Localisation of light in disordered lattices

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A short review of the basic concepts underlying localisation of light through multiple scattering in disordered media is provided, in conjunction with the ideas related to the universal features occurring during transport of waves and Anderson localisation. Progress in the area is described, including the recent experimental observation of localisation in photonic lattices upon which random perturbations are superimposed, which has constituted the first observation of Anderson localisation in any perturbed periodic system. Subsequently, some of the new intriguing concepts in the field of localisation of light are discussed, among them the combination of nonlinearity and disorder, and their effects on waves transport. Finally, being somewhat speculative, future directions in the area and their potential impact on the basic understanding of the universal phenomena associated with transport of waves are suggested.

In our first steps of learning solid-state physics, we are being taught that many solid materials, such as metals or semiconductors, are periodic arrays of atoms, with a crystalline structure repeating itself periodically [1]. The wave-functions of the electrons in these structures are the well-known Bloch functions, which are extended all over the lattice. The energies the electrons can possess are divided into 'allowed' bands separated by 'forbidden' gaps. The relation between the energy band-structure and the momentum (so-called dispersion curve) of the electron in the crystal determines its transport properties. In reality, however, the picture is more complex – disorder always exists, and no real material is perfectly periodic. Classically, random impurities in the crystalline structure scatter the electron and give rise to a 'random walk' motion of the electron, as if they were classical billiard balls. This is the mechanism behind diffusion and Ohm's law, where the drift velocity of the electron is proportional to the field applied to the crystal.

In 1958, Philip Anderson revolutionised the whole understanding in the field, predicting that interference effects among multiple scattering events may alter the eigenmodes of a disordered lattice, from extended states into exponentially-localised states [2]. Consequently, when an electron is initially placed on one atom, its wave-function will no longer diffuse to cover the whole crystal, but will rather remain localised around its initial position. In other words, the material will cease to conduct charge and will become an insulator. Anderson's prediction was been awarded with the Nobel prize in 1977, and today the phenomenon of Anderson localisation is a basic concept in solid-state physics. However, in spite of that, Anderson localisation has never been unequivocally observed in atomic lattices. This is because Anderson localisation occurs only if the potential (periodicity + fluctuations) is stationary in time. In atomic crystals, there are important deviations from the Anderson model owing to electron–electron interactions and due to phonons – vibrations of the crystalline structure – that are excited at finite temperatures. These interactions modify the behaviour of the electrons, causing the potential experienced by the electrons to vary in time, to the extent that Anderson localisation probably cannot be isolated in atomic lattices. In fact, the closest any experiment was getting to this 'holy grail' was to demonstrate the arrest, caused by disorder, of transport via hopping of charges between randomly-distributed impurities in a crystal [3].

In 1984, Sajeev John recognised that the idea of localisation goes far beyond solid-state physics. It is actually general to any wave system with disorder, and applies specifically also to electromagnetic waves [4]. One year later, Phillip Anderson himself continued this line of thought and published another seminal paper, attempting to write down the theory of white paint [5]. Realising there is a relation between transport of charges in crystals containing disorder and the scattering of light in random media is actually far from being straightforward, because charge-carriers are necessarily fermions whereas photons are bosons. Nonetheless, linking the two concepts has raised some basic questions about phenomena ubiquitous in everyday life, such as scattering of light from clouds, milk or grains of sugar. These media do not absorb visible light, yet clouds, milk and sugar are all opaque: light does not penetrate very far through them, because of multiple scattering.

Using electromagnetic waves to study the generic question of waves scattering in disordered potentials contributed a great deal to isolate the universal phenomenon of localisation from the other effects. More specifically, coherent light propagating in a medium with static scatterers remains coherent throughout propagation, not suffering from 'dephasing' (unlike electrons, which can lose phase coherence due to inelastic scattering). Moreover, light is comprised of photons, which are neutral bosons, and therefore do not interact with each other in a linear medium. As such, the issue of wave localisation by random disorder can be fully isolated and understood with EM waves. Indeed, shortly after Anderson related white paint to localisation, several papers appeared proposing and demonstrating coherent backscattering (enhanced reflection due to interference effects), the so-called 'weak localisation' [6–9], which is largely considered a precursor to Anderson localisation ('strong localisation').

Roughly a decade later, several experiments observed and studied strong localisation effects in random optical media – powders or suspensions of dielectric material [10–12]. These experiments studied the transmission properties of the highly scattering media, where localisation is manifested as an exponential decay of the transmitted intensity with the thickness of the sample. The challenge in such experiments is to obtain a sample with high enough scattering, but with extremely low absorption, since absorption will also lead to exponential decrease of transmission. It is therefore difficult to discriminate between the two effects, and special means were necessary [12]. The structure of the media in all of these experiments was completely random, lacking the underlying periodic potential of the Anderson model. The observation of localisation in disordered crystals, that is, periodic structures with disorder superimposed upon them, as Anderson originally predicted, was still a goal to accomplish [13]. Localisation in periodic structures fundamentally differs from localisation in completely random media, because transport phenomena in crystals display a variety of effects with no counterpart in homogeneous media. Examples include negative effective mass (anomalous dispersion), tunnelling between bands (Zener tunnelling), and the formation of gap solitons, to name a few. Moreover, photonic crystals, which are now designed for various optical devices [14, 15], would inevitably possess some degree of disorder, and therefore the influence of randomness on the transport of light is relevant also from the point of view of applications.

As described above, the traditional view on localisation of light was to observe the reduction in the intensity of the light transmitted through the disordered media. However, an alternative approach was suggested in 1989 [16]. Under the paraxial approximation, the propagation of a monochromatic light beam can be modelled by the archetypical Schrödinger equation:

$$i\frac{\partial A}{\partial z} + \frac{1}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) + \frac{k}{n_0}\Delta n(x, y)A = 0 \qquad (1)$$

Here z is the propagation direction, x and y the transverse dimensions, $k = \omega n_0/c$ is the carrier wave-number, ω the frequency of the EM wave and c the vacuum speed of light. The index of refraction is described by the bulk (linear) refractive index n_0 with a weak modulation Δn around it. $A(x, y, z)$ is the evolving envelope of the wave, such that the electric field component of the optical wave is just $E = A(x, y, z)\exp[i(kz - \omega t)]$. The similarity to the Schrödinger equation of quantum mechanics means that the propagation of light can be viewed as a propagation-dependent evolution of a two-dimensional wave-packet, in the transverse plane x–y, as the beam is propagating primarily in the z direction. Hence, the evolution of the beam behaves just like the wave-packet of a quantum particle (i.e. a single electron) in a two-dimensional potential, but with the propagation direction z replacing time, as illustrated in Fig. 1*a*. The most simple and intuitive example is that of a diffracting beam. At a particular plane, say, at the plane $z = 0$, the beam is focused and narrow, but after that plane ($z > 0$), the beam expands and its width increases linearly with the propagation distance (Fig. 1*a*, top). The idea suggested originally in [16] relies on this analogy: if we somehow construct a medium with an index of refraction that varies randomly in x and y, but uniform along the propagation direction z, the diffraction of the beam will be eliminated, and it will no longer be able to expand, just like an electron in a disordered potential, which remains pinned around its original location. In this situation the light will be localised, but rather than being localised in the propagation direction z (as in the transmission experiments [10–12]), it will be localised in the transverse plane, as shown at the bottom of Fig. 1*a*. Hence the name 'transverse localisation'.

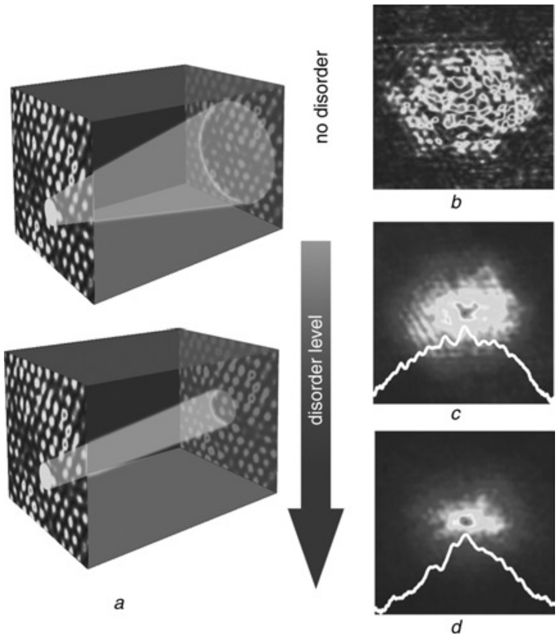

Fig. 1 *Transverse localisation of light, and experimental results*

a Transverse localisation of light: a probe beam is propagating in photonic lattice of hexagonal symmetry and a controlled level of disorder, where the index of refraction is invariant along propagation direction. Without disorder, the beam exhibits ballistic transport: its width is increasing linearly with propagation distance (top). Under influence of disorder the beam is localised in transverse plane, and maintains its width throughout propagation (bottom)
b–d Experimental results, showing (ensemble-averaged) intensity distribution at the output face of lattice. Results show gradual transition from ballistic transport (*b*), where the diffraction pattern acquires the hexagonal symmetry of the lattice, to diffusion (*c*) in presence of disorder (designated by a Gaussian shape of the intensity profile, plotted in logarithmic scale) and, at stronger disorder, to transverse localisation with exponentially-decaying intensity profile (*d*) (in a logarithmic scale as well)

Our recent paper [17] has utilised the transverse localisation scheme in order to construct an experimental realisation of Anderson's original model, and demonstrate, for the first time, Anderson localisation in disordered lattices [17]. We use the optical induction technique to induce a two-dimensional optical lattice [18–20]. Then, relying on this method, we induced propagation-invariant disorder superimposed upon the lattice. That is, we introduced random fluctuations in the refractive index in the $x-y$ plane, which were invariant in the propagation direction z, superimposed upon an otherwise-periodic photonic lattice. The propagation-invariance of the disorder plays the same role as the 'frozen disorder' in Anderson's model. Otherwise, allowing the disorder to vary noticeably during propagation prohibits the localisation phenomenon. The optical induction technique facilitates precise, real-time control over the level of the disorder and its statistics. Using this system to perform statistical (ensemble-average/expectation value) measurements, we were able to demonstrate the crossover from diffusive transport of light to Anderson localisation, as the disorder level is increased.

Our experiments are shown schematically in Fig. 1*a*. We launched a focused probe beam into the disordered lattice, and let it propagate and evolve inside the structure. In our system, in contrast to solid-state physics where only global quantities can be measured (e.g. conductance), here we have the advantage that the actual wavepacket can be monitored directly, by imaging the intensity cross-section of the probe beam exiting the lattice with a CCD camera. This provides a direct means to study the transport properties of the disordered lattice. The results are shown in Figs. 1*b–d*. In the absence of disorder (Fig. 1*b*), the beam diffracts in the periodic structure by ballistic transport, manifested in the hexagonal symmetry of the intensity pattern. When disorder is introduced, this symmetry is lost, and the intensity tunnels randomly among the lattice sites. We repeat the measurement 100 times, inducing a different realisation of the disordered lattice (with the same strength of disorder), and recording the output intensity structure of the probe beam in each iteration. Fig. 1*c* shows the ensemble-averaged intensity cross-section with moderate disorder (15%). Here, the transport is diffusive, as is evident by the Gaussian intensity profile of the intensity (whose profile, in logarithmic scale, is given by the white curve). When we increase the level of disorder, the output intensity profile narrows down (Fig. 1*d*), as the beam acquires exponentially-decaying tails (since its logarithm now decays linearly with the distance from the beam's centre). This exponential decay means that the (transverse) transport of light stops: after a short propagation distance, along which the beam expands diffusively, the (ensemble-averaged) beam diameter reaches the localisation regime and its diffraction broadening is arrested. The exponentially-decaying tails of the ensemble-averaged beam are a direct indication of Anderson localisation ('strong localisation') [17].

A most significant question related to Anderson localisation, which is intensively studied these days, is the influence of nonlinearities on the localisation process. In essence, the nonlinearity can couple the localised modes of the disordered potential in a nontrivial manner, resulting in effects that are much more complex than the linear ones (of interference between multiple reflections from scattering events). Such nonlinear interactions may appear in various forms in different systems – Coulomb or spin-exchange interactions among the electrons in solids, dipole–dipole interaction between cold atoms in Bose-Einstein condensates, to name a few. In optics, nonlinear response of the disordered medium gives rise to indirect interaction between photons, by an intensity-dependent contribution to index of refraction. One question that immediately comes to mind is what happens to the localisation process itself, under weak or strong nonlinear conditions. In our study [17], we used our system as a well-controlled tool to study this fundamental issue. The simplest way to introduce nonlinearity into our system is by increasing the intensity of the probe beam, such that it creates a nonlinear index change on top of the disordered photonic lattice. The experimental results are shown in Fig. 2. We measure the ensemble-averaged width of the beam at the output of the lattice, as a function of disorder level (Fig. 2*a*) with a nonlinearity of the self-focusing type (marked by crosses), and compare with the results of linear propagation (dots). Our experiments show that, under self-focusing (positive, attractive) nonlinearity the localisation is enhanced. Not only does the ensemble-averaged beam narrow down, but also the characteristic exponential decay of localisation appears at a lower level of disorder, for which the linear transport is still diffusive [17]. This is revealed by examining the average intensity profile, shown in Fig. 2*b* (on a logarithmic scale) for the linear case, and with various strengths of nonlinearity. Thus, self-focusing nonlinearity promotes the localisation process, at least at finite propagation distances and when the strength of the nonlinearity is not too high (i.e. the nonlinear index change is smaller than the linear index change defining the disordered lattice). We note a more recent work [21] on a 1D disordered array of coupled waveguides, which also has transverse localisation effects. That work demonstrates the existence of several localised modes individually, by scanning a probe beam across the array. In that work, nonlinear effects were also shown to promote localisation [21]. Similar results were actually pioneered in [22] for some specific realisation of disorder.

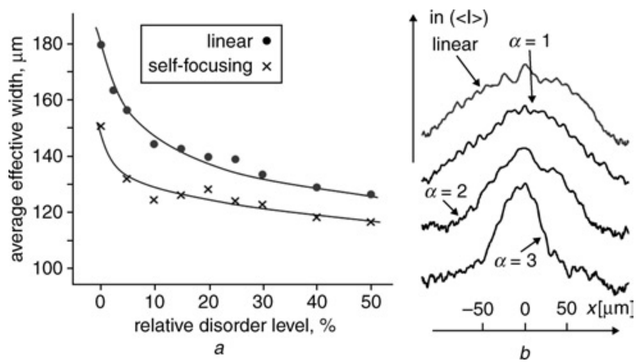

Fig. 2 *Experimental results for nonlinear propagation in disordered lattices*

a Output width against disorder strength with self-focusing nonlinearity (crosses), compared with results of linear propagation (dots)
b Ensemble-averaged intensity profiles (in logarithmic scale) at the output of lattice, showing transition from diffusion in linear propagation (upper curve) to localisation with exponential decay in nonlinear propagation (second curve from the top curve), demonstrating that self-focusing nonlinearity promotes localisation (α scales nonlinearity strength)

However, when we think of the transport in periodic structures and their associated band-structure, recalling that dispersion can be either normal or anomalous (same concept as the positive or negative effective mass of the electron in a crystal), one may expect that localisation effects would behave differently in these two regimes, when nonlinearity combines with disorder. In the anomalous dispersion regime, close to the Bragg surface (the edge of the first Brillouin zone), a wavepacket tends to narrow under a negative (self-defocusing, repulsive) nonlinearity, whereas a positive nonlinearity causes broadening of the wavepacket. For such a wavepacket in the negative effective mass regime, one would expect that negative nonlinearity would enhance the localisation process in the presence of weak disorder. This is indeed the case (at least for finite propagation distances); however, as the disorder is made stronger, the lattice potential becomes so deformed that the concept of effective mass (anomalous dispersion) no longer holds. This raises intriguing questions on the interplay between the periodic structure, disorder and nonlinearity. One such question is as follows. Launch a narrow wavepacket from the anomalous dispersion into the lattice, set a positive nonlinearity of some value smaller than the potential depth (say, by a factor of ten), and increase the level of disorder gradually. At weak disorder, the combined action of positive nonlinearity and anomalous dispersion act to broaden the wavepacket. But how would a positive nonlinearity act at high levels of disorder, when the disorder is strong enough to make the potential completely random? Preliminary studies [23] indicate that, at finite observation distances, sometimes the nonlinearity acts to broaden the wavepacket (when the nonlinear index change is still much smaller than that of the disorder), while a high enough positive nonlinearity would act in just the opposite way, narrowing the wavepacket down. How sharp is the transition from the broadening tendency to a narrowing-down tendency of a wavepacket in the presence of a positive nonlinearity and an increasing level of disorder? Could it be abrupt? Could it be discontinuous, indicating a phase transition? Then, thinking further for very long propagation distances, what would happen? Some papers claim that, after long enough distances ('infinite times' in the Schrödinger equation), the end result of nonlinear transport combined with disorder is always the same – converging to the outcome of linear Anderson localisation. Some other papers claim exactly the opposite. What happens in the transverse localisation scheme? Would the linear and nonlinear behaviours converge, or would this happen only under experimentally-unrealistic propagation distances, implying that, in practice, nonlinearity does affect transport in disordered lattices, even at large distances? These and related questions are now within experimental reach.

A different aspect of nonlinear disordered systems is the behaviour of solitons in disordered potentials. Solitons are self-trapped entities, which form in a nonlinear medium when a wavepacket induces a potential well through the nonlinearity and then captures itself in it, propagating without changing its shape. Solitons are very robust against scattering, and behave like classical particles, for instance, when interacting with each other [24–26]. On the other hand, Anderson localisation in random media relies on multiple scattering and the interference effects of waves. When these solitons will propagate in a nonlinear medium with random scatterers embedded within it, which one of the soliton aspects will prevail, the particle-like nature or the wave nature? Can a soliton form and survive in the presence of disorder at all? Several recent papers study this issue theoretically, and show some of the rich variety of phenomena which solitons in a random potential exhibit, such as diffusion of solitons and soliton percolation [27], and the arrest of soliton transport [28]. Our experimental techniques can provide an optimal platform to observe and study these phenomena, among others.

Localisation of light waves has attracted considerable research efforts in the past decade, but it is not the only direction pursued nowadays by researchers studying localisation phenomena. Localisation of waves owing to disorder can appear, in principle, in any wave systems in nature. In recent years there has been considerable effort to observe localisation of matter waves – cold atoms sharing a macroscopic wavefunction, known as Bose-Einstein condensate (BEC) [29–33]. The methods we have presented here are directly relevant to matter-waves systems, since the disordered potential acting on the BEC can also be created optically, using speckled light beam [29, 30]. In this situation, the spatial expansion of the atom cloud, as it evolves in time, will be arrested by disorder [32, 33]. In this case, the importance of doing localisation experiments in BEC goes beyond merely repeating optics experiments with matter-waves: the BEC system can be made three-dimensional, hence it offers an experimentally-viable means to observe the phase-transition associated with Anderson localisation in 3D.

To summarise, our recent observation of Anderson localisation in disordered photonic lattices opens a whole variety of experimental possibilities and provides a new path to study many questions related to localisation and nonlinear effects in disordered media. The realisation of transverse localisation using a real-time optical induction technique provides a well-controlled tool to study localisation. In particular, many new ideas have now become experimentally accessible, including effects arising from the underlying periodicity with its band-structure, the influence of nonlinearity, and more. Transverse localisation effects may also show up in the traditional transmission configuration [34]. We have discussed some of these ideas here, in the context of optics and matter-waves. However, as always happens in science when a new experimental technique is invented, most probably, the best ideas are yet to be suggested, and they will reveal new information on the universal phenomena associated with the transport of waves in random media.

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