beam. (b) The intensity of the beam at $z = 0$ (red dashed line) and $z = 283$ mm (black solid line). (c) The linear diffraction curves β^L_κ vs. κ for $p_0 = 0$ (no lattice, dashed line) and $p_0 = 3.4 \times 10^{-4}$ (solid line). (d) The d_κ-spectrum of the noise intensity at $z = 0$ (red dotted line) and $z = 236$ mm (black solid line). (e) The exponential growth of the maximally destabilizing perturbations d_κ; $\kappa + 2\pi n/D$, $n = -1, 0, 1$. (f) The growth rates $g(\alpha)$ reduced to the 1st BZ, as calculated from the numerical evolution (red dashed line) and Eq. (9) (black solid line).

shows the intensity of $B_0 + B_1$ after propagation for $z = 283$ mm. This apparent instability, which occurs when the lattice is sufficiently deep, can be explained by observing the diffraction curves [see Fig. 1(c)] of the linear system for $p_0 = 0$ (dashed line) and $p_0 = 3.4 \times 10^{-4}$ (solid line). The curvature close to $\kappa = 0$, where most of the c_κ-spectrum is located, is smaller for a deeper lattice. Hence, diffraction is smaller. Because MI occurs when nonlinear self-focusing overcomes incoherent diffraction [2, 3], it follows that the stationary states B_0, with a spectrum similar to that of Fig. 1(a), and corresponding to deeper lattices, will be more unstable.

In order to characterize the growing eigenmodes B_α, we project the perturbation intensity $I_1(x,z) = B_1(x,x,z)$ onto the FB waves of the linear lattice: $I_1(x,z) = \sum_\kappa d_\kappa(z)\phi^L_\kappa(x) + c.c.$; the d_κ-spectrum of the perturbation at $z = 0$ and $z = 236$ mm is shown in Fig. 1(d). Clearly, the power within certain linear FB modes grows, indicating the instability. This is underpinned in Fig. 1(e) which shows the exponential growth of the d_κ-spectrum with z. We find that the growth rates corresponding to modes $\phi^L_{\kappa\pm 2\pi n/D}$, $n = 0, 1, \ldots$, are identical. For this reason, we plot the growth rates $g(\alpha)$ [red dashed line in Fig. 1(f)] in the reduced BZ scheme. This numerical observation suggests that the diagonal structure of the eigenmodes B_α is $I_\alpha(x) = B_\alpha(x,x) = \sum_n d_{\alpha+2\pi n/D}\phi^L_{\alpha+2\pi n/D}$ (n integer). The accuracy of our calculations is checked by comparing the gain curves obtained from straightforward numerical evolution and from Eq. (9); the gain

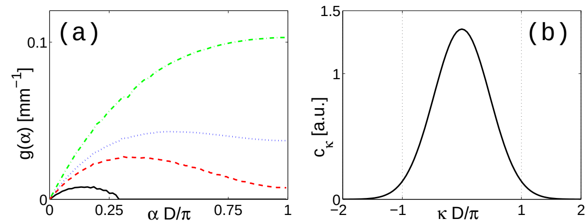

Fig. 2. (a) Gain curve for various values of the nonlinearity γI_0: 3.48×10^{-5} (black solid line), 3.74×10^{-5} (red dashed line), 3.87×10^{-5} (blue dotted line), and 4.2×10^{-5} (green dashed line). The most unstable mode locks at the edge of the first BZ, for sufficiently high values of the nonlinearity. (b) The power spectrum of the extended stationary state (see text for details).

curve obtained from Eq. (9) is shown with black solid line in Fig. 1(f). The two gain curves in Fig. 1(f) obtained with two different methods coincide.

6. Locking of the most unstable mode

The process of MI is enhanced by increasing the strength of the nonlinearity. In the case of a homogeneous medium, an incoherent beam of uniform intensity is stable (unstable) below (above) a nonlinearity threshold, whose value is determined by the degree of incoherence [2, 3]. In a similar fashion, we find such a threshold to exist in the case of incoherent lattice MI; however, the behavior of the instability for high values of the nonlinearity is fundamentally changed by the presence of the lattice.

Figure 2(a) shows the gain curves for various values of the nonlinearity. The parameters are identical to those of Fig. 1 ($p_0 = 3.4 \times 10^{-4}$) except that the c_κ-spectrum is somewhat broader [shown in Fig. 2(b)]. We clearly see that the gain of the maximally-unstable perturbation decreases for smaller values of the nonlinearity, and that there is a threshold value, below which the stationary state B_0 is stable. Just above this threshold, the gain curve (black solid line) looks similar to that of incoherent MI in homogeneous media [2], because the largest unstable (cut-off) α value is smaller than π/D. That is, the interval of unstable mode values is smaller than the extent of the 1st BZ. However, by increasing the nonlinearity further, the cut-off value reaches the edge of the BZ, and it stays *locked* at the edge for larger values of the nonlinearity. This result is a consequence of the periodic potential, and does not have its counterpart in homogeneous media. By increasing the nonlinearity further, the most unstable mode is that for $\alpha = \pi/D$, and it stays *locked* there for any higher value of the nonlinearity. It should be noted that the same locking of the most unstable mode was observed within a discrete nonlinear Schrödinger model [18]. We have obtained the same result by using a continuous model. Here we show this effect for an incoherent beam, but the same result holds in the coherent limit as well.

7. Influence of the spatial coherence; MI of a multi-band incoherent beam

The degree of spatial coherence of a beam has profound influence on its stability [2, 3], which is manifested in the existence of a threshold for incoherent MI (see Sec. 6). The degree of incoherence is associated to *the width of the spatial power spectrum*; the wider the c_κ-spectrum, the larger is the degree of incoherence. It should be noted that beside the width, the shape of the power spectrum can also affect the MI process [8]. In the case of incoherent lattice MI, the

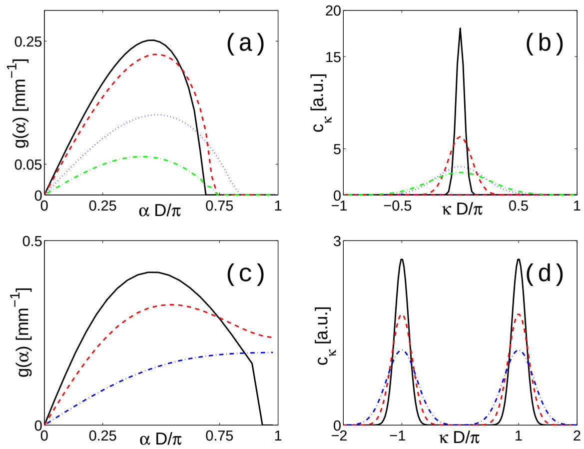

Fig. 3. Gain curves for two types of c_κ-spectrum and various degrees of the coherence; a more incoherent beam has a broader c_κ-spectrum. (a) The gain curves correspond to different widths of the bell-shaped and symmetric c_κ-spectra in (b). (c) The gain curves correspond to multi-band beams with different widths of the two-humped c_κ-spectra shown in (d).

position of the c_κ-spectrum in κ-space is important as well, because by shifting the spectrum from $\kappa = 0$, the excitation corresponds to higher bands, and in fact it can be a multi-band excitation [34, 35].

Figure 3(a) shows the gain curves for extended stationary states with c_κ-spectra shown in Fig. 3(b). Other parameters are identical to those of Fig. 1 ($p_0 = 3.4 \times 10^{-4}$). The maximal value of the gain curve [Fig. 3(a)] decreases with the decrease of coherence [broader c_κ-spectra, Fig. 3(b)], as expected from the homogeneous studies of the incoherent MI process [2, 3]. It is interesting to note that the interval of the unstable α-modes is approximately the same for all of the graphs in Fig. 3(a), while the Bloch wave of the most unstable perturbation is roughly unchanged.

A different behavior of the gain curves occurs for the multi-band excitations depicted in Figs. 3(c) and (d). The parameters are $p_0 = 2.9 \times 10^{-4}$ and $\gamma I_0 = -7.9 \times 10^{-5}$; note that in this case the nonlinearity is of the self-defocusing type. Fig. 3(d) shows the c_κ-spectra of a few extended stationary states; the spectra have two side-bands. These states excite Bloch waves from both the first and the second band, i.e., they are multi-band excitations. In addition, about half of the power excites Bloch waves from the normal, and the other half from anomalous diffraction regions. When each side-band of the spectrum is narrower (i.e., the beam is more coherent), the maximal gain value is higher, that is, the instability is stronger. However, by increasing the width of each side-lobe of the c_κ-spectrum, the most unstable mode shifts towards the edge of the Brillouin zone and gets locked there. This behavior is qualitatively different from that shown in Figs. 3(a) and (b), which points at the complexity of the incoherent lattice MI phenomena. The complexity is enhanced by the fact that specific properties of the excitation (such as the FB power spectra) can have significant influence on the process of incoherent lattice MI.

8. Incoherent lattice MI in self-defocusing nonlinear potential

It has been experimentally demonstrated that coherent lattice MI is possible in a self-defocusing medium, when a nonlinear Bloch mode is excited in the anomalous diffraction region [20]. In a

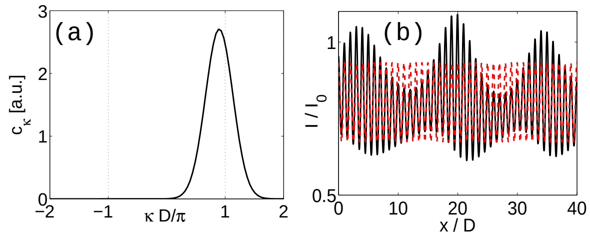

Fig. 4. Incoherent MI in self-defocusing nonlinear medium. (a) The c_κ spectrum, and (b) the intensity structure of an unstable beam at $z = 0$ (red dashed line) and $z = 263$ mm (black solid line).

similar fashion, we demonstrate the existence of incoherent lattice MI within a self-defocusing nonlinearity, with the c_κ-spectrum exciting Bloch waves mostly from the anomalous regions. Let us consider an extended incoherent beam with Gaussian c_κ-spectrum displayed in Fig. 4(a). A beam with such a spectrum in a nonlinear homogeneous medium ($p_0 = 0$) with a self-defocusing nonlinearity of the strength $\gamma I_0 = -5.25 \times 10^{-5}$ is stable (other parameters are identical to those of Fig. 1). However, within a nonlinear photonic lattice, a solution with such a c_κ-spectrum can become unstable. Figure 4(b) displays results of the numerical evolution of such extended steady state with small initial noise placed upon the beam; the depth of the lattice is $p_0 = 1.05 \times 10^{-5}$. Figure 4(b) shows the intensity structure of the beam at $z = 0$ (red dashed line), and at $z = 263$ mm (black solid line), when the instability has already developed.

9. Discussion, Conclusion, and Outlook

In this paper we have studied the stability of extended stationary incoherent beams in nonlinear photonic lattices. The intensity and coherence structure of these stationary states posses the periodicity of the lattice, and they self-consistently obey the equations of motion (see Sec. 3). The incoherent lattice MI phenomenon studied here fundamentally differs in many aspects from coherent lattice MI, as well as from incoherent MI in nonlinear homogeneous media. Quite generally, we have found that the structure of the unstable modes and their growth rates are governed by the structure of the periodic potential. A straightforward consequence of this effect is that the most unstable mode is locked at the edge of the Brillouin zone for sufficiently high value of the nonlinearity (see Sec. 6)

The extended incoherent beam is comprised of an ensemble of modes, and it can excite modes from different bands, and from regions of both normal and anomalous diffraction. This feature does not have a counterpart in the coherent lattice MI case where a single mode is excited. This fact makes the phenomena of incoherent lattice MI richer, but at the same time harder to explore (see Sec. 7). The complexity of the incoherent lattice MI phenomena is enhanced by the fact that diffraction curves change considerably by the change of the lattice parameters, which can profoundly affect MI. This is demonstrated by enhancing the lattice MI by flattening the diffraction in Sec. 5. This opens the possibility to tailor the diffraction curves to stimulate or inhibit MI. It is important to emphasize that these conclusions hold generally for both coherent and incoherent MI in continuous periodic photonic lattices.

We envision several further research themes following our current study. Recent studies of incoherent white-light solitons in nonlinear photonic lattices [29] suggest that MI in photonic lattices is possible with incoherent white light [6, 7] as well. It would be interesting to see whether all frequencies would behave collectively, as they do in a homogeneous medium [6,

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7], or certain departures from this behavior would occur because diffraction curves differ for different temporal frequencies [29]. Finally, the similarity between the dynamics of partially coherent optical- and matter-waves [36] suggests that a type of instability similar to incoherent MI should occur for partially condensed interacting bosons (either in a cigar-shaped harmonic trap [37] or in an optical lattice [38]).

Acknowledgments

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