nature Vol 440|27 April 2006|doi:10.1038/nature04722

LETTERS

Wave and defect dynamics in nonlinear photonic quasicrystals

Barak Freedman[1], Guy Bartal[1], Mordechai Segev[1], Ron Lifshitz[2], Demetrios N. Christodoulides[3] & Jason W. Fleischer[4,5]

Quasicrystals are unique structures with long-range order but no periodicity. Their properties have intrigued scientists ever since their discovery[1] and initial theoretical analysis[2,3]. The lack of periodicity excludes the possibility of describing quasicrystal structures with well-established analytical tools, including common notions like Brillouin zones and Bloch's theorem. New and unique features such as fractal-like band structures[4–7] and 'phason' degrees of freedom[8] are introduced. In general, it is very difficult to directly observe the evolution of electronic waves in solid-state atomic quasicrystals, or the dynamics of the structure itself. Here we use optical induction[9–11] to create two-dimensional photonic quasicrystals, whose macroscopic nature allows us to explore wave transport phenomena. We demonstrate that light launched at different quasicrystal sites travels through the lattice in a way equivalent to quantum tunnelling of electrons in a quasiperiodic potential. At high intensity, lattice solitons are formed. Finally, we directly observe dislocation dynamics when crystal sites are allowed to interact with each other. Our experimental results apply not only to photonics, but also to other quasiperiodic systems such as matter waves in quasiperiodic traps[12], generic pattern-forming systems as in parametrically excited surface waves[13], liquid quasicrystals[14], and the more familiar atomic quasicrystals.

We produce two-dimensional photonic quasicrystals by using the optical induction technique[9,10] that has recently become a useful tool in studying solitons in two-dimensional nonlinear optical lattices[11]. A similar technique was used in the past to trap cold atoms in quasiperiodic potentials[12]. Other theoretical[15] and experimental techniques, based on direct fabrication at the level of individual structural elements, have been used for creating photonic quasicrystals for the general study of their optical properties[16], nonlinear frequency conversion applications[17,18], or the generation of complete photonic bandgap materials[19]. The recent discovery of self-assembled dendrimer liquid quasicrystals[14] may lead to another interesting method, but optical induction is currently the only approach that allows for full structural control and real-time manipulation of the generated photonic quasicrystals.

The optical induction technique relies on the interference of several monochromatic light beams, whereby the resulting intensity pattern is translated into a (periodic or quasiperiodic) change in the refractive index of a photosensitive nonlinear material, in our case photorefractive SBN:75 ($Sr_{0.75}Ba_{0.25}Nb_2O_6$, see refs 9, 11). For photonic quasiperiodic structures, we overlap five coherent monochromatic laser beams with wave vectors $\boldsymbol{k}_1, \dots \boldsymbol{k}_5$ separated by angles of $2\pi/5$. The experimental image of the field-intensity pattern is shown in Fig. 1a. It consists of a d.c. $\boldsymbol{k} = 0$ component and exactly 20 harmonic components with wave vectors of the form $\boldsymbol{k}_i - \boldsymbol{k}_j$ $(i, j = 1\dots5)$, as indicated by the appearance of 20 Bragg peaks in the calculated diffraction image (Fig. 1c). We emphasize that the induced refractive index change in our material is of a saturable nature. We intentionally work in the saturated regime, so the induced photonic quasicrystal contains additional higher-harmonic Bragg peaks, as clearly visible in the experimental diffraction diagram (Fig. 1b). Evidently, the refractive index structure impressed into the volume of the nonlinear material is a fully-fledged photonic quasicrystal. From the relative phases of these Fourier components, we infer that the induced quasicrystal has decagonal (ten-fold, as opposed to only pentagonal) symmetry. Furthermore, one can tile the induced decagonal pattern using rhombic Penrose tiles, producing a

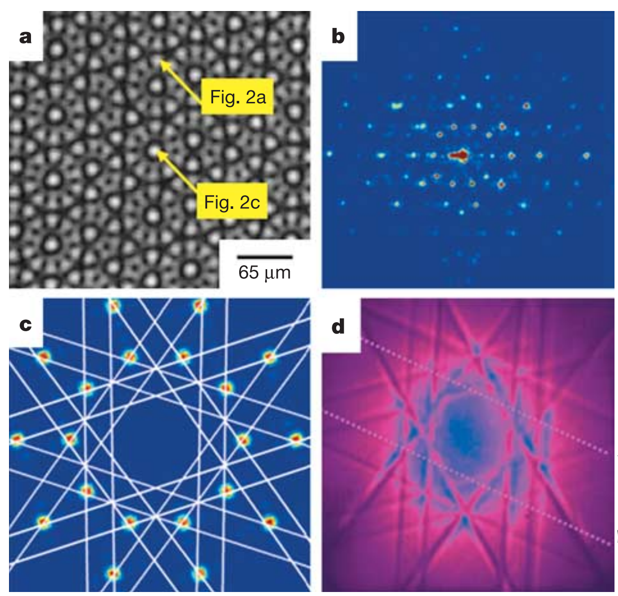

Figure 1 | Photonic quasicrystal and its extended Brillouin zone map. **a**, Experimental image of the decagonal field-intensity pattern generated by five writing beams with wave vectors $k_1, \dots k_5$ separated by angles of $2\pi/5$. (Yellow arrows mark the probe beam excitation sites for Fig. 2.) **b**, Experimental diffraction pattern showing additional higher-harmonic Bragg peaks, confirming the creation of a fully-fledged photonic quasicrystal. **c**, **d**, Theory (**c**) and experiment (**d**) showing the effective Brillouin zones of the optically induced decagonal quasicrystal. Panel **c** shows the 20 Bragg peaks (after subtraction of the $\boldsymbol{k} = 0$ component) corresponding to all wave vectors of the form $\boldsymbol{k}_i - \boldsymbol{k}_j$ $(i, j = 1\dots5)$ that make the real space intensity patterns. Arrows at right of **d** indicate lines appearing 'weak' in the experiment. Bright colours indicate higher intensity.

[1]Physics Department and Solid State Institute, Technion–Israel Institute of Technology, Haifa 32000, Israel. [2]School of Physics & Astronomy, Tel Aviv University, Tel Aviv 69978, Israel. [3]College of Optics and Photonics, University of Central Florida, Florida 32816-2700, USA. [4]Electrical Engineering Department, [5]Princeton Institute for the Science and Technology of Materials, Princeton University, New Jersey 08544, USA.

decagonal tiling that contains ten-fold star configurations, sometimes referred to as the 'anti-Penrose' tiling.

There are two fundamentally different types of lattices that can be created using the optical induction method: fixed and deformable. In the first case, the induced array is essentially rigid, while in the second case the individual sites can move and interact with each other. Technically, these two cases differ in the way the writing beams are polarized, but the end result is far more important. The photorefractive crystal has the property that arbitrarily polarized light induces changes in its refractive index according to the field intensity. On the other hand, only light that is polarized in a certain direction (extraordinary polarization in our SBN:75 crystal) experiences a spatially-varying index of refraction. Light that is polarized in the orthogonal direction (ordinary polarization in SBN:75) propagates through the medium as if its refractive index were homogeneous. The first optical induction scheme involves ordinarily polarized writing beams that induce the quasicrystal but do not interact with each other, and an additional extraordinarily polarized probe beam[9–11]. The quasicrystal produced in this way is ideally suited to studying the linear and nonlinear tunnelling transport of a wavepacket in a fixed quasiperiodic lattice (Fig. 2), in a way similar to previous studies in square and hexagonal lattices[11]. The second scheme of optical induction uses extraordinarily polarized writing beams, which are affected by the induced changes in the refractive index and therefore interact with each other as the crystal is induced, giving rise to complex nonlinear dynamics[20]. That is, the sites interact with one another through the optical nonlinearity, in a way similar to the interactions between an array of solitons in homogeneous media. This kind of induced crystal can be used for the study of dynamical processes such as defect healing, dislocation motion, and phason strain relaxation (Figs 3 and 4). (An explanation of the term 'phason' appears below.)

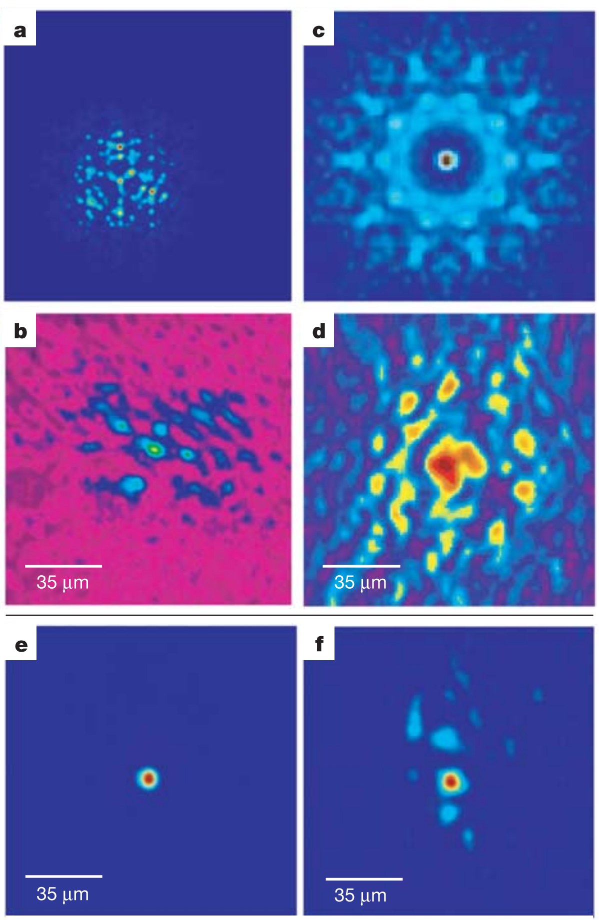

Figure 2 | Discrete diffraction and lattice solitons in a photonic quasicrystal. **a**–**d**, Linear tunnelling ('discrete diffraction') in a decagonal photonic quasicrystal: **a**, **c**, simulation results of the probe beam exiting the photonic quasicrystal for an input beam launched at the two different sites marked by yellow arrows in Fig. 1a; **b**, **d**, the corresponding experimental results (note the nicely resolved ring of 10 spots in **d**). **e**, **f**, Formation of a soliton in a quasicrystal when the intensity of the probe beam (that linearly diffracted in **d**) is sufficiently increased (with the lattice conditions unchanged). **e**, Input face and **f**, output face of the soliton beam.

We start our experiments by mapping the extended effective Brillouin zones (Jones zones) of the induced photonic structure (Fig. 1c, d). This is done by employing the recently developed Brillouin zone spectroscopy method[21], which relies on visualizing the power spectra of the optical Bloch modes[22] propagating through the photonic crystal. Similar results, showing the effective Brillouin zones of an icosahedral photonic quasicrystal, yet via very many point-by-point measurements, have been reported recently[16]. The excellent agreement between the experimental momentum-space picture (Fig. 1d) and Bragg diffraction peaks (Fig. 1b), and the expected effective Brillouin zones predicted by the positions of the Bragg peaks (Fig. 1c), proves not only that what we have induced is a photonic quasicrystal, but also that this quasicrystal is capable of carrying optical Bloch modes, extended along its axis of ten-fold symmetry.

The standard benchmark experiment to study wave dynamics in lattices is 'discrete diffraction', which is the optical equivalent of quantum tunnelling in a periodic potential[23,24]. A focused probe beam is launched into a single site and experiences lateral transport within the lattice as it propagates along its axis, resulting in a diffraction pattern characteristic of the local structure that is probed[11,25]. In a simple periodic crystal, with a single 'atom' per unit cell, each site looks like any other and discrete diffraction looks the same irrespective of the initial site of excitation. In quasicrystals, however, there are always a number of different local environments, so that transport behaviour is expected to vary significantly from site to site. We observe this behaviour in the decagonal photonic quasicrystal numerically (Fig. 2a, c) and experimentally (Fig. 2b, d), by comparing the discrete diffraction from two different sites (marked in Fig. 1a). The two points chosen differ from one another by their local environments, one having a higher local symmetry than the other, as clearly observed in the numerical and experimental linear propagation (discrete diffraction) images.

Unlike this linear phenomenon of discrete diffraction, in many physical systems (such as photonic quasicrystals and matter waves in a quasiperiodic potential) the underlying quasiperiodic structure behaves nonlinearly to waves of large-enough amplitude, and can support lattice solitons (discrete solitons)[9–11,23–25]. We therefore conducted a series of experiments (Fig. 2e, f), and observed nonlinear localization of the discrete diffraction pattern. In this set of experiments, we used a positive bias field, hence the nonlinearity is of the self-focusing type[11]. At sufficiently high intensity of the probe beam (while keeping the photonic quasicrystal unchanged), we obtained the first observation of lattice solitons in a quasiperiodic waveguide structure (Fig. 2f). To verify the robustness of the lattice soliton, we conducted several more nonlinear localization experiments, by changing the intensity of the probe beam while keeping all other parameters fixed; that is, by varying the intensity ratio of the soliton/lattice from 1:3 to 1:4. Throughout this range, the output intensity distribution of the soliton remains unchanged (Fig. 2f). We have also tested the robustness of the soliton to the lattice depth, by varying the bias field. The soliton is robust to such variations within $\sim$5%. Altogether, soliton formation and propagation is a robust phenomenon in the quasicrystal, with an experimentally large basin of attraction.

We now move on to study the dynamics of the photonic quasicrystal, using interacting extraordinarily polarized beams, so it is essential to maintain a stable ground-state structure of the interacting sites of the crystal. Such behaviour, of a stable interacting square lattice of the self-focusing type (where all the lattice sites are the same), was recently

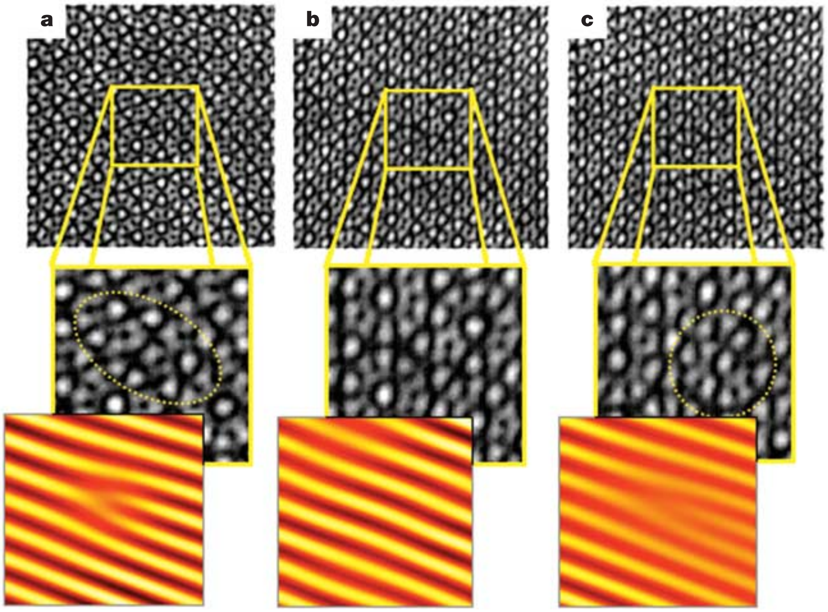

Figure 3 | Creation and healing of a dislocation in a decagonal quasicrystal. a, Non-interacting quasicrystal with a (1,0,0,0) dislocation. **b**, With nonlinear interaction in the same crystal the dislocation disappears. The resulting 'healed' quasicrystal is again a ground state of the system, but is in general a ground state that differs from the original one (in Fig. 1b) by phason rearrangements throughout the whole plane (long-range effect). **c**, With increased power to the vortex beam the nonlinearity can no longer overcome the defect within the propagation distance, and the dislocation is again visible. The top panels show the actual quasicrystal. The middle panels are a magnification of the middle section of the top panels. The bottom panels show one of the filtered harmonic components clearly visualizing the dislocation.

observed[20]. However, we find, numerically and experimentally, that an interacting decagonal photonic quasicrystal of the self-focusing type disintegrates rapidly when the waves inducing it are extraordinarily polarized. On the other hand, when the nonlinearity is of the self-defocusing type (when a negative field is applied to the photorefractive SBN:75 crystal), the interacting quasicrystal is much more stable, and the structure does not disintegrate. We pursue this avenue and, for the interacting lattice experiments, we induce a photonic quasicrystal exhibiting the photorefractive self-defocusing nonlinearity[26–28]. Repeating the Brillouin zone spectroscopy and Bragg diffraction experiments reveals pictures very similar to Fig. 1b and d, thus ensuring that we still have a fully-fledged photonic quasicrystal when the lattice is dynamically interacting.

Having created a stable interacting photonic quasicrystal, we can use it to study the dynamics of defects[29] by observing the motion of dislocations and the relaxation of phason strain, as these propagate in the quasicrystal. We induce defects in the photonic quasicrystal by changing one of the plane-wave writing beams into a 2π vortex beam. This new combination of writing beams results in an intensity pattern, and therefore a photonic quasicrystal, with a dislocation with Burgers vector (1,0,0,0). When the quasicrystal is formed by non-interacting ordinarily polarized waves, the dislocation is visible even when the vortex beam is very weak and even after a considerable propagation distance (5 mm in our experiment; Fig. 3a). This is due to the fact that, without nonlinear interaction, there is no dynamics and the photonic crystal remains fixed and cannot 'heal' itself. However, when the crystal is induced by interacting extraordinarily polarized waves, 'atoms' (crystal sites) can change their positions and even merge or split, so the quasicrystal can rearrange itself—through 'atomic' diffusion and local configurational rearrangements—back into the lower-energy structure of a perfect defect-free quasicrystal (Fig. 3b). Finally, when the vortex beam inducing the dislocation is of high intensity, the nonlinear interaction between the sites of the quasicrystal can no longer heal the dislocation within the propagation distance in the crystal, and the dislocation remains all the way to the output (Fig. 3c).

At this point, it is essential to identify whether or not the defect indeed completely disappears, and to find its exact location. In a

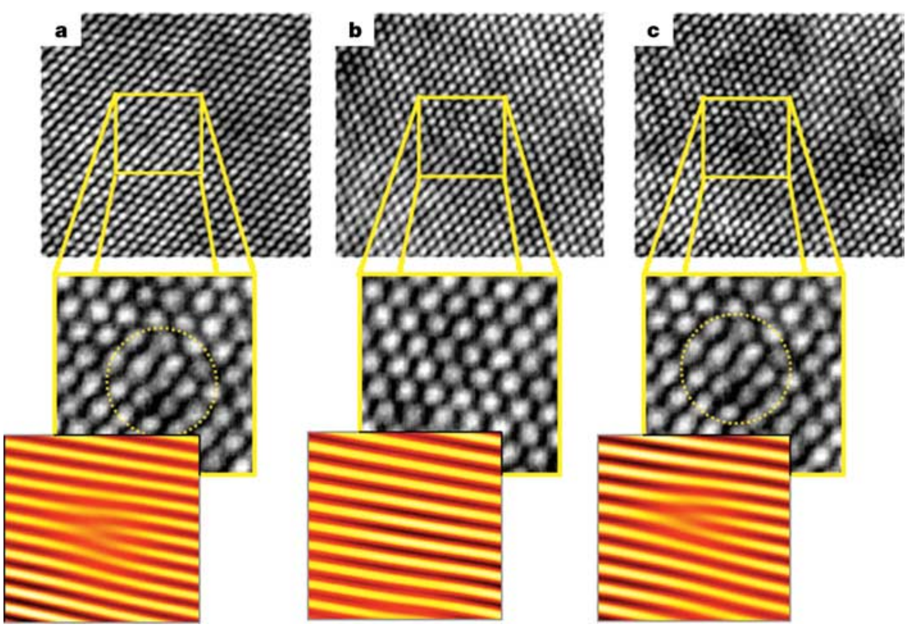

Figure 4 | Creation and healing of a dislocation in a periodic hexagonal crystal. Panels **a**–**c** are as for Fig. 3. **a**, The non-interacting periodic crystal with a (1,0) dislocation. **b**, Note that as in the quasiperiodic case (Fig. 3), the resulting 'healed' crystal is in a different ground state, but here ground states can differ only by a simple shift of the pattern. **c**, As for Fig. 3c.

periodic crystal, it is rather easy to identify a dislocation, but in a quasicrystal it is not as apparent. We therefore identify the dislocations by filtering pairs of Bragg peaks in the Fourier transform image of the quasicrystal, one at a time, and then perform an inverse Fourier transform to visualize the dislocations present in each individual wave[29]. The results, depicted in the bottom panels of Fig. 3, show clearly the defect and its location in Fig. 3a and c, and confirm that the dislocation has fully disappeared in Fig. 3b. Finally, for illustrating the difference from periodic crystals, we use three writing beams to create a hexagonal crystal with an embedded dislocation with Burgers vector (1,0), and repeat the defect dynamics experiments under similar nonlinear conditions (Fig. 4).

We recall that far away from a dislocation, both in the periodic crystal (Fig. 4a) and in the quasicrystal (Fig. 3a), the pattern is in a minimum-energy ground state, but it is in a different ground state in every direction away from the dislocation. In a periodic crystal, ground states can differ only by a shift of the crystal (a non-lattice translation). When going around the dislocation, the crystal is gradually shifted, and upon completion of a full circle and a return to the starting point, one finds that the crystal has shifted by an integer number of lattice translations (a Burgers vector). In a quasicrystal, there is greater freedom. Ground states can differ by more than simple shifts; they can also differ by rearrangements of the relative positions of the atoms (or 'tile flips' had the crystal been decorated with tiles). These are the so-called phason flips ('phasons')—a terminology that reflects the fact they arise from changing the relative phases, under certain constraints, of the different waves making up the crystal. In Fig. 3b, we clearly observe the long range consequence of phason flips that are induced in different directions around the dislocation. Particularly interesting is the predicted fact that, when a dislocation glides or climbs away, the quasicrystal cannot be as easily patched up as the periodic crystal, and a phason trail of local atomic rearrangements is left behind, which must then 'heal' through a sequence of phason flips until all 'atoms' are back in their proper positions.

The resulting 'healed' quasicrystal is again a decagonal ground state of the system, as can readily be observed in Fig. 3b, but is in general a ground state that differs from the original one by phason rearrangements throughout the whole plane (a 'long-range' effect). The comparison with the periodic hexagonal lattice of Fig. 4 is remarkable: in the periodic case, half a line of 'atoms' can simply be shifted to replace the line next to it; thus, as the defect in the hexagonal lattice of Fig. 4 moves, the crystal rearranges locally (through the so-called Peierls mechanism) and no long-range memory of the defect is left behind. Our results, depicted in Fig. 3, clearly demonstrate experimentally the creation and healing of a dislocation in a quasicrystal, and the consequence of phason flips, under controlled and engineered conditions—a 20-year-old theoretical prediction[2,3].

This work paves the way for a variety of experiments on linear and nonlinear wave dynamics in quasicrystals: these include direct imaging of the fractal band structure of quasicrystals, linear and nonlinear localization experiments, experiments with gap solitons, vortex dynamics in quasicrystals, modulation instability, and wave collapse in a quasiperiodic crystal. Especially interesting is the possibility of studying the dynamics of multiple defect states in interacting nonlinear optical quasicrystals and the relaxation of phason strain by observing individual phason flips. Finally, as in three-dimensional photonic crystals[30], we envision a three-dimensional realization of our current experiments as a means to study the details of wave dynamics in photonic structures that are quasiperiodic in all three dimensions.

Received 14 November 2005; accepted 9 March 2006.

1. Shechtman, D., Blech, I., Gratias, D. & Cahn, J. W. Metallic phase with long-range orientational order and no translational symmetry. *Phys. Rev. Lett.* **53,** 1951–1953 (1984).
2. Levine, D., Lubensky, T. C., Ostlund, S., Ramaswamy, S. & Steinhardt, P. J. Elasticity and dislocations in pentagonal and icosahedral quasicrystals. *Phys. Rev. Lett.* **54,** 1520–1523 (1985).
3. Socolar, J. E. S., Lubensky, T. C. & Steinhardt, P. J. Phonons, phasons, and dislocations in quasicrystals. *Phys. Rev. B* **34,** 3345–3360 (1986).
4. Rotenberg, E., Theis, W., Horn, K. & Gille, P. Quasicrystalline valence bands in decagonal AlNiCo. *Nature* **406,** 602–605 (2000).
5. Kaliteevski, M. A. *et al.* Two-dimensional Penrose-tiled photonic quasicrystals: from diffraction pattern to band structure. *Nanotechnology* **11,** 274–280 (2000).
6. Dal Negro, L. *et al.* Light transport through the band-edge states of Fibonacci quasicrystals. *Phys. Rev. Lett.* **90,** 055501 (2003).
7. Chan, Y. S., Chan, C. T. & Liu, Z. Y. Photonic band gaps in two dimensional photonic quasicrystals. *Phys. Rev. Lett.* **80,** 956–959 (1998).
8. Edagawa, K. & Suzuki, K. High resolution transmission electron microscopy observation of thermally fluctuating phasons in decagonal Al-Cu-Co. *Phys. Rev. Lett.* **85,** 1674–1677 (2000).
9. Efremidis, N. K., Sears, S., Christodoulides, D. N., Fleischer, J. W. & Segev, M. Discrete solitons in photorefractive optically induced photonic lattices. *Phys. Rev. E* **66,** 046602 (2002).
10. Fleischer, J. W., Carmon, T., Segev, M., Efremidis, N. K. & Christodoulides, D. N. Observation of discrete solitons in optically induced real time waveguide arrays. *Phys. Rev. Lett.* **90,** 023902 (2003).
11. Fleischer, J. W., Segev, M., Efremidis, N. K. & Christodoulides, D. N. Observation of two-dimensional discrete solitons in optically induced nonlinear photonic lattices. *Nature* **422,** 147–150 (2003).
12. Guidoni, L., Triché, C., Verkerk, P. & Grynberg, G. Quasiperiodic optical lattices. *Phys. Rev. Lett.* **79,** 3363–3366 (1997).
13. Lifshitz, R. & Petrich, D. M. Theoretical model for Faraday waves with multiple-frequency forcing. *Phys. Rev. Lett.* **79,** 1261–1264 (1997).
14. Zeng, X. *et al.* Supramolecular dendritic liquid quasicrystals. *Nature* **428,** 157–160 (2004).
15. Clausen, C. B., Kivshar, Y. S., Bang, O. & Christiansen, P. L. Quasiperiodic envelope solitons. *Phys. Rev. Lett.* **83,** 4740–4743 (1999).
16. Man, W., Megens, M., Steinhardt, P. J. & Chaikin, P. M. Experimental measurement of the photonic properties of icosahedral quasicrystals. *Nature* **436,** 993–996 (2005).
17. Bratfalean, R. T., Peacock, A. C., Broderick, N. G. R. & Gallo, K. Harmonic generation in a two-dimensional nonlinear quasicrystal. *Opt. Lett.* **30,** 424–426 (2005).
18. Lifshitz, R., Arie, A. & Bahabad, A. Photonic quasicrystals for nonlinear optical frequency conversion. *Phys. Rev. Lett.* **95,** 133901 (2005).
19. Zoorob, M. E., Charlton, M. D. B., Parker, G. J., Baumberg, J. J. & Netti, M. C. Complete photonic bandgaps in 12-fold symmetric quasicrystals. *Nature* **404,** 740–743 (2000).
20. Martin, H., Eugenieva, E. D., Chen, Z. & Christodoulides, D. N. Discrete solitons and soliton-induced dislocations in partially coherent photonic lattices. *Phys. Rev. Lett.* **92,** 123902 (2004).
21. Bartal, G. *et al.* Brillouin-zone spectroscopy of nonlinear photonic lattices. *Phys. Rev. Lett.* **94,** 163902 (2005).
22. Russell, P. St J. Optics of Floquet-Bloch waves in dielectric gratings. *Appl. Phys. B* **39,** 231–246 (1986).
23. Christodoulides, D. N. & Joseph, R. I. Discrete self-focusing in nonlinear arrays of coupled waveguides. *Opt. Lett.* **13,** 794–796 (1988).
24. Christodoulides, D. N., Lederer, F. & Silberberg, Y. Discretizing light behaviour in linear and nonlinear waveguide lattices. *Nature* **424,** 817–823 (2003).
25. Eisenberg, H. S., Silberberg, Y., Morandotti, R., Boyd, A. R. & Aitchison, J. S. Discrete spatial optical solitons in waveguide arrays. *Phys. Rev. Lett.* **81,** 3383–3386 (1998).
26. Segev, M., Valley, G. C., Crosignani, B., DiPorto, P. & Yariv, A. Steady-state spatial screening solitons in photorefractive materials with external applied field. *Phys. Rev. Lett.* **73,** 3211–3214 (1994).
27. Christodoulides, D. & Carvalho, M. Bright, dark, and gray spatial soliton states in photorefractive media. *J. Opt. Soc. Am. B* **12,** 1628–1633 (1995).
28. Efremidis, N. K. *et al.* Two-dimensional optical lattice solitons. *Phys. Rev. Lett.* **91,** 213906 (2003).
29. Barak, G. & Lifshitz, R. Dislocation dynamics in a dodecagonal quasiperiodic structure. *Phil. Mag.* **86,** 1059–1064 (2006).
30. Joannopoulos, J. D., Meade, R. D. & Winn, J. N. *Photonic Crystals: Molding the Flow of Light* (Princeton, Princeton Univ. Press, 1995).

Acknowledgements This work was supported by the Israeli Science Foundation, the Israel-USA Binational Science Foundation and the German-Israeli DIP Project.

Author Information Reprints and permissions information is available at npg.nature.com/reprintsandpermissions. The authors declare no competing financial interests. Correspondence and requests for materials should be addressed to M.S. (msegev@techunix.technion.ac.il).