Spatial Four Wave Mixing in Nonlinear Periodic Structures

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We present the first experimental study of spatial four-wave mixing in photonic lattices, demonstrating universal aspects of nonlinear processes in periodic media, such as engineered phase matching, Bloch-wave folding, and continuous control over the band at which the interaction products emerge.

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Nonlinear optics plays a significant role in many areas of science and technology. One of its most important manifestations is the process of four waves mixing (FWM): the interaction of four coherent optical fields through third-order susceptibility [1,2]. Over the years, FWM has been used in many important applications, ranging from energy level spectroscopy [3], real-time holography and correction of distortions [4], generation of coherent sources in UV and IR regions [5], and more. Going beyond optical waves, FWM was also observed with plasma density waves [6] and with matter waves in Bose-Einstein condensates [7]. In all of these processes, FWM is significant when the interaction is phase matched [8]. Otherwise, when the interaction is not phase matched, the FWM effects do not accumulate and their efficiency is very small.

Generally, FWM can be described as nonlinear mixing between the modes of the underlying linear system. In this sense, FWM in homogeneous media occurs between plane waves at different frequencies and/or with different k vectors. Thus, FWM in a periodic system takes place among the Floquet-Bloch (FB) waves. Fundamentally, the spatial structures of the FB waves, as well as their propagation constants (eigenvalues), are different from the uniform distribution and simple dispersion relation characterizing plane waves. Moreover, the FB waves are organized into bands, which are separated by gaps in which no propagating modes exist. Consequently, FWM in nonlinear periodic structures [e.g., 1D waveguide arrays [9,10] and 2D photonic lattices [11]] offers new possibilities for phase-matched FWM interactions, while displaying new phenomena that cannot be observed in homogeneous media.

Here, we show that, by combining one of the main nonlinear optics themes (four-wave mixing) and the fundamentals of waves propagation in periodic structures (the Floquet-Bloch theory) we can predict and experimentally demonstrate new phenomena that have no equivalent in homogeneous systems. We study phase-matched spatial FWM interactions in photonic lattices, and demonstrate universal aspects associated with FWM in lattices, such as FB wave folding of the interaction products, "engineered" phase matching, and "band control" over the FWM interaction facilitating continuous tuning determining in which band and at which momentum the new waves emerge. Our experimental study is carried out in nonlinear photonic lattices, yet its consequences are universal, applicable to any nonlinear periodic system.

Efficient FWM occurs when phase matching is satisfied [8]. However, there are many FWM configurations that are inherently not phase matchable in homogeneous systems. Consider the generic example of semidegenerate FWM between two plane waves at frequency ω_1 and a third plane wave at ω_2, resulting in a new wave at $\omega_4 = 2\omega_1 - \omega_2$. This interaction cannot be phase matched in a homogeneous (bulk) medium when the waves are collinear and have the same polarization, because $n(\omega_4) \neq 2n(\omega_1) - n(\omega_2)$ if $\omega_1 \neq \omega_2$. Here, we are interested in the spatial version of this semidegenerate FWM, where the waves are at the same frequency and polarization, but with different propagation angles [12].

Consider two plane waves at wavelength λ and wave vectors $\bar{k}_1$ and $\bar{k}_2$, propagating at angles $\theta_{1,2}$ characterized by their transverse wave vectors $k_{1x} = k_0 \sin\theta_1$ and $k_{2x} = k_0 \sin\theta_2$, with $k_0 = 2\pi n/\lambda$. In a third-order nonlinear system, these waves interact and generate new waves propagating at angles $\theta_4^{(a)}$ and $\theta_4^{(b)}$ associated with $k_{4x}^{(a)} = 2k_{2x} - k_{1x}$ and $k_{4x}^{(b)} = 2k_{1x} - k_{2x}$, provided that phase matching is satisfied, i.e., $\Delta k_z^{(a)} = 2k_{2z} - k_{1z} - k_{4z}^{(a)} = 0$ and $\Delta k_z^{(b)} = 2k_{1z} - k_{2z} - k_{4z}^{(b)} = 0$. In homogeneous media, each transverse momentum k_x is associated with a single eigenmode of the system: a plane waves represented by $Ae^{i(k_x x + k_z z)}$. Under the paraxial approximation, the dispersion (diffraction) relation (k_z versus k_x) is given by $k_z = k_0 - k_x^2/2k_0$. Consequently, in such a configuration of spatial FWM, perfect phase matching ($\Delta k_z^{(a)} = 0$ or $\Delta k_z^{(b)} = 0$) can *never* take place if all interacting waves have the same frequency and polarization [differing only in propagation angles; see Fig. 1(a)].

Consider now the equivalent semidegenerate spatial FWM interaction in a lattice. Transport properties in the lattice are determined by the propagation constants of the FB waves, $\beta(k_x)$, which are defined by the lattice periodicity and potential depth [13]. These lattice parameters can be engineered to enable perfect phase matching between FB waves from *different* bands [Fig. 1(b)], giving rise to efficient generation of new waves. Furthermore, according to Bloch Theorem, every FB wave with transverse momentum outside the first Brillouin zone ($|k_x| > \pi/D$, D being

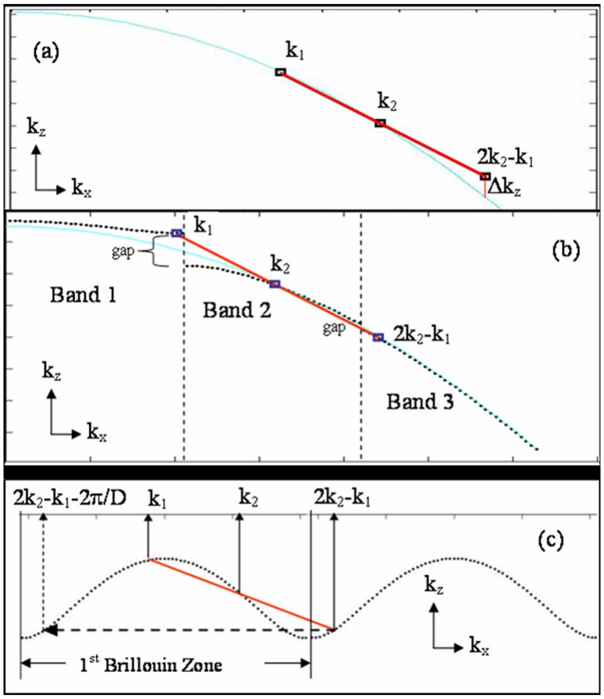

FIG. 1 (color online). Spatial FWM processes represented in the momentum space (k_x, k_z). Efficient FWM that satisfies the phase matching condition is presented as a straight line connecting k_1, k_2, and k_4. (a) Under the diffraction relations in homogeneous media, perfect phase matching is not possible. (b) The presence of a lattice enables perfect phase matching between input waves from Band 1 and Band 2, and a new wave emerging at Band 3. (c) Perfect phase matching between three FB waves from Band 1. The newly generated wave resides outside the 1st BZ, yet its propagation constant (k_z) belongs to Band 1. Consequently, the FWM interaction product "folds" back to the first BZ by one reciprocal lattice vector, and displays Band-1 characteristics (i.e., propagating at an angle smaller then $k_x = \pi/D$).

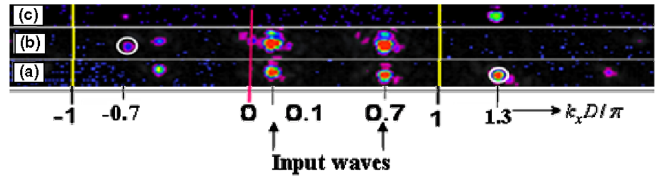

FIG. 2 (color online). Observation of Bloch-wave folding via FWM in a lattice. The edges of the first Brillouin zone are marked by the yellow lines. (a) In the absence of a lattice, one of the FWM products lies at an angle larger than π/D ($k_x = 1.3\pi/D$; circled beam). (b) With the lattice "on," that FWM product folds back by a reciprocal lattice vector into the first band and appears between $-\pi/D$ and π/D ($k_x = -0.7\pi/D$; circled beam). (c) A plane wave launched at the same angle as the newly generate wave (at $k_x = 1.3\pi/D$), results in a (linear) Bragg-reflected wave at $k_x = -0.7\pi/D$, at very low diffraction efficiency.

the lattice period) is "folded" back into the first BZ by an integer number of reciprocal lattice vectors $K_L = 2\pi/D$. Consequently, nonlinear mixing of four FB waves, all from Band 1 (three input waves and one new wave), can be perfectly phase matched. An example is shown in Fig. 1(c), where the input waves are at k_{1x}, k_{2x}, and k_{2x}, the new wave at $2k_{2x} - k_{1x} - 2\pi/D$, and $\Delta k_z = 2\beta_2 - \beta_1 - \beta_4 = 0$, i.e., the process is perfectly phase matched [14].

Our experiments are carried out in an optically induced photonic lattice in a nonlinearly anisotropic photorefractive SBN crystal [10,15]. We use a pair of plane waves to excite the two FB waves that initiate the FWM process, and monitor this interaction in momentum space by performing an optical Fourier transform [16]. Typical results demonstrating Bloch-wave folding through lattice FWM are shown in Fig. 2. The input waves are at angles corresponding to $k_x = 0.1\pi/D$ and $k_x = 0.7\pi/D$ [17]. In the absence of the lattice one of the newly generated waves lies at an angle larger than π/D [$k_x = 1.3\pi/D$; circled beam in Fig. 2(a)], and the other at $kx = -0.5\pi/D$. These waves reflect a FWM process with imperfect phase matching. The lattice alters the diffraction relations (hence the phase matching) considerably, where the change in the propagation constant β increases with k_x [Fig. 1(b)]. Consequently, the imperfect phase matching of the wave at $k_x = 1.3\pi/D$ is spoiled completely, leading to a total disappearance of that wave, while giving rise to a new wave appearing between $-\pi/D$ and π/D [at $k_x = -0.7\pi/D$, circled beam in Fig. 2(b)]. That is, the FWM product has been folded back into the first BZ by one reciprocal lattice vector. At the same time, β of the wave at $k_x = -0.5\pi/D$ is less affected by the presence of the lattice. Hence, the phase mismatch, albeit further increased, does not cause total disappearance of the wave at $k_x = -0.5\pi/D$. It is now essential to verify that the (circled) beam emerging at $k_x = -0.7\pi/D$ is indeed a nonlinear product of lattice FWM, and not just the linear Bragg-diffraction of a wave at $k_x = 1.3\pi/D$ that would have been the FWM product in the absence of a lattice. To test this, we perform a control experiment in the same lattice without the nonlinear interaction [Fig. 2(c)]: we launch a plane wave at $k_x = 1.3\pi/D$, and find that its Bragg reflection (at $k_x = -0.7\pi/D$) is at a low diffraction efficiency (5%), with the incident wave at $k_x = 1.3\pi/D$ left almost non-depleted. Thus, the circled beam in Fig. 2(b) is a truly nonlinear product of lattice four-wave mixing, which has no equivalent in homogeneous media.

Things get even more interesting for a 1D lattice in a 3D space, where the system is periodic in one transverse direction (x), and uniform both in the second transverse direction (y) and in the propagation direction (z) [Fig. 3(a)]. Here, the dispersion curve (k_z versus k_x and k_y) consists of bands and gaps in the direction of periodicity k_x, but is parabolic along the continuous dimension k_y [Fig. 3(b)]. Such systems inherently lack a complete gap, because the dispersion relation in y is continuous. Hence, linear and nonlinear coupling between bands can be more easily achieved. In particular, spatial FWM initiated by three waves with fixed FB wave vectors (e.g., one at k_{x1}

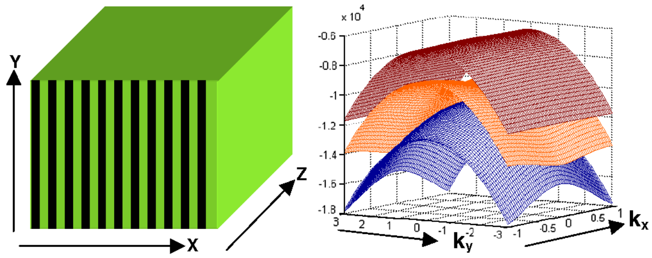

FIG. 3 (color online). (a) 1D lattice in a 3D bulk material; the index of refraction is periodic in x and uniform in y and z. (b) Band structure of a 1D lattice in 3D space. The three lower bands (higher propagation constants) are presented.

from Band 1, and two at k_{x2} from Band 2), can be manipulated by varying the angle between the input waves in the continuous direction y (thus controlling k_y). In what follows, we describe how this setting allows for selecting the band in which the FWM interaction product emerges.

We begin by defining the k vectors of the interacting waves. We let the propagation angle of the first wave be in the x-z plane only, with $\bar{k}_1 = (k_{1x}, 0, k_{1z})$, while the second wave is launched with some small angle in y, having $\bar{k}_2 = (k_{2x}, k_{2y}, k_{2z})$. The semidegenerate FWM gives rise to a new wave with $\bar{k}_4 = (k_{4x}, k_{4y}, k_{4z})$. Transverse momentum conservation implies that the new wave will emerge at $k_{4x} = 2k_{2x} - k_{1x} - 2m\pi/D$, where m is an integer, and with $k_{4y} = 2k_{2y}$. Since $k_{4y} = 2k_{2y}$, the new wave emerges at an angle in y which is (paraxially) twice the launch angle in y of the second wave. Conservation of momentum along z (phase matching) necessitates $\Delta k_z = 2k_{2z} - k_{1z} - k_{4z} = 0$. However, the propagation constant of the second wave now has two contributions, one arising from the lattice dispersion in x, and the other from the paraxial propagation in y: $k_{2z} = \beta_2 - k_{2y}^2/2k_0$. That is, the rate at which the second FB wave is accumulating phase during propagation is equal to the sum of the propagation constant in the lattice β_2 (which depends on k_{2x} and is associated with a specific band) and the paraxial addition $-k_{2y}^2/2k_0$. Likewise, the propagation constant of the (new) fourth wave is $k_{4z} = \beta_4 - k_{4y}^2/2k_0 = \beta_4 - (2k_{2y})^2/2k_0 = \beta_4 - 4k_{2y}^2/2k_0$. Consequently, the phase matching condition is now $\Delta k_z = 2k_{2z} - k_{1z} - k_{4z} = 2\beta_2 - \beta_1 - \beta_4 - 2k_{2y}^2/2k_0 + 4k_{2y}^2/2k_0 = 0$. Hence, the new wave is propagating with $\beta_4 = 2\beta_2 - \beta_1 + k_{2y}^2/k_0$, highlighting the fact that one can select the band in which the new wave will appear (manifested in β_4) by continuously tuning the angle in y of the second wave (determined by k_{2y}). Evidently, having a 1D lattice in a 3D space introduces an additional degree of freedom, where the momentum mismatch caused by lattice dispersion (from periodicity in x) can be balanced by the dispersion arising from the homogeneous direction y. This additional degree of freedom enables perfect phase matching between FB waves from different bands, even when the longitudinal momentum difference, $\Delta\beta = 2\beta_2 - \beta_1 - \beta_4$, is very large. Moreover, since the angle in y is fully tunable, it facilitates complete tunability over the FWM interaction, and the ability to choose the band in which the interaction product will emerge, for practically any choice of input FB waves (and bands) initiating the interaction.

We now demonstrate this idea experimentally (Fig. 4). We excite two FB waves by launching two plane waves at the appropriate angles [17]; the Band-1 FB wave is excited at $k_{1x} = 0.37\pi/D$, while the Band-2 FB wave with $k_{2x} = 1.45\pi/D$ (Fig. 4, middle columns). Throughout this experiment, we maintain k_{1x} and k_{2x} fixed (thus keeping the transverse momenta and the band number of the two input FB waves fixed), and tune the angle of the Band- 2 FB wave in the y direction continuously. Consider first the simplest case, where $k_{2y} = 0$ (or very small), for which the system acts in a way similar to a pure 1D lattice (with no dynamics in the y dimension). In this case, the FWM interaction leads to a new FB wave with $k_{4x} = 2k_{2x} - k_{1x}$ from Band 3, with perfect phase matching $2k_{2z} - k_{1z} - k_{4z} = 2\beta_2^{(2)} - \beta_1^{(1)} - \beta_4^{(3)} = 0$, as sketched in Fig. 1(b) for a pure 1D lattice. The experimental demonstration of such an interaction is shown in Fig. 4(a). The Band-2 FB wave is tilted by a small angle $k_{2y} = 0.3\pi/D$. Hence, the phase matching condition is satisfied for a Band-3 wave, and the newly generated wave emerges at $k_{3x} = 2.53\pi/D$ [circled beam in Fig. 4(a), which resides in the third band], and $k_{3y} = 0.6\pi/D$.

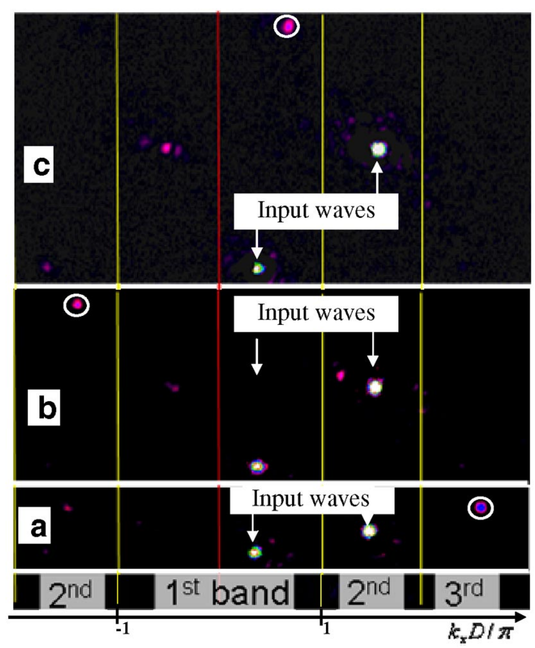

FIG. 4 (color online). Band control of the FWM product. The edges of the first Brillouin zone are marked by the yellow lines. The FWM products are marked by white circles. (a) For a small tilt angle in y, phase matching is satisfied for a Band-3 FB wave, hence the FWM product resides in the third band. (b) For a larger tilt angle in y, phase matching occurs for a Band-2 FB wave. (c) An even larger angular shift results in a FWM product residing in Band 1.

Next, we increase k_{2y} to be π/D, while keeping k_{1x} and k_{2x} the same as in the previous experiment. The FWM interaction is now able to generate a new FB wave whose transverse momentum still is $k_{4x} = 2k_{2x} - k_{1x}$ (up to an integer number of reciprocal lattice vectors). This wave emerges at a y angle corresponding to $k_{4y} = 2k_{2y}$, and the phase matching condition is now $\beta_4 = 2\beta_2 - \beta_1 + k_{2y}^2/k_0$. Consequently, the new wave necessarily has a higher propagation constant than the $k_{2y} = 0$ case. For a proper range of k_{2y} values, the new wave resides in Band 2. This is demonstrated in Fig. 4(b), with the new wave appearing at $k_{3x} = 1.53\pi/D$, which corresponds to the Band 2, and with $k_{4y} = 2\pi/D$ [circled beam in Fig. 4(b)].

Finally, we demonstrate how a FWM product of input waves from Band 1 and Band 2 can emerge in Band 1 [Fig. 4(c)]. To do that, we further increase the y angle of the Band-2 FB wave to $k_y = 1.6\pi/D$, while keeping k_{1x} and k_{2x} the same as in the previous experiments. The larger angular shift results in a larger contribution to phase matching, $\beta_4 = 2\beta_2 - \beta_1 + k_{2y}^2/k_0$, and leads to a new wave with a large β_4 that resides in Band 1. Consequently, the FWM product appears at $k_{3x} = 0.53\pi/D$ (Band 1), with $k_{4y} = 3.2\pi/D$ [circled beam in Fig. 4(c)].

Before closing, we compare the basic ideas presented above and temporal frequency mixing processes in periodic optical structures, where it is known that the periodicity is able to enhance the phase matching [18]. In such systems, the refractive index is periodic along the propagation direction (i.e., has a grating vector along z, K_z) or along the transverse direction x (having a grating vector K_x). The wave-mixing process occurs between temporal frequencies ω, hence the dispersion relation, $k_z(\omega)$, is modified by the presence of the grating, which alters phase matching. However, for temporal frequency conversion processes the unique features associated with the periodicity (band structure, Bloch modes, etc.) are *not* manifested in the dispersion curve. Hence, temporal frequency mixing in photonic lattices fundamentally cannot give rise to Bloch-wave folding, control over the band at which the FWM products emerge, etc. In contradistinction to our *spatial* system, where the mixing is between spatial frequencies, k_x, where x is the same direction as the direction of the periodicity, hence the dispersion curve $k_z(k_x)$ exhibits bands and gaps, etc, This is what enables the new phenomena reported here, having no equivalent in nonlinear mixing between temporal frequencies in periodic systems.

In conclusion, we explored various phenomena associated with spatial four-wave mixing in photonic lattices, such as Bloch-wave folding, and continuous control over the band at which the interaction products emerge. To our knowledge, this is the first experimental study on spatial FWM in photonic lattices and crystals. The physics studied here is universal to all nonlinear periodic systems in nature. Indeed, recent works on cold atoms have predicted [19] and experimentally demonstrated [20] third-order parametric interactions between matter waves in optical lattices. The experimental work with Bose-Einstein condensates [20] was dedicated solely to interactions among Band-1 FB waves. The new concepts presented here, such as interactions between FB waves from different bands, and the utilization of a homogeneous dimension to control the FWM interaction product ("band control"), introduce new possibilities that are realizable with cold atoms and with other nonlinear periodic systems.

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[17] A plane wave launched into a lattice is naturally projected onto a set of FB waves from different bands, all having the same quasimomentum $k_x = 2\pi \sin\theta/\lambda$, θ being the angle of incidence. When k_x is not near a band edge, a plane wave projects almost solely onto a single FB wave: the one which would coincide with a plane wave when the lattice depth is taken to zero. For example, a plane wave launched at nearly normal incidence will excite almost solely a FB wave from Band 1, and a plane wave launched at $\pi/D < k_x < 2\pi/D$ (not too close to the band edges) will excite primarily a FB wave from Band 2. On the other hand, a plane wave launched at the Bragg angle will always excite FB waves from multiple bands. In these experiments, we always launch waves at angles sufficiently displaced from the Bragg angles, so as to maximize the projection onto pure FB waves from single bands.

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