Nondiffracting beams in periodic media

Ofer Manela and Mordechai Segev

Department of Physics and Solid State Institute, Technion, Haifa 32000, Israel

Demetrios N. Christodoulides

School of Optics–CREOL, University of Central Florida, Orlando, Florida 32816-2700

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We identify nondiffracting beams in two-dimensional periodic systems, exhibiting symmetry properties and phase structure characteristic of the band(s) they are associated with. © 2005 Optical Society of America

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Diffraction, the spatial broadening of wave packets, is a fundamental feature of wave dynamics, occurring universally in any branch of physics that involves waves: optics, acoustics, quantum mechanics, etc. However, there are well-known examples of localized wave packets that can propagate without diffraction even in linear systems. Such wave packets are commonly referred to as "nondiffracting beams" (NDBs). In optics, the most widely known NDBs are the Bessel beams,[1,2] which were first demonstrated experimentally by Durnin *et al.*[3] Another renowned family of NDBs is that of X-waves.[4] In the spatiotemporal domain, X-waves are localized in both space and time, and are free of spatial and temporal broadening. NDBs, apart from being theoretically interesting, can also be useful in various settings, such as second-harmonic generation,[5] creation of X-shaped light bullets,[6] and Bessel lattice solitons.[7] We note that so far NDBs have been investigated primarily in homogeneous media. Nevertheless, NDBs can also exist in media displaying anisotropic diffraction. For example, acoustic NDBs and X-waves can exist in anisotropic elastic materials.[8] Even more interestingly, X-waves were predicted in periodic structures,[9–11] which in recent years have witnessed rapid advances in the exploration of phenomena related to light propagation in photonic crystals and to Bose–Einstein condensates in optical lattices.

Here, we demonstrate NDBs in 2D linear photonic lattices. We show that the symmetry properties and the phase structure of such a NDB are related to the band the beam is associated with, to the symmetry point of the Brillouin zone (BZ) it encircles in **k**-space, and to the beam's vorticity.

First, let us recall the reason why Bessel beams do not diffract. The spatial (plane-wave) spectrum of a Bessel beam (in homogeneous media) lies on a ring in **k**-space. That is, a Bessel beam is a coherent superposition of plane waves, all of which having transverse wave vectors of the same magnitude.[2] The propagation constant β is related to the wavenumber k and to the transverse wave vector $\mathbf{k}_\perp=(k_x,k_y)$ through the relation $\beta=k_z=\sqrt{k^2-\|\mathbf{k}_\perp\|^2}$. Thus, the propagation constants of all the plane waves that make up a Bessel beam are identical, i.e., the rate at which phase is accumulated by each plane wave is the same.[12] As a result, the interference pattern of all the plane waves that make up the beam does not change during propagation, and the beam as a whole does not diffract.

The above notion can be generalized to any linear system or medium that is translation invariant in the propagation direction. Wave propagation in such a structure may be described in terms of the system's eigenmodes—the analogs of plane waves in free space. These modes are waves that propagate without changing their transverse structure, merely accumulating a $\beta\Delta z$ phase, while propagating a distance of Δz. Any linear combination of modes that share the same propagation constant is also a mode with the same propagation constant. Mathematically, this notion can be stated as follows: Let us assume that the equation describing wave propagation in the system is of the form $i[\partial\psi(x,y,z)/\partial z]=L\psi(x,y,z)$, where ψ is the wave amplitude; L is a linear, Hermitian, z-independent operator with eigenmodes $\varphi_{\beta,\gamma}$ and their eigenvalues β, i.e., $L\varphi_{\beta,\gamma}=\beta\varphi_{\beta,\gamma}$, $\gamma\in\Gamma(\beta)$. The set of indices $\Gamma(\beta)$ reflects the degeneracy of the eigenvalue β. For example, the set $\Gamma(\beta)$ may be a finite discrete set, as for propagation in (1+1)D free space or in a (1+1)D periodic structure. In both cases $\Gamma(\beta)$ consists of two points: $(k_x, -k_x)$. Another example, in which $\Gamma(\beta)$ is a continuous set, is (2+1)D propagation in free space. In this case, under the paraxial approximation, $\beta=-(k_x^2+k_y^2)/(2k)$, so the surface formed by β as a function of k_x and k_y is a paraboloid with circular cross sections forming the sets $\Gamma(\beta)$.

As noted above, any linear combination of the eigenmodes $\varphi_{\beta,\gamma}$, $\psi(x,y)=\Sigma_{\gamma\in\Gamma(\beta)}A_\gamma\varphi_{\beta,\gamma}(x,y)$, where A_γ is the complex amplitude of the mode $\varphi_{\beta,\gamma}$, is also an eigenmode of L with the same eigenvalue β. [The sum could be a finite sum, an infinite sum, an integral, or a combination of them, depending on the set $\Gamma(\beta)$.] Any such linear combination, as any other mode, propagates without any change in its intensity: $I(x,y,z>0)=|\Sigma_{\gamma\in\Gamma(\beta)}A_\gamma\varphi_{\beta,\gamma}(x,y)|^2=I(x,y,0)$. As an example, in free space, by superimposing a finite number of modes (plane waves), one obtains an extended, periodic or quasi-periodic nondiffracting beam. Taking A_γ as a continuous function one can obtain the known Bessel beams or other families of NDBs in free space, such as the Mathieu beams.[13]

Consider now the case of a 2D periodic medium, where the propagation constant as a function of k_x

and k_y displays a band structure [Fig. 1(a)], i.e., a collection of surfaces that may be separated by gaps with normal eigenmodes being the Floquet–Bloch (FB) modes.[14] Fixing some propagation constant β_0, the set of eigenmodes with propagation constant β_0 defines a curve in the (k_x, k_y) plane on which $\beta(k_x, k_y) = \beta_0$ (assuming that β_0 is not in a gap or an extremum of a band). If there is an overlap between bands for this β_0, there will be one such curve for each overlapping band. The curve(s) defined by the relation $\beta(k_x, k_y) = \beta_0$ form the sets $\Gamma(\beta)$.

A new feature of 2D periodic systems is the existence of "accidental" degeneracies that cannot be attributed to the system's symmetries. In contrast, in free space (1D or 2D) and in 1D periodic systems all degeneracies are solely due to the symmetries of the system. For instance, in 2D free space all degeneracies are due to symmetries under rotations around the propagation axis, hence $\Gamma(\beta)$ is always a circle encompassing the $\mathbf{k}_\perp = 0$ point. This basic fact makes all the Bessel beams "look alike." In contrast, in 2D periodic systems, in addition to degeneracies related to the symmetry under 90° rotations and under reflections, there are also continuous sets of accidental degeneracies defining curves in **k**-space. As a result, for every β_0, the curve has a different shape [see, e.g., Figs. 1(b) and 1(c)], and thus every NDB in a lattice looks different. Moreover, due to the band structure one can construct families of NDBs, relating them to the properties of the FB modes from which they are composed: **NDBs** from different bands, with $\Gamma(\beta)$ encircling the Γ-, X-, or M-point in the **k**-space [inset in Fig. 1(a)]. As in the case of free space, one can add the FB modes with phase differences between them. This can lead to a vortex NDB, i.e., a NDB with a phase singularity.

For concreteness, we construct some examples of NDBs in 2D periodic systems. Consider the paraxial propagation of linearly polarized light modeled by the following dimensionless equation:

$$i\frac{\partial A}{\partial z} + \nabla_\perp^2 A + V(\mathbf{r})A = 0, \tag{1}$$

where $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$, $A(\mathbf{r}, z)$ is the slowly varying amplitude of the field, $\mathbf{r} = x\hat{x} + y\hat{y}$ is the vector of the transverse coordinates, and $V = \{\cos[\pi(x+y)/D] + \cos[\pi(x-y)/D]\}^2$ is a square periodic potential. We calculate the band structure $\beta(\mathbf{k}_\perp)$ using the Bloch theorem: $A = \exp[i(\beta z + \mathbf{k}_\perp \cdot \mathbf{r})]u(\mathbf{r})$, where $u(\mathbf{r})$ is periodic with the same period as the potential. Then, we choose a value for β_0 and find the section of the calculated band structure with this particular β_0. That is, we find a large set of pairs (k_x, k_y) evenly spaced on the $\Gamma(\beta_0)$ curve for which $\beta(k_x, k_y) = \beta_0$. To get the wave function of the NDB, we add up all the FB modes of these pairs with the appropriate amplitudes A_γ. Finally, we verify that these beams indeed do not diffract by numerically propagating them for tens of diffraction lengths using a fast Fourier transform beam propagation method.

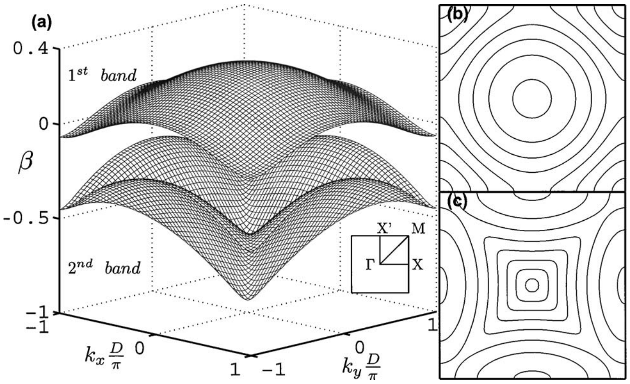

Fig. 1. (a) First two bands in the band structure of a 2D square lattice with $D = 5.5$. Inset, high symmetry points of the reciprocal lattice. (b), (c) Curves of equal propagation constant of the first band (b) and the second band (c).

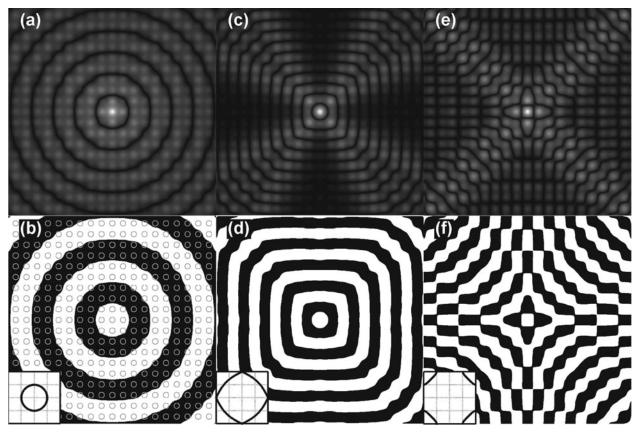

Fig. 2. Absolute value (a), (c), (e) and phase (b), (d), (f) of first-band NDBs, with the wavenumber (a), (b) encircling the Γ point, (c), (d) passing through the X points, (e), (f) encircling the M points. The insets show the loci of the FB modes in the first BZ from which the NDBs were constructed. The gray circles in (b) indicate the boundaries of the waveguides.

Let us describe some specific examples of NDBs in a square lattice with $D = 5.5$, whose band structure is depicted in Fig. 1(a). To isolate the most symmetric cases, we assign the same amplitude to all the modes, e.g., $|A_\gamma| = 1$. A first-band NDB encircling the Γ-point looks like a Bessel beam, showing almost circular symmetry with a π phase shift between adjacent rings and some lattice modulation [Figs. 2(a) and 2(b)]. As the propagation constant β_0 decreases, the $\Gamma(\beta_0)$ curve approaches the X-points and the beam acquires squarish symmetry both in real space and in **k**-space [Figs. 2(c) and 2(d)]. Decreasing β_0 further, below the propagation constant at the X-point, the $\Gamma(\beta_0)$ curve encircles the M-point and the phase of the beam attains a hyperbolic shape [Figs. 2(e) and 2(f)]. [Recalling that the propagation constant is periodic in (k_x, k_y), with the BZ being topologically equivalent to a 2-torus, the curves of the shape in the inset of Fig. 2(f) are actually encircling the M symmetry point.] Hence, the symmetry of a NDB is related to the locus of its FB components in **k**-space. Second-band NDBs show different structures. A second-band

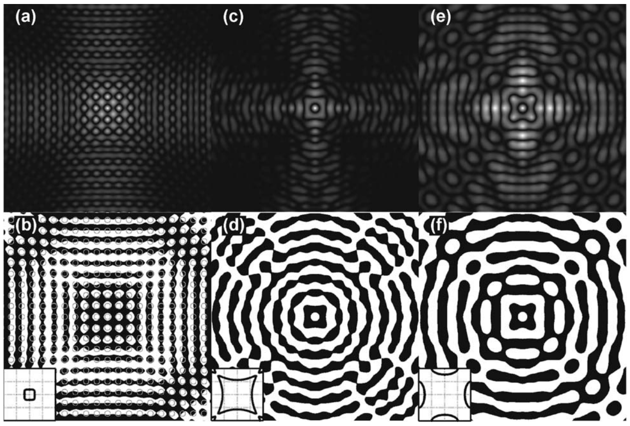

Fig. 3. Absolute value (a), (c), (e) and phase (b), (d), (f) of the second-band NDB, with the wavenumber (a), (b) encircling the Γ point, (c), (d) encircling the Γ point with greater distance, (e), (f) encircling the X points. The insets show the loci of the FB modes in the second BZ from which the NDBs were constructed. The gray circles in (b) indicate the boundaries of the waveguides.

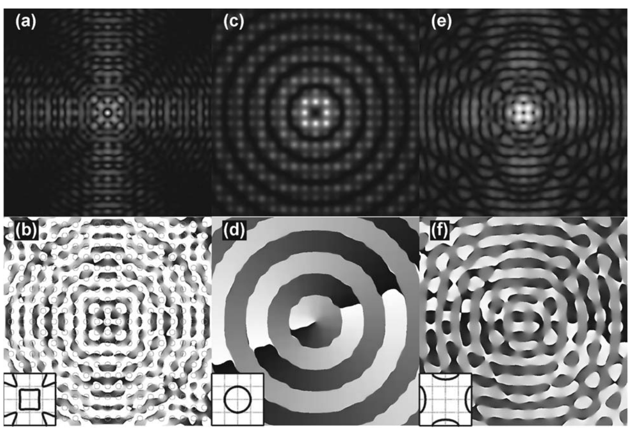

Fig. 4. Absolute value (a), (c), (e) and phase (b), (d), (f) of NDBs. (a), (b) Combination of second-band Γ-point NDB and third-band M-point NDB phase retarded by π rad, with both NDBs sharing the same β; (c), (d) first-band vortex NDB encircling the Γ point, (e), (f) second-band vortex NDB encircling the X points. The insets show the loci of the FB modes in the corresponding BZs from which the NDBs were constructed. The gray circles in (b) indicate the boundaries of the waveguides.

NDB encircling the Γ-point has squarish symmetry [Figs. 3(a) and 3(b)], which turns concave and is even disconnected in **k**-space as β_0 is decreased, encircling both the Γ- and the M-points [Figs. 3(c) and 3(d)] or the two X-points [Figs. 3(e) and 3(f)]. The specific lattice considered here has an overlap between its second and third bands, thus one can construct two different beams, one from the second band and the other from the third band, with different symmetries, while both share the same propagation constant. In addition, any linear combination of these two beams is a multiple-band NDB. The example given in Figs. 4(a) and 4(b) is a combination of a second-band Γ-point NDB and a third-band M-point NDB phase retarded by π rad. This example exhibits complex phase structure but actually possesses vanishing angular orbital momentum.

The examples in Figs. 4(c)–4(f) exemplify vortex NDBs that carry angular momentum. The first-band vortex NDB shown in Figs. 4(c) and 4(d) is composed of the same FB modes as the NDB shown in Figs. 2(a) and 2(b), but to make it a vortex NDB we take the amplitudes to be $A_\gamma = \exp(i2\pi t)$ where t is a parameter varying linearly from zero to one along the curve. The symmetry of this beam does not change; however, a phase singularity with zero intensity appears at the center of the beam. The second-band vortex NDB shown in Figs. 4(e) and 4(f) is composed of the same FB modes as in Figs. 3(e) and 3(f). The phase of this example has the structure of a counter-rotating vortex array, similar to the phase structure of the 2D second-band vortex lattice soliton.[15,16]

In conclusion, we have presented nondiffracting beams in 2D linear periodic systems, explaining why these beams do not diffract, showing how to construct them, giving some concrete examples, and discussing their symmetries. We have shown that these beams can exhibit various forms, including Bessel-like, X-like, and square-like structures, depending on the pathway the $\Gamma(\beta_0)$ curve takes with the Brillouin zone. Experimental realization of NDBs can be achieved by modulating a plane wave with an amplitude–phase mask. Such masks can be implemented using either computer-generated holograms or spatial light modulators. The ideas presented here can be further generalized to 3D systems or interpreted in other contexts, such as electrons in a lattice or Bose–Einstein condensates in optical lattices.

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