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Anderson localization of light

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Over the past decade, the Anderson localization of light and a wide variety of associated phenomena have come to the forefront of research. Numerous investigations have been made into the underlying physics of how disorder affects transport in a crystalline lattice incorporating disorder. The physics involved relies on the analogy between the paraxial equation for electromagnetic waves and the Schrödinger equation describing quantum phenomena. Experiments have revealed how wavefunctions evolve during the localization process, and have led to discoveries of new physics that are universal to wave systems incorporating disorder. This Review summarizes the phenomena associated with the transverse localization of light, with an emphasis on the history, new ideas and future exploration of the field.

The scattering of waves due to disorder is ubiquitous throughout nature. Examples range from the scattering of light as it passes through clouds or sugar to the scattering of sound waves in the presence of fluctuations. These phenomena have intrigued scientists for centuries. However, a major breakthrough in the understanding came in 1958 from a seemingly unrelated context: the coherent scattering of electrons passing through crystals that contain disorder. Phillip Anderson proposed that scattering from disorder can bring transport to a complete halt[1]. Almost 20 years later, Anderson made a speech on the Nobel podium, stating: "Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it."

Before Anderson's discovery, scientists modelled crystal disorders as perturbations that scatter electrons randomly, treating electrons as point-like particles. This logic led to the description of transport in such media as Brownian motion, which underlies Ohm's law. However, in his paper, Anderson revisited the effect of disorder on the evolution of an electron's wavefunction in an otherwise periodic crystal[1]. Anderson analysed the problem in the quantum regime, thus fundamentally accounting for the wave nature of the electron, and found that the classical diffusive motion of the electron breaks down as the electronic wavefunction becomes exponentially localized, under a broad range of conditions. Consequently, when the electron is initially placed on one atom, its wavefunction will no longer expand to cover the whole crystal with time, but it will rather remain localized around its initial position. Thus, the material will cease to conduct charge, and eventually become an insulator. This localization phenomenon is a direct consequence of interference between different paths arising from multiple scattering of the electron by lattice defects.

One of the inherent assumptions of Anderson's model is that the potential is time-invariant. In reality, however, temporal variations in the potential (lattice + disorder) tend to dimish localization effects. Moreover, the presence of temporal fluctuations (such as phonons) reduces the coherence of the scattering process, which destroys the interference effects and eventually leads to the recovery of Ohm's law. Perhaps even more importantly, Anderson's model represents a single particle, or an ensemble of non-interacting particles. But electrons are fermions, which fundamentally interact (through Coulomb's law or spin exchange, for example). When interactions are included, the scenario changes dramatically and localization generally does not occur. These two preconditions underlying Anderson's model — time-invariance of the potential and the absence of interactions — posed great difficulties for observing Anderson localization in atomic crystals. Nevertheless, as localization is in essence a wave-mechanics phenomenon, scientists later realized that it is universal to all wave systems, and in particular it should occur in optics[2–4]. In fact, the random scattering of light is ubiquitous throughout nature, occurring, for example, in clouds, milk and sugar. These media are all microscopically transparent to light; however, they appear opaque owing to the multiple scattering of light travelling through them. Optics seems an ideal framework to study localization effects, as coherence is naturally preserved and photons are inherently non-interacting bosons. This was the logic behind pioneering experiments that studied the transmission of electromagnetic waves through random media, which showed exponential decay of transmittance with sample length[5–8].

Anderson localization in photonic lattices containing disorder

In the late 1980s, De Raedt, Lagendijk and de Vries proposed the 'transverse localization scheme' for studying the effects of disorder on the transport of light, but this proposition was not adopted for a long time[9]. Interestingly, a similar suggestion had been made almost a decade earlier by Abdullaev[10], but as it appeared in a Russian journal and was omitted from the translated edition, it remained practically unknown. Major advances came from a different area altogether: research into discrete solitons. In 1988, the existence of solitons in waveguide arrays was first suggested. This problem, under some approximations can be modelled by a discrete cubic Schrödinger-type equation, hence solitons found in this setting are called discrete solitons[11]. It took a decade for scientists to observe discrete solitons experimentally[12], and another five years to demonstrate them in two transverse dimensions[13], but by 2005 this area of nonlinear waves and solitons in photonic lattices (waveguide arrays) had become a major research area in nonlinear optics and soliton science (see recent reviews by Christodoulides *et al.*[14] and Lederer *et al.*[15]). It was therefore natural to search for localization effects in paraxial disordered photonic systems. However, early experiments seemed rather discouraging[16] or very preliminary[17], partly because some of the concepts were missing (for example, the necessity for ensemble averaging to obtain meaningful results), and partly because the transverse localization scheme[9] was unknown to researchers in the area of solitons. It was not until 2007 that the first successful transverse localization experiments were performed[18,19], and

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many experiments followed (see, for example, refs 20–22). The transverse localization of light has become one of the most convenient and direct schemes for observing localization effects in all research areas. More importantly, many new ideas have emerged from these experiments, such as disorder-enhanced transport in quasiperiodic systems, super-ballistic transport (hyper-transport) and the quantum aspects of localization with entangled states.

The transverse localization scheme is explained in Box 1. In this scheme, the evolution of a light beam in space behaves like the wave packet of a quantum particle, with the propagation coordinate z replacing time. It is important to emphasize the role of ensemble-averaging in the transverse localization scheme, as demonstrated by the first experiments[18]. Namely, to obtain meaningful experimental results in the photonic system, one must average over multiple realizations of the disorder because the typical propagation distance is too short to support self-averaging. The most important observable in the transverse localization scheme is therefore the intensity structure of the beam, $\langle|\Psi(x,y,z = L)|^2\rangle$, emerging from the photonic system after propagating a distance L, averaged over multiple (~100) experiments with different spatial realizations of disorder.

The analogy between the Schrödinger equation and paraxial optics has been applied in many recent experiments to demonstrate concepts from solid-state physics in an optical setting[14,15]. In the study of localization, this approach has opened the possibility of following the evolution of wave packets in disordered media at the microscopic level. Together with the inherent flexibility and controllability of optical systems, this approach led to the first direct observations of localization[18], as originally described by Anderson: the expansion of an initially confined wave packet comes to a complete halt and attains exponentially decaying tails due to multiple scattering in a disordered crystal. In addition, this approach made it possible to observe other predictions of the theory, such as how individual Bloch modes become localized[19]. Absorption and thermal vibrations of the underlying potential are not problematic in this system; rather, they can be introduced in a controlled manner. This approach, which was originally conceived and implemented in the context of optical waves, was later echoed in experiments on the Anderson localization of matter waves[23,24]. It is important, however, to point out that the transverse localization scheme is associated with transport effects in one or two dimensions, and cannot describe transport phenomena in three dimensions, such as the phase transition associated with Anderson localization in three dimensions. Such phenomena must be studied using short pulses propagating in a volume whose disorder is frozen in time (see examples in optics[6,8,25] and with cold atoms[26,27]). This Review is primarily dedicated to phenomena associated with transverse localization.

An important additional aspect of these optical set-ups is that they were the first to enable experimental studies of a related new problem: the interplay between nonlinear interactions and Anderson localization. The phenomenon Anderson predicted is a non-interacting linear interference effect. It therefore corresponds to disordered systems containing a single particle, or many identical non-interacting particles. This, however, is not generally the case for real systems, which usually contain many particles with non-negligible mutual interactions. The problem of the interplay between disorder and interactions is currently a great challenge in modern solid-state physics[28,29]. Nonlinear interactions may appear in various forms in different systems, such as Coulomb or spin-exchange interactions among the electrons in a solid, and dipole–dipole interactions between cold atoms. In optics, the nonlinear response of a disordered medium gives rise to indirect interactions between the photons through various mechanisms, such as intensity-dependent contributions to the refractive index. This mechanism corresponds to the case of cold atoms with pair-wise attractive or repulsive interactions. This kind of optical nonlinearity enters the evolution equation as a nonlinear term $\Delta n_{\mathrm{NL}} = f(|\Psi|^2)$ added to the index change Δn (the 'potential' term) in equation (1). Hence, this simple system of the Anderson localization of light in the presence of nonlinearity provides a basis for obtaining a better understanding of complex quantum many-body systems.

Figure 1 shows the configuration used by Schwartz *et al.* to observe Anderson localization in two-dimensional (2D) disordered lattices[18], together with characteristic photographs of the ensemble-averaged intensity patterns depicting the transition from ballistic to diffusive propagation when disorder is introduced, and eventually leading to localization when the strength of the disorder is increased. A narrow probe beam is launched into the photonic lattice. In the absence of disorder, the beam expands during propagation (Fig. 1a, left) due to coupling between adjacent waveguides. In this scheme, the disorder halts the broadening of the beam (Fig. 1a, right) such that it becomes exponentially localized in that transverse plane (hence the term 'transverse localization'). The experimental observation is shown in Fig. 1b–d. In the absence of disorder, the beam diffracts in the periodic structure by ballistic transport, which is manifested in the triangular symmetry of the intensity pattern (Fig. 1b) and in the fact that the width of the expanding beam grows proportionally with the propagation distance. When weak disorder is introduced, this symmetry is lost, and the intensity tunnels randomly among the lattice sites (Fig. 1c). Here, the transport is diffusive, as is evident by the Gaussian profile of the ensemble-averaged intensity pattern. When the level of disorder is increased, the output intensity profile narrows and the beam acquires exponentially decaying tails (Fig. 1d). This exponential decay indicates that the (transverse) transport of light stops; after a short propagation distance, during which the beam expands diffusively, the (ensemble-averaged) beam diameter reaches the localization regime and its diffraction broadening is arrested.

Shortly after the experiments by Schwartz *et al.*[18], Lahini *et al.* carried out similar experiments in 1D lattices (1D arrays of waveguides)[19]. These experiments demonstrated that, in one dimension, as the disorder level is increased, transport changes from ballistic to localized without the intermediate diffusive regime observed in two dimensions. The experiments of Lahini *et al.*[19] enabled the direct observation of how extended states (Bloch modes associated with

Box 1 | Transverse localization scheme

The evolution of optical waves in the transverse localization scheme is described by the Schrödinger-type paraxial equation for monochromatic light:

$$i\frac{\partial\Psi}{\partial z} = \left[-\frac{1}{2k}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) - \frac{k}{n_o}\,\Delta n(x,y,z)\right]\Psi \equiv \hat{H}\Psi \qquad (1)$$

where z is the propagation coordinate, x and y are the transverse dimensions, Ψ is the slowly varying envelope of the field $E(\mathbf{r}, t) = \mathrm{Re}[\Psi(x, y, z)e^{i(kz-\omega t)}]$ of frequency ω and wavenumber $k = \omega n_0/c$, n_0 is the bulk refractive index and Δn is the local change in refractive index (lattice plus disorder), with $|\Delta n| << n_0$. Equation (1) has the form of the Schrödinger equation when $z \rightarrow$ t and $-\Delta n \rightarrow$ V. Hence, the evolution of a light beam in space behaves like the wave packet of a quantum particle, with z replacing time. Anderson localization requires a stationary potential, which implies that the index change Δn in equation (1) must be propagation invariant; that is, $\Delta n(x, y)$ must be z independent.

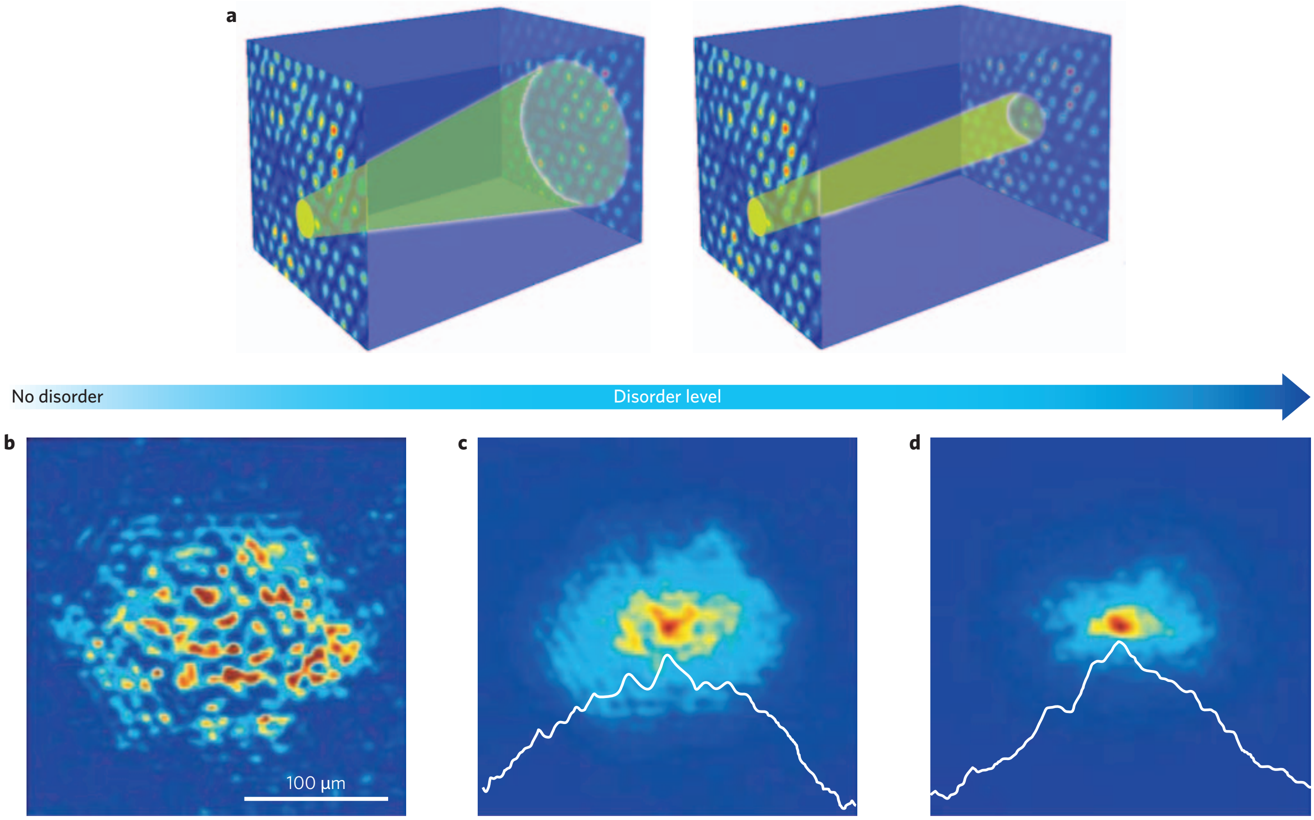

Figure 1 | Transition from ballistic transport to diffusive transport, and eventually to Anderson localization. a, Transverse localization scheme. A probe beam propagates in a photonic lattice with a controlled level of disorder. Without disorder, the beam exhibits ballistic transport; its width increases linearly with propagation distance (left). Under the influence of disorder, the beam becomes exponentially localized in the transverse plane, maintaining its mean width throughout propagation (right). **b**–**d**, Ensemble-averaged intensity distribution at the output face of the lattice. The results reveal a gradual transition from ballistic transport (**b**), where the diffraction pattern reflects the lattice symmetry, to diffusion (**c**) in the presence of disorder (intensity profile has a Gaussian shape, plotted in logarithmic scale), and, at stronger disorder, to Anderson localization with exponentially decaying tails (**d**). Figure reproduced with permission from ref. 18, © 2007 NPG.

the periodic lattice) transform into exponentially localized states (Anderson modes). The localized eigenmodes of a disordered lattice have one particularly interesting property: modes near the band edges (of the spatial spectrum) are tightly localized, whereas those near the middle of the band are typically broader[19]. This creates a situation in which, for a finite system, the modes from the band edges are localized, while the width of mid-band modes is larger than the size of the system. In such a case, the mid-band modes behave as extended states (Bloch modes). Hence, starting from an ideal periodic lattice and increasing the strength of the disorder transforms more of the extended Bloch modes into highly localized Anderson states, with the band-edge modes first becoming localized. Lahini *et al.* observed such transformation of Bloch modes into localized modes[19].

Nonlinearity and disorder

The interplay between disorder and nonlinearity is an important issue relating to Anderson localization. One obvious question is what happens to the localization process itself, under weak or strong nonlinear conditions. For years, this issue has been controversial in studies examining the theoretical aspects of Anderson localization. One early study[30] conjectured that asymptotically localization prevails. Tight-binding simulations in 1D systems, specifically for Kerr-type nonlinearity, revealed that nonlinearity strongly affects the localization process[31,32], thus leading to sub-diffusive transport. Analytic (perturbative) attempts[33] to resolve the issue were able to describe only the early stages of evolution, and supported the conjecture of asymptotic localization. However, the problem remains unsolved to this day (see review by Fishman *et al.*[34]).

Experimentally, the first attempts to address Anderson localization in the presence of nonlinearity were presented by Schwartz *et al.*[18] and Lahini *et al.*[19]. The simplest way to introduce nonlinearity into an optical system is by increasing the intensity of the probe beam, such that it creates a nonlinear index change at the top of the disordered lattice. The experiments of Schwartz *et al.*[18] and Lahini *et al.*[19] showed that the localization process is enhanced under self-focusing nonlinearity. Not only does the ensemble-averaged beam become narrower, but also the characteristic exponential decay of localization appears at a lower disorder level.

The influence of nonlinearity on transport in disordered lattices can be rather complex, in a manner similar to the band structure associated with periodic media. Because dispersion can be either normal or anomalous (analogous to positive or negative effective mass of the electron in a crystal), one may expect localization effects to behave differently in these two regimes, when nonlinearity combines with disorder. In the anomalous dispersion regime, a wave packet tends to narrow under self-defocusing nonlinearity, whereas a self-focusing nonlinearity causes broadening. For a wave packet in the negative effective mass regime, one would expect self-focusing to delay the localization process in the presence of weak disorder. This is indeed the case for short propagation distances; however, for strong disorder the concept of effective mass no longer holds, and hence at some disorder level the system should change its behaviour abruptly. These ideas raise intriguing questions regarding the interplay between the periodicity of the structure, disorder and nonlinearity.

Localization and enhanced transport in quasicrystals

Quasicrystals (QCs)[35,36] (see also the associated Review by Vardeny in this issue[37]) form a class of structures that constitutes an intermediate phase between fully periodic and fully disordered media. They do not have a unit cell and do not exhibit translational symmetry, yet they possess long-range order and display Bragg diffraction. The eigenstates of QCs are multifractal critical states, which may be normalizable (and thus localized) or not (in which case they act as extended states). Many of the properties of QCs are now well understood, but some fundamental questions remain. Perhaps one of the most intriguing questions is associated with transport, which is directly related to the critical nature of a QC's eigenstates, particularly in the presence of disorder[38]. In contrast with crystals, in which disorder always acts to arrest transport, it has been suggested that disorder can enhance transport in QCs[39]. Indirect experiments have shown that, in some regime, increasing disorder can enhance transport[40]. However, until recently, transport in QCs containing disorder was not understood, and the experiments, being indirect, did not help much to unravel the physics involved.

Researchers have studied photonic QCs in the domain of electromagnetic waves for some time now[41–45]. Studies of disordered photonic QCs were therefore expected sooner or later. Indeed, two recent studies explored this topic: one investigated disorder-enhanced transport and localization effects[18], while the other[20] demonstrated the phase transition associated with the 1D potential described by the Aubry–André model[46] (see below).

Studies of disorder-enhanced transport in photonic QCs[22] were carried out in a system similar to that of Schwartz *et al.*[18], but employing a quasicrystalline lattice. Because this system facilitated imaging of the propagating wavefunction, it enabled the first direct experimental observation of disorder-enhanced transport in QCs[22]. Indeed, disorder considerably enhances the transport of wave packets associated with eigenstates in the proximity of a pseudo-gap (the region of the Fermi energy in electronic systems). These experiments helped explain the underlying physical reason for this enhancement: disorder-enhanced transport occurs because disorder acts to couple highly localized states near the pseudo-gap, and consequently the states become more extended. When disorder is further increased, experiments revealed finite-range, diffusive-like transport. On increasing the disorder even further, localization eventually prevails: the width of the wave packet shrinks, and its tails display exponential decay.

Another interesting connection between QCs and localization is displayed in a 1D system known as the Aubry–André model. In this system, the onsite energy (or, equivalently, the tunnelling coefficient) is modulated spatially with a periodicity that is incommensurate with the lattice periodicity[46]. This system is known to be a 1D QC. An interesting feature of the spectrum of this system is the existence of a threshold value for the modulation strength, beyond which all eigenstates convert from extended to localized. Such a transition to localization in a standard Anderson localization system occurs only in three dimensions, and hence is not accessible to transverse localization experiments. Lahini *et al.* reported the first realization of the Aubry–André system and observation of the localization transition, together with a description of the nonlinear effects on the localization in this system[20].

Finite quasiperiodic Aubry–André systems, even below the localization transition, have been found to support localized edge states on one of their boundaries. In recent work, this observation has provided a new understanding of the connection between QCs and the class of novel states known as topological insulators[47]. This work not only showed that a 1D QC can support non-trivial topological states, which were previously believed to exist only in two dimensions or higher, but also gave the first experimental demonstration of the process of adiabatic pumping — known sometimes as Laughlin pumping — in which the localized states shift from one side to the other in an adiabatically modified structure.

Hyper-transport stochastic acceleration by evolving disorder

Since Anderson's initial study in 1958, scientists have known that localization requires the potential to be constant in time ('frozen'). Otherwise, if the disorder evolves dynamically (for example, fluctuates in time), localization breaks down and transport resumes. But would such transport be diffusive, or would disorder increase the transport rate beyond diffusion? A recent study presented experiments[48] supported by simulations, and subsequently by a semi-analytic model[49], showing that an evolving random potential can give rise to hyper-transport. In this regime, the spatially disordered fluctuating potential causes stochastic acceleration, which makes an initial wave packet expand at a faster rate than ballistic, while its transverse momentum spectrum expands continuously. Although these experiments were carried out in a disordered photonic system in the transverse localization scheme, the phenomenon is relevant to all waves systems containing disorder.

Anderson localization has been traditionally studied in periodic systems containing disorder[18,19] (see also references in ref. 50 on electronic systems) and in fully random potentials[5–8,51], but in both cases the disorder is 'frozen'. Some researchers have also explored transport in potentials that are random in space and fluctuate in time. However, only a handful of studies — all strictly theoretical — have suggested hyper-transport, transport mechanisms through which the region within which a particle can be found expands faster than ballistic expansion[52–56]. A picture of such motion, in terms of resonances between the particle and the potential, was developed in 1972[52]. Ten years later, a related quantum model[53] showed that the root-mean-squared displacement of the particle in a temporally fluctuating spatially random potential grows with an exponent of 3/2 in time (whereas the exponent is 1/2 for diffusion and 1 for ballistic transport). Later theoretical studies identified the hyper-transport of particles[54–56] in fluctuating potentials with correlated disorder (that is, when the bandwidth of the disorder is finite). Finally, the first experimental proof that evolving disorder can give rise to hyper-transport was presented by Levi *et al.* in 2012[48].

The hallmark of ballistic transport is that the expansion rate of a wave packet is proportional to time, while the width of its spectrum in momentum space remains constant. In the hyper-transport regime, the wave packet expands at a much faster rate than that of ballistic evolution, while at the same time its width in momentum-space also expands dramatically. Experiments performed by Levi *et al.* were carried out in the transverse localization scheme described by equation (1): a probe beam was launched into a photonic medium containing spatial disorder that also fluctuated dynamically in the propagation direction. The experiments were repeated many times with different realizations of the disorder, and meaningful results were obtained by ensemble averaging[48]. The ensemble-averaged beam intensity and its spatial spectrum are analogous to the probability amplitudes of finding a quantum particle or measuring its momentum, respectively. This direct analogy with transport in quantum systems made the findings relevant for many wave systems containing disorder.

Typical experimental results are shown in Fig. 2. First consider two established cases: free (ballistic) diffraction and localization. Figure 2a shows the intensity cross-section of the beam in the absence of disorder: the 514 nm wavelength Gaussian beam of 15 μm (full-width at half-maximum) diffracts freely for 1 cm, experiencing ballistic transport in the medium, to an output width of 166.42 μm. Figure 2b shows the ensemble-averaged intensity structure of the

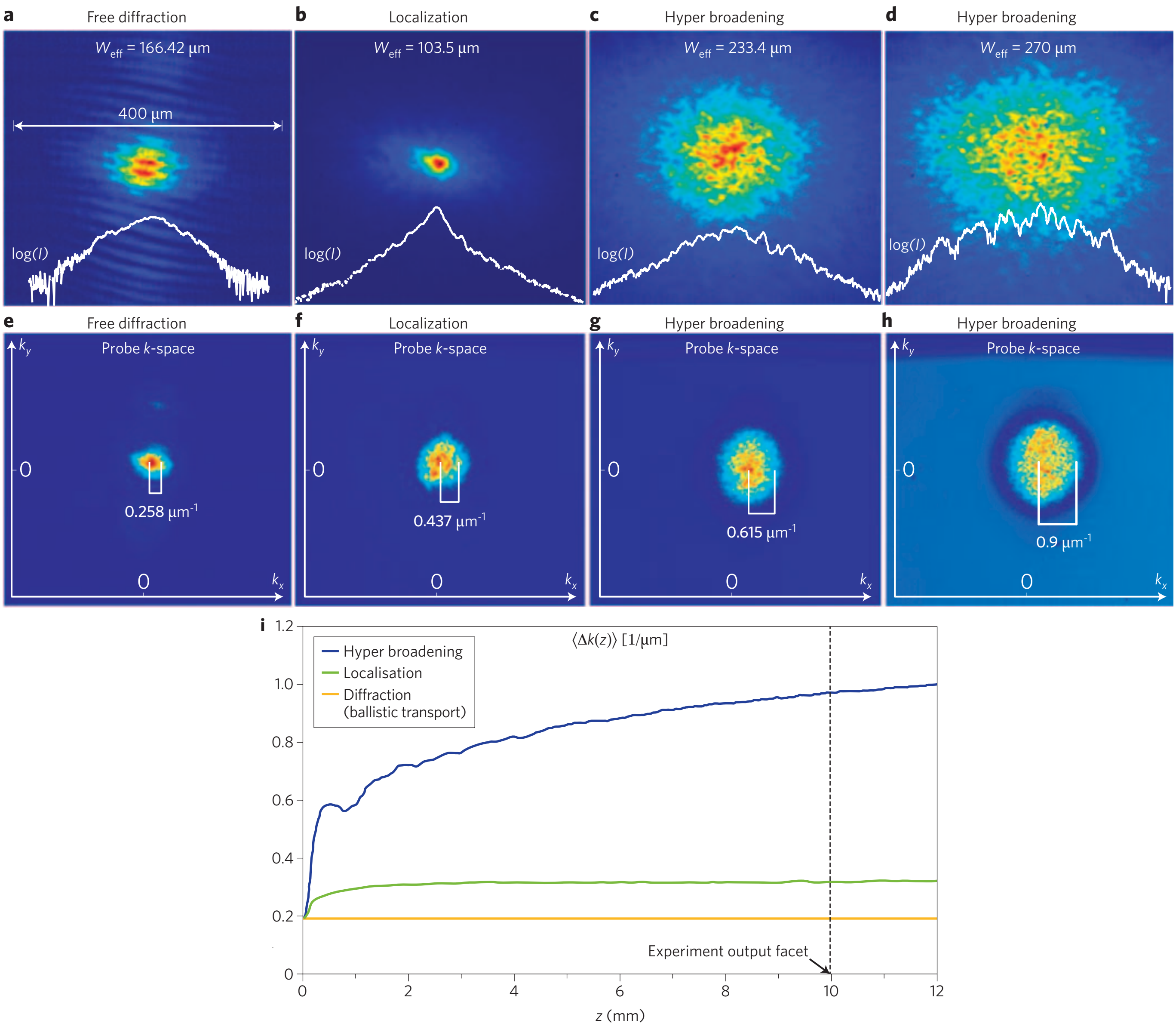

Figure 2 | Hyper-transport of a light beam propagating through fluctuating spatial disorder. a–d, Ensemble-averaged shape of the beam exiting the medium (cross-sections displayed in a logarithmic scale, with their corresponding width W_{eff}). **a**, Without disorder, the beam undergoes ballistic transport. **b**, When disorder is propagation-invariant, the beam displays Anderson localization, which is manifested in its exponential structure. **c**,**d**, When disorder evolves during propagation, the beam expands faster than it would in the ballistic transport regime. **e**–**h**, Corresponding spatial power spectra of the beams displayed in panels **a**–**d**. **i**, Simulation results showing the width of the ensemble-averaged power spectrum of the beams, undergoing ballistic transport (homogeneous medium; lower curve), localization (propagation-invariant disorder; middle curve) and hyper-transport (evolving disorder; upper curve). For ballistic transport, the spectral width is conserved. For localization, the spectrum initially expands but once localization is reached the width remains unchanged. In contrast, the spectrum of a beam undergoing hyper-transport expands continuously. Figure reproduced with permission from ref. 48, © 2012 APS.

beam exiting the medium, after propagating through the z-invariant spatial disorder. This beam is exponentially localized with a width of around 103 μm — much narrower than the freely diffracting beam of Fig. 2a.

Consider now experiments with dynamically evolving (fluctuating) disorder, and examine what happens when the rate of dynamic fluctuations is increased. Figure 2c,d shows the ensemble-averaged intensity structure for the same initial wave packet, after propagating in the presence of evolving disorder. The disorder evolves faster in Fig. 2d than in Fig. 2c. The widths of the (ensemble-averaged) beams experiencing dynamic disorder are considerably larger than that of the freely diffracting beam: ~230 μm and ~270 μm, compared with the ~166 μm beam of Fig. 2a. The beams propagating through the rapidly fluctuating spatial disorder exhibit hyper-transport. The widths of these beams increase with evolution rate. It is also interesting to examine the shape of the beams undergoing hyper-transport: their cross-sections (Fig. 2c,d) display increasing deviation from the exponential structure that characterizes the localized beam of Fig. 2b.

These experimental and numerical findings raise fundamental questions regarding the evolution of the spectrum of wave packets undergoing hyper-transport. Figure 2e displays the spatial power of the spectrum of the freely diffracting beam shown in Fig. 2a. The power spectrum of this beam is the same as the power spectrum of

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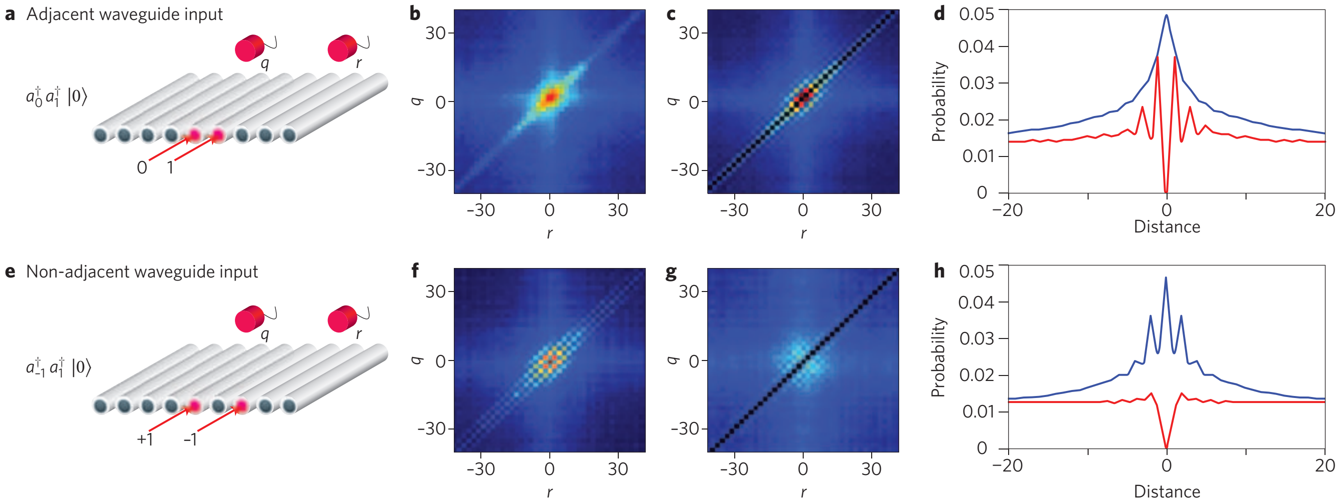

Figure 3 | Simulations of the correlations between the positions of two particles co-localizing in a disordered lattice. a, Particles are launched in adjacent sites, labelled 0 and 1. **b**,**c**, Correlation map for detecting two bosons (**b**) and fermions (**c**) at locations *r* and *q*. Note the checkered pattern for fermions. **d**, Probability distribution for the distance between two co-localizing bosons (blue) and fermions (red). **e**–**h**, Simulations as in **a**–**d**, but for two particles launched at sites −1 and +1. Figure reproduced with permission from ref. 60, © 2010 APS.

the input beam (see also lower curve in Fig. 2i). Consider now the case in which the beam is propagating through *z*-invariant disorder, where the beam becomes localized (Fig. 2b). The ensemble-averaged power spectrum of this localized beam is displayed in Fig. 2f: its spectral width Δk is wider than that of the freely diffracting beam (Fig. 2e). Simulations of this case, presented in Fig. 2i, reveal that Δk for the Anderson-localized beam increases during the early stages of propagation, where the ensemble-averaged beam reshapes due to multiple scattering, but once the wave is localized the spectral width no longer varies. Finally, and most interestingly, the power spectrum of a beam undergoing hyper-transport is found to expand throughout propagation (Fig. 2g–i). The randomly fluctuating spatially random potential causes stochastic acceleration, which is manifested in the spectral expansion of the wave packet undergoing faster-than-ballistic transport.

Localization with quantum-correlated photons

The new experimental playground provided by optics not only enables direct observations of several key phenomena related to localization, but also raises many new questions that were experimentally irrelevant beforehand. One particularly intriguing question relates to the co-localization of several particles simultaneously.

Anderson localization is primarily a wave phenomenon, and hence it could be realized and investigated using classical light. However, one may question what would change if these experiments were performed with non-classical light. Would the quantum nature of the field affect localization? A single photon travelling down a waveguide array performs a quantum random walk; that is, it hops randomly from one waveguide to its neighbour, and the interference of all possible paths leads to the discrete diffraction pattern that characterizes ballistic expansion[57]. Indeed, the propagation of a single quantum particle should not show any deviations from classical wave propagation, as the wave intensity profile represents the probability of arrival at any particular position. However, when two or more indistinguishable particles co-propagate in the system, even without any interactions, intriguing correlation properties appear that reflect quantum statistics[58]. Such quantum correlations can be detected through intensity–intensity correlations of classical waves (known as Hanbury Brown and Twiss correlations[58]) or coincidence counting with single-quantum detectors[59].

The correlations of two indistinguishable photons travelling in a periodic lattice were first discussed by Bromberg *et al.*[58], who also measured Hanbury Brown and Twiss correlations for various input conditions. These studies were then extended to true single-photon systems[59] and placed in the context of quantum random walks for two indistinguishable particles. The correlation maps were predicted to be different for bosonic and fermionic particles. For example, two bosons initially on two neighbouring sites show bunching (that is, they tend to propagate ballistically in the same direction), whereas fermions in the same situation tend to antibunch[58,60]. Most interestingly, fermions and even anions can be simulated by sending entangled photon pairs with appropriate phases[61].

How are such correlated quantum walks affected by disorder? A theoretical study by Lahini *et al.* had surprising results[60]. On short time scales, localization of one of the particles determines whether the other particle will be localized. On longer time scales, when both particles are localized, new and surprising results were predicted to appear. In particular, two particles that co-localize in such systems often exhibit an oscillatory behaviour in their average separation. Consider, for example, the distance between two bosons that were initially launched on the same site. Naively, one would expect the (mean) distance between them to decay exponentially,

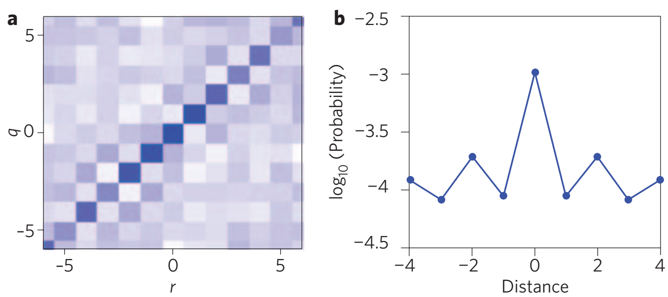

Figure 4 | Experimental measurements of quantum correlations in a 1D photonic lattice. a, Intensity–intensity correlations measured for light launched into site 0. **b**, Intensity correlation as a function of distance between the waveguides. The oscillatory correlation echoes the quantum distribution for indistinguishable bosons. Figure reproduced with permission from ref. 62, © 2011 APS.

with an exponent that is related to the localization length. However, surprisingly, the distance between the bosons favours an even number of sites. Similarly, two fermions launched in two neighbouring sites tend to localize to locations separated by an odd number of sites. Figure 3 shows the correlation maps and distance distribution predicted for bosons and fermions with two different input conditions. Note in particular the checkered patterns, which reflect an oscillatory distance distribution for the co-localizing particles. These effects are strong in systems in which the coupling strength is randomized (off-diagonal disorder). Although no experiment has yet verified this prediction with non-classical light, intensity–intensity correlation experiments with classical input fields have indeed reproduced the oscillatory correlation function for lattices with off-diagonal disorder[62] (Fig. 4). The oscillatory pattern reflects the symmetry of the spectrum characterizing non-diagonal disorder.

Outlook

Bringing the concepts of Anderson localization to the domain of optics has greatly enhanced our understanding of fundamental processes such as transport and multiple scattering. The first experimental efforts were intended to observe the principal localization phenomena, which had been predicted decades earlier. However, this new experimental approach gives rise to a wealth of completely new ideas, some specific to optics but many universal to all wave systems that contain disorder. Two examples of such ideas are recent studies on hyper-transport and on localization with entangled photons. Other examples (not described here) include random lasing[63], amorphous photonic lattices[64] and nonlinear optics in fractal structures[65]. Many of the concepts described in this Review are directly relevant to matter–wave systems that contain disorder, in which the main challenge is carrying out localization experiments in domains where optics cannot provide answers, such as the localization of interacting fermions[66] or a Tonks–Girardeau gas[67]. We have provided a contemporary summary of the Anderson localization of light. However, as often happens in science, when a new experimental paradigm is proposed, the best ideas are most probably yet to be suggested; such ideas will surely reveal new information on the universal phenomena associated with the transport of waves in random media.

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Competing financial interests

The authors declare no competing financial interests.