Physics

VIEWPOINT

Non-Hermitian Topological Systems

A theoretical framework tries to sort out where topological phases may arise in non-Hermitian systems—which are systems with gain and loss.

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Two emerging fields—non-Hermitian systems and topological phases—have recently begun to merge together, but just how much they overlap remains an open question. On one side, "non-Hermitian" typically applies to systems that experience gain and loss. On the other side, a "topological phase" is a state of matter that is characterized by a property that remains invariant during continuous deformations of the system. At first blush, it is not at all obvious that a topological invariant could arise within a non-Hermitian (NH) system, which can be out of equilibrium and even unstable. Experiments have given evidence for topological phases in 1D and 2D NH systems, but researchers have yet to place these results in a broader context that might reveal other NH topological systems. Zongping Gong from the University of Tokyo and colleagues present a new general framework for classifying topological phases of non-Hermitian systems [1]. They create a "periodic table" of NH topological phases based on the symmetries of the system and the number of dimensions. Under their classification, 2D non-Hermitian systems should not have topological phases, which would seem to conflict with recent experimental and theoretical work [2, 3]. However, the apparent discrepancy is a result of different definitions for what constitutes a NH topological phase, which means the book on non-Hermitian topological systems isn't yet closed.

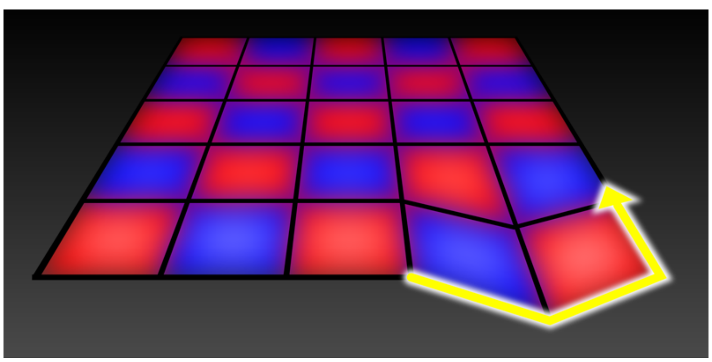

Figure 1: A non-Hermitian system is represented here in two dimensions with different regions experiencing gain (red) and loss (blue). New theoretical work addresses conditions under which the system supports topological phases. Such a phase can allow edge states (shown as a yellow arrow) that move in one direction around the material and are immune to defects and disorder. (APS/Alan Stonebraker)

In quantum mechanics, an operator—a mathematical function that acts on a wave function—is typically assumed to be Hermitian, which means it returns only real values. These values are called observables, as they are the potential outcomes of measurements. By contrast, a non-Hermitian operator can allow imaginary (or complex) values. NH operators have proved useful in describing loss mechanisms, open systems, finite lifetime, and dephasing—all phenomena that would otherwise have to be described by coupling to degrees of freedom outside the system of interest. While the NH version of quantum mechanics is helpful in simplifying calculations and for identifying resonances, the assumption has long been that NH operators are not physically meaningful, as one can't have complex valued observables. Twenty years ago, this assumption was shown to be wrong, when researchers found that the observables of certain NH operators that obey parity-time (*PT*) symmetry are real [4]. It took another decade for these concepts to be introduced into optics [5], and shortly thereafter *PT* optics produced the first experimental realizations of NH *PT*-symmetric systems [6], earning it recognition as one of the top physics discoveries in recent years [7].

In parallel to *PT* optics, another major research effort has taken off recently: topological photonics. This field started off as a way to emulate condensed-matter topological insulators, which are insulators in their bulk but conduct electricity on their surfaces in a way that is unidirectional and robust against disorder. Following early work with microwaves [8], the first optical-frequency topological insulators were demonstrated in 2013 [9]. In these systems, light travels along the surface in one direction and with immunity to defects and disorder (Fig. 1). This "topologically protected transport of light" could help researchers build nanoscale optical isolators and make the transport of quantum information more robust.

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Right from the very beginning, researchers realized that topological photonics could do more than emulate condensed-matter phenomena. Unlike condensed-matter systems, photonic systems can be highly nonlinear, and their dynamics can be far away from thermal equilibrium. Photonic systems also open up possibilities of combining topology with NH physics by making use of gain (stimulated emission) and loss, which are common to any laser. In 2015, the first NH topological system was demonstrated in an experiment with a 1D lattice of coupled optical waveguides engineered with loss [10]. But the experiments were thought to be specific to a particular system, and several theory papers questioned whether NH systems were compatible with topological invariants.

Some of the controversy was resolved by subsequent *PT*-optics experiments that demonstrated 1D topological phases [11]. But light cannot propagate along the edge of a 1D system (since it's just a single point), so the question remained open as to whether NH topological systems could exhibit topologically protected transport. The issue came up again earlier this year, when we and our colleagues demonstrated experimentally and theoretically a 2D topological insulator laser [2], which exploits topological immunity to defects and disorder to enhance the lasing efficiency and maintain single-mode lasing even high above the lasing threshold. The topological laser can be described by a non-Hermitian model derived by introducing gain, loss, and nonlinearity into the Haldane model, which is the archetypical model for topological phases [2]. While this work shows that a 2D NH system can exhibit topologically immune transport, it remains unclear how to extrapolate this result to other systems. For this reason, Gong *et al.* [1], as well as other groups [3, 12], have aimed to create a theoretical basis for topological phases in NH systems.

Gong *et al.* developed a general methodology that classifies a NH topological system by examining how it can be mapped from one system to another by smoothly changing its parameters. If such a mapping requires the energy bands in the calculated spectrum to cross a "base energy" point, then the system is said to have a topological phase, characterized by a topological invariant. By contrast, if the mapping can occur without any crossing, then the system is defined to be topologically trivial. Based on these criteria, Gong *et al.* constructed a periodic table, which lists NH systems based on the number of dimensions and what symmetries are present (chiral, particle-hole, etc.). This table, which borrows from a similar classification for topological materials in Hermitian systems, identifies where topological phases should be found, according to the definition Gong *et al.* proposed.

The most profound conclusion from Gong *et al.* is that 2D NH systems do not have topological phases. How does one reconcile this with the topological insulator laser [2], as well as with recent derivations of topological invariants of 2D NH systems [3]? The answer lies in the criterion for classification. Gong *et al.* apply a very strict definition of topological invariant, which is separate from topologically protected transport. In other words, a system that exhibits topologically protected transport may be classified in Gong *et al.*'s framework as not having a topological phase.

Gong *et al.*'s classification scheme offers a deep perspective on NH topological systems that may guide future work. But at the end of the day, the interest in topological phases stems from their robust transport properties, which offer fascinating possibilities for quantum computing, optical-fiber networks, and much more. The marriage of topology and NH physics is already opening up exciting new applications, and revealing the underlying principles of NH topological physics could greatly enrich the development of the field. The step taken here by Gong *et al.* undoubtedly marks important progress, but their way of classification does not yet answer the question of what physical mechanisms underlie topologically protected transport. This fundamental question has been addressed for some specific cases [3, 12], but the jury is still out on how to formulate a more general answer.

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