Long-range interactions between optical solitons

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Solitons are self-localized wave packets arising from a robust balance between dispersion and nonlinearity. They are a universal phenomenon, exhibiting properties typically associated with particles. Thus far, interactions between solitons have been observed only between neighbouring solitons at close proximity. Here, we study interactions between solitons in highly non-local nonlinear media, and demonstrate experimentally attraction between solitons propagating in different samples, where their optical fields never overlap and the interaction is mediated solely by a non-optical wire. This increases the soliton interaction range by orders of magnitude, and breaks the close-proximity and nearest-neighbour limitations on soliton interactions. We also experiment with three-dimensional interactions between solitons that are far apart, where the solitons capture each other into a spiralling motion with a circular orbit, and a tangential velocity that does not depend on the separation between solitons. Our study suggests that these phenomena could be used in the construction of novel model systems for studying the behaviour of complex nonlinear networks.

Solitons are self-trapped, localized, wave packets that do not broaden while propagating in a dispersive environment[1]. They interact with each other and exhibit properties normally associated with particles, hence the name 'solitons'[2]. Solitons are ubiquitous in nature, and can be found in a variety of systems: from water waves, sound waves and charge-density waves to matter waves and electromagnetic waves[1]. However, in spite of the large diversity of the systems in which solitons exist, the basic properties of solitons always follow the same trends, conserving quantities such as power and momentum. Undoubtedly, the most fascinating features of solitons are their particle-like interaction phenomena, as solitons can exert forces on each other, and the interactions between solitons can result in soliton fusion, fission and annihilation[3–5], as well as spiralling[6], breakup into multiple filaments[7–10] and so on. So far, the vast majority of experiments on soliton interactions have been carried out in nonlinear media with a local response; that is, the nonlinear effect at a given location is a function of the field only at that same location. In 'local' nonlinearities, the forces between solitons are determined by the overlap between their wavefunctions[11]. Hence, the forces decay quickly as the separation between solitons is increased, becoming negligible when the soliton separation is approximately three times the soliton width.

Here, we experiment with solitons in highly non-local nonlinear media, and demonstrate attraction between solitons propagating in different samples, thereby increasing the soliton interaction range by orders of magnitude, and breaking the close-proximity and the nearest-neighbour limitations on soliton interactions. Our experiments are carried out in materials exhibiting the optical thermal nonlinearity, for which the interaction is mediated by heat transfer[12]. Nonetheless, the new concepts presented here are universal and can be implemented in other highly non-local nonlinearities, such as, for example, semiconductor amplifiers, where the interaction is mediated by transport of charge carriers[13]. Our study suggests that these phenomena could be used in the construction of novel model systems for studying the behaviour of complex nonlinear networks[14].

In contrast to 'local nonlinearities', the nonlinear response in non-local media is carried to regions beyond the range of localized wave packets. That is, the nonlinear effect at a given location is a function of the field at some non-locality range surrounding

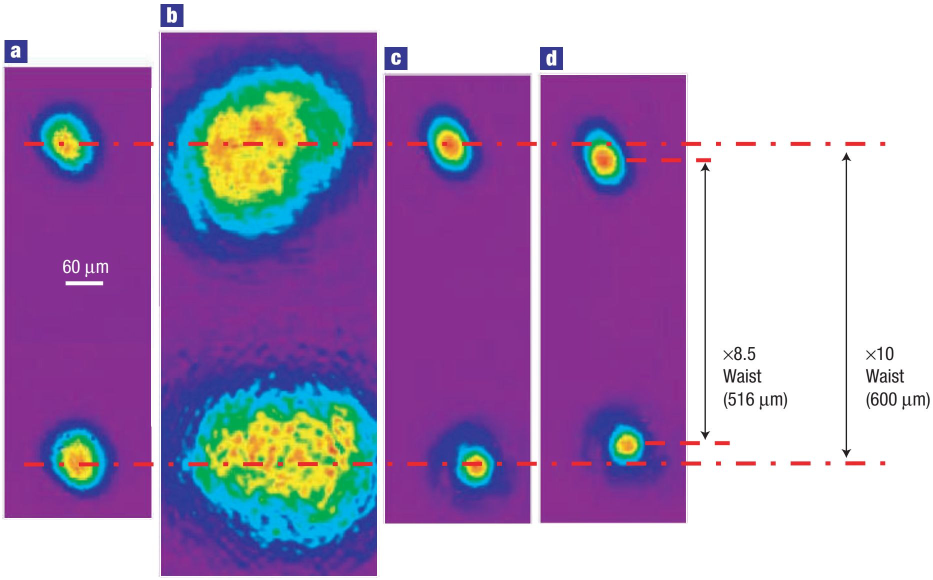

Figure 1 Experimental observation of planar interactions between solitons that are far apart in non-local nonlinear media. a, Input beams. **b**, Broadened output beams after propagating in the medium at low power (linear propagation). **c**, Individually launched output solitons. **d**, Simultaneously launched output solitons exhibiting attraction from afar.

that location. Non-local optical nonlinearities are inherent in many systems, when the underlying mechanism involves transport (of heat[15,16], atoms in a gas[17], charge carriers in dielectrics[18] and semiconductors[13,19], and so on), long-range forces (for example, electrostatic interactions in liquid crystals[20–22], photon attraction[23]) and many-body interactions (for example, plasma[24,25] and matter waves[26]). Interestingly, in spite of the widening effect of non-locality, even highly non-local nonlinearities can support solitons[12,27,20,28]. These observations raise a fundamental question: is there an upper limit on the distance at which solitons can interact? In non-local media, the range of soliton interactions is set by the range of non-locality, and can get much larger than the soliton width[29,30]. In optics specifically, Snyder and Mitchell suggested that two solitons in highly non-local nonlinear media can attract each other from a very large distance, with the interaction mediated solely by the non-local property of the medium[29]. The interaction, in this case, is determined solely by the power of the solitons, and is independent of their relative phase[29]. In many cases, however, the range of non-locality is limited by saturation effects, as happens with photorefractives where the Debye length sets the range of non-locality[18], and with liquid crystals where the molecules' orientation angle has a maximum value[20]. For this reason, soliton interactions in non-local media were thus far demonstrated with solitons separated by up to approximately three soliton widths[31]. Naturally, for long-range interaction effects, it is desirable to have a non-saturable transport mechanism underlying the nature of the nonlinearity. The simplest non-saturable transport processes are those described by a Poisson-type (diffusion) equation, with the light acting as a 'source'[28]. In this spirit, our experiments are carried out in lead glass exhibiting an optical thermal nonlinearity. The heat, generated by the (small) light absorption, diffuses with some thermal conductivity, thereby increasing the temperature in a large area surrounding the illuminated region. The temperature increase raises the refractive index. Hence, the lead glass acts as a non-local nonlinear medium of the self-focusing type[28]. As there is no saturation in the heat transport process, the non-locality range is limited only by the finite size of the sample[28].

We begin by demonstrating attraction between solitons with an initial separation 10 times larger than the soliton width. The details of our experiments are explained in the Methods section. A 2 W laser beam at 488 nm wavelength is split in two, and each beam is focused onto the input face of the lead glass sample, at normal incidence, so that the beams initially have parallel trajectories. Figure 1a shows the two parallel-launched 60 µm full-width at half-maximum (FWHM) input beams, which are initially separated by 600 µm. At low power (∼10 mW), these beams propagate linearly and broaden to ∼160 µm at the sample output (Fig. 1b). At high power (2 W), each individually launched beam forms an ∼60 µm FWHM soliton. The output separation between individually launched solitons is identical to their input separation (Fig. 1c). However, when the high-power beams are launched simultaneously, the solitons attract each other, in spite of their large separation. The attraction bends the soliton trajectories, and they emerge at the output separated by 516 µm, that is, an 84 µm change in the separation between their centres (Fig. 1d).

From planar interactions between solitons, we move on to full 3D spiralling interactions. In the past, such 3D soliton collisions were explored in nonlinear media with a local response, where it was demonstrated that two solitons launched with initial trajectories that are not in a single plane can capture each other into a spiralling motion[6,32]. For those experiments in local nonlinearities, the spiralling orbits were generally elliptic[6], and the motion was always accompanied by power flow from one soliton to the other[32]. Moreover, in local nonlinearities, the forces between solitons, and consequently the tangential velocities, decrease exponentially as the soliton separation is increased. In non-local nonlinearities, on the other hand, we find that two solitons can capture each other into a spiralling motion with a circular orbit, and rotate around each other from a very large distance, with a tangential velocity that does not depend on the separation between the solitons.

For the 3D soliton interaction experiments, we launch two solitons at small inclination angles relative to the propagation direction, providing the system with angular momentum (Fig. 2a).

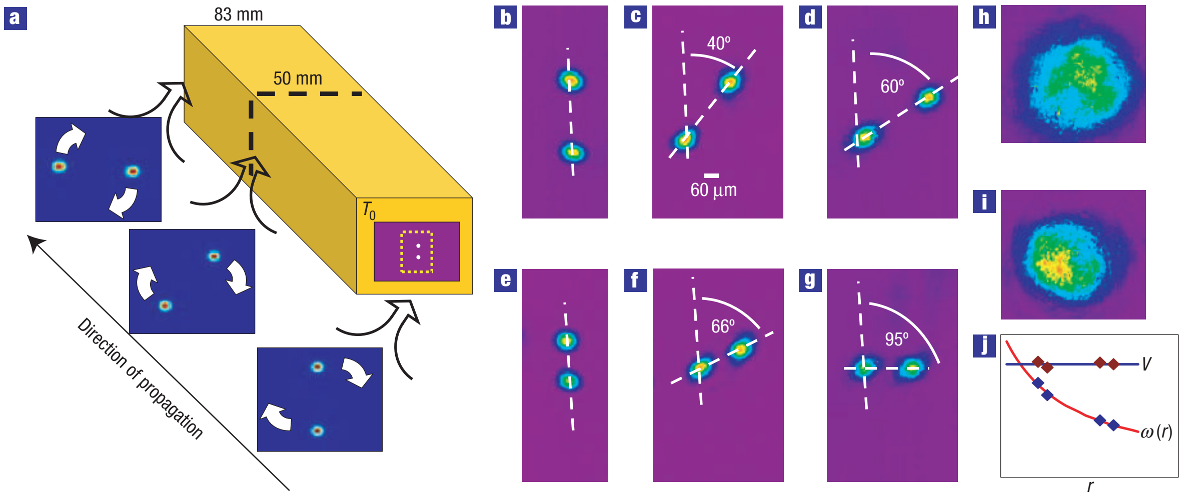

Figure 2 Experimental observation of three-dimensional spiralling interactions between widely separated solitons. a, Experimental scheme. **b–d**, Two input beams separated by five times their FWHM (**b**), form 60 μm FWHM solitons at high power, and experience 40° and 60° rotation after 50 mm and 83 mm of propagation (**c** and **d**, respectively). **e–g**, Two input beams separated by three times their FWHM (**e**), form 60 μm FWHM solitons at high power, and experience 66° and 95° rotation after 50 mm and 83 mm of propagation (**f** and **g**, respectively). **h**,**i**, At low power, the beams linearly diffract and broaden. **j**,The experimentally measured tangential velocity and the angular velocity of the interaction.

The conservation of angular momentum and the radial attraction between the (widely separated) solitons lead to a spiralling motion, where the solitons orbit around each other. Typical examples are shown in Fig. 2, where the spiralling is in a circular orbit. In the first example, we launch two solitons, each 60 μm FWHM, separated by five times their width (Fig. 2b), and observe their mutual rotation, of 40° and 60° after 50 and 83 mm of propagation, respectively (Fig. 2c,d). In this experiment we have tuned the parameters to yield rotation in a circular orbit, that is, the initial separation between solitons remains unchanged. We then decrease the initial separation between the solitons to three times their FWHM, while maintaining all other parameters (launch angles, laser power and boundary temperatures) unchanged. As the attraction between the solitons has increased, the solitons now experience rotation at a higher angular velocity, exhibiting 66° and 95° rotation after propagation distances of 50 mm (Fig. 2f) and 83 mm (Fig. 2g), respectively. Note that in this second experiment, the initial separation between the solitons remains unchanged throughout propagation without any further tuning. That is, setting the conditions of the first experiment to support a circular orbit and then modifying only the separation between solitons, maintains the circular nature of the spiralling orbit. This feature occurs for any separation distance larger than ∼2 soliton widths. Moreover, plotting the experimentally measured tangential and angular velocities as a function of soliton separation (Fig. 2j), reveals that the tangential velocity is constant, independent of the separation between solitons, whereas the angular velocity is inversely proportional to the separation. Finally, for comparison, when the power of the input beams is low, they broaden considerably to a ∼155 μm output width (Fig. 2h,i).

Soliton interactions in our non-local thermal nonlinearity are described by the nonlinear wave equation coupled to the heat-diffusion equation[28]. The latter is essentially the Poisson equation describing electrostatic forces, where the optical power of the beam plays the role of the source (charge). By virtue of this equivalence, we find (see the Methods section), that the attraction between solitons that are far apart is proportional to the product of their optical powers, divided by the distance between them, $F = -kp_1p_2/r(\hat{r})$, where p is the power in each soliton, r is the distance between the solitons and k is a constant reflecting the medium parameters. A manifestation of this interaction force is nicely demonstrated in the 3D spiralling interaction between solitons. When the solitons spiral around each other in a circular orbit (as in Fig. 2), the attraction force is balanced by the centripetal force, $F = kp_1p_2/r = V^2/r$, with both forces being inversely dependent on the separation r. Thus, the tangential velocity V does not depend on the soliton separation r, whereas the angular velocity $\omega = V/r$ is inversely proportional to r. These features are highlighted by the experiments and plot in Fig. 2 (see also the discussion at the end of the Methods section on the many-body aspect of our systems).

It is important to emphasize that the particle-like behaviour is associated with solitons, and cannot be applied to non-soliton beams. Generally, optical beams propagating at close vicinity in a nonlinear medium always interact, whether they are solitons or not. However, such interaction is complex, and generally cannot be described by a finite set of interaction rules. Beams (fields) generally have an infinite number of degrees of freedom. Thus, the information contained in the outcome of an interaction between non-soliton beams is embedded in an infinite number of degrees of freedom. For example, when the initial information is embedded in the position of the beams, the interaction between non-soliton beams deforms the beams, hence the information contained in the interaction product is no longer embedded solely in the beams' position. This causes the information to 'fade away' in the system in an irretrievable fashion. Solitons, on the other hand, have conserved quantities, setting a finite limit on the possible degrees of freedom. The interactions between solitons are therefore tractable, and the information about their interactions remains contained in a small number of degrees of freedom. To exemplify this fundamental difference between soliton interactions and interactions among other nonlinear beams, we carry out a set of experiments on 3D spiralling interactions of solitons and of high-power non-soliton beams, under comparative conditions. We set the experimental parameters to be identical (same nonlinear medium, boundary conditions, input beam trajectories and optical power), with a

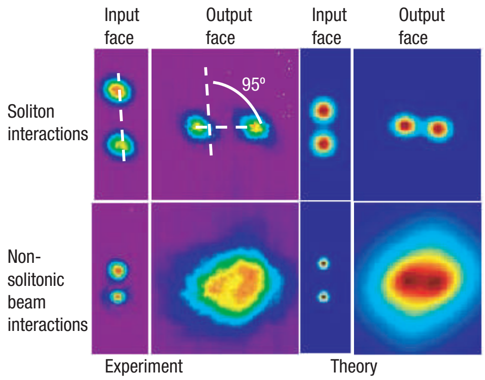

Figure 3 Experimental and theoretical comparisons of 3D interactions between solitons and between high-power non-soliton beams. All are in the same settings (same medium, boundary conditions, optical power and input beam trajectories). The only difference between the initial conditions is that, for the solitons, the initial beam width is appropriate to form solitons at the particular power level, whereas the initial width of the non-soliton beams is too narrow to form solitons at the power level used. The interacting solitons (upper level) maintain their identity and shape, with the only parameter modified by the interaction being their rotation angle around a common centre. On the other hand, the interacting non-soliton beams deform, merge and lose their individual identities.

single difference: the width of the non-soliton beams is too narrow to form solitons at the power level we use. The experimental and numerical results (see the Methods section) are shown in Fig. 3. The upper row shows the 3D interaction between two 60 μm FWHM solitons, which spiral around each other by 95° after a propagation distance of 83 mm. In this example, the information about the soliton interaction is embedded only in the relative position of the solitons, because each soliton conserves its power and shape. The lower row in Fig. 3 shows the interaction between two beams launched with 35 μm width (FHWM), of the same power (1 W each) and the same trajectories as the solitons. These high-power beams self-focus and attract each other, but their self-focusing rate is too weak to balance diffraction, hence they broaden. The nonlinear interaction distorts these beams: a portion of each beam reaches the centre, rendering the output beams indistinguishable. Hence, the 'usable' information about the interaction (the relative position of the beams) is lost when the beams are not solitons. Moreover, even the basic 'unit cell' (the separate identity of each beam) is not conserved. Thus, the information about the interaction must be described by the change of shape, separation and rotation of the beams. That is, the interaction between such non-soliton beams is described by a continuous set of variables, rather than a change in a single parameter (rotation angle), as demonstrated by the soliton interaction shown in the upper panel of Fig. 3, under the same launch conditions. This example is not unique in any way: nonlinear interactions are described by a continuous set of variables, whereas soliton interactions can be described by a finite set of rules.

Under such reasoning, solitons have been suggested as the building blocks of computation systems[33,34], in the spirit of soliton-like behaviour in cellular automata[35]. In fact, experiments have demonstrated that solitons can indeed transfer and cascade information from one collision event to another[36,37]. However, thus far, all soliton interaction experiments (in local or non-local media) have been between nearest-neighbour solitons, thus posing a severe problem on the 'connectivity' of a 'soliton network'.

In the final section of our article, we demonstrate how soliton interactions in non-local nonlinear media can be 'wired' from one sample to another, thus breaking the nearest-neighbour limitation on soliton interactions. The range of soliton interactions can be extended far beyond the sample size by transporting the physical property responsible for the nonlinearity (heat, in this case) by some non-optical means over a large distance. We realize such interactions between solitons propagating in different samples by connecting the samples with a copper foil with a high heat conductivity (Fig. 4a). The nonlinear response is now transported between different samples that could be placed very far apart from each other. In fact, the only limitation on the soliton interaction range in such settings is the finite heat conductivity of the foil, hence, in principle, the soliton interaction in this system is of 'infinite' range. The solitons in each of these samples 'feel' each other, because the heat generated by each soliton diffuses to the boundaries, and through the foil to the other sample. In symmetric settings, heat accumulates on the foil, causing a temperature increase on the boundaries of both samples connected to it. This creates a small asymmetry in the induced index-change in each sample, pulling the soliton beams towards the connected boundaries. Thus, the solitons, each propagating in a different sample, attract each other from a very large distance, with the interaction mediated by the metal foil.

For the experiments on soliton interaction mediated by a metal wire, we use two samples, as described in the Methods section. In each sample, three transverse boundaries are thermally connected to a heat sink at fixed temperature. The remaining transverse faces, one in each sample, are connected at their centres with a 100-μm-thick copper foil (Fig. 4a). We use two laser beams, each of 1.5 W power, to generate a soliton in each sample, with a 2 mm initial separation between the centres of the solitons, which is ∼50 times their width. Figure 4 shows typical experimental results, where Fig. 4b,c shows the two 38 μm FWHM input beams, each launched in its own sample. At high power (1.5 W), each individually launched beam forms an ∼38 μm FWHM soliton (Fig. 4d,e). However, when these high-power beams are launched simultaneously, the solitons attract each other, and bend their trajectories towards each other, decreasing their separation by ∼90 μm (Fig. 4f,g). For completeness, at low power (10 mW), the beams broaden to ∼188 μm at the sample output (Fig. 4h,i). In this experiment, the solitons attracted each other from a distance ∼50 times their width. This constitutes the first experimental demonstration of essentially 'infinite-range' forces between solitons. We emphasize that the interaction length in our setting is limited only by the power of the laser, the heat conductance of the foil and the thermal insulation of the foil in the regions between the samples. Such limitations are not fundamental, and can be overcome by packaging techniques. Moreover, no threshold or saturation occurs in such long-range interaction; a higher laser power will yield narrower solitons and a stronger heating, hence the attraction force will be acting from a larger distance (in terms of soliton width) at the same efficiency.

The technique of using 'wiring' to mediate interactions between solitons propagating in different samples overcomes the nearest-neighbour limitation on soliton interactions: a soliton is no longer restricted to interact only with its nearest-neighbouring soliton. This technique, which can also be implemented in other non-local nonlinear systems (for example, semiconductors[13] and liquid crystals[21]), opens up a variety of new possibilities. The ability to 'transport' the soliton interactions over large distances, connecting to multiple 'addressees', allowing feedback, gating and amplification of the 'interaction signal' (heat, in our particular case), suggests fascinating directions that have never been explored in soliton science. It might be possible to harness such non-local soliton interactions to construct model systems of complex nonlinear networks[14], in ways that have been never explored

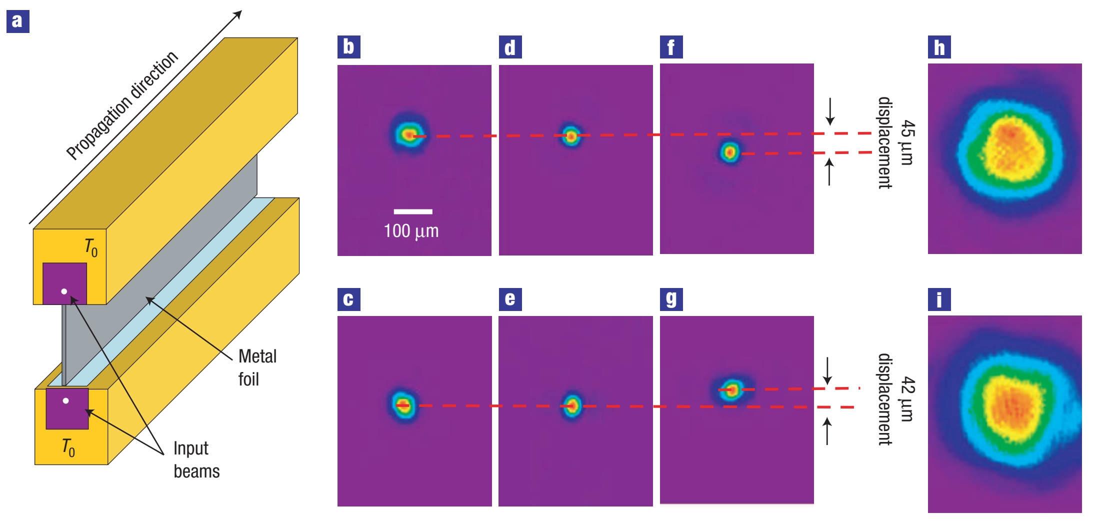

Figure 4 Experimental demonstration of the interaction between two solitons propagating in separate samples, with the interaction mediated by wiring. **a**, Experimental scheme. **b**,**c**, Input beams. **d**,**e**, Individually launched output solitons. **f**,**g**, Simultaneously launched output solitons exhibiting strong attraction. **h**,**i**, At low power, the beams diffract and broaden.

experimentally. In such a complex network of solitons, solitons would be the building blocks, the predesigned wiring architecture ('circuitry') and optical feedback defining the information flow in the network, and the nonlinear optical interaction providing the switching, 'programming' and embedded logic. It might be possible to study self-synchronization of a network of solitons, 'small-world' networks of solitons[14], fractals from solitons[38–40] and possibly chaotic networks of solitons.

METHODS

Here, we explain the experimental and theoretical methods used in our article, and discuss the many-body aspects of soliton interactions in optical thermal nonlinear media.

EXPERIMENTAL METHODS

All our experiments are carried out in lead glass samples, with a square 2 mm × 2 mm cross-section, which are 83-mm-long in the propagation direction. In the experiments on soliton interactions in a single sample (Figs 1–3), the four transverse boundaries of the sample are thermally connected to a heat sink and maintained at room temperature. In the experiments on interactions between solitons propagating in different samples, the samples are connected with a heat-conducting metal foil, as shown in Fig. 4a. In all the experiments we use 488 nm wavelength laser beams, which are focused onto the input face of the samples with their trajectories according to the specific experiment. The intensity distribution of the beams at the input and output faces of the sample are monitored by charge-coupled device cameras.

The parameters of the lead glass samples used in our experiments and calculations are: refractive index of $n_0 = 1.8$, index change per temperature degree $\beta = 14\times10^{-6}\ \mathrm{K}^{-1}$, optical absorption coefficient $\alpha \approx 0.01\ \mathrm{cm}^{-1}$ and thermal conductivity of $\kappa = 0.7\ (\mathrm{W\,m^{-1}\,K^{-1}})$.

THEORETICAL METHODS

The optical thermal nonlinearity in lead glass acts in the following manner. A light beam gets slightly absorbed and heats the glass, acting as a heat source, and the heat diffuses with a thermal conductivity κ. This yields a non-uniform temperature distribution T induced by the light intensity I, satisfying the heat equation in (temporal) steady state[28]:

$$\kappa\nabla^2 T(x,y,z) = -\alpha I(x,y,z), \quad (1)$$

where α is the absorption coefficient. The change in temperature, ΔT, results in a proportional increase in the refractive index $\Delta n = \beta\Delta T = \beta(T - T_0)$, with β being the thermal coefficient of the refractive index. Here, T_0 is the temperature when $I = 0$, which is imposed by the temperature at the sample boundaries. These boundary conditions directly affect the temperature distribution everywhere in the sample, and hence affect Δn induced by I as

$$\nabla^2\Delta n(x,y,z) = -\tilde{\kappa} I(x,y,z), \quad (2)$$

where $\tilde{\kappa}$ is a constant of the medium. Equation (2) is formally equivalent to the Poisson equation in electrostatics with a 'source term': $\nabla^2\Phi(x,y,z) = -\rho(x,y,z)$, where Φ is the potential and ρ is the charge density. By virtue of this equivalence, and the dimensionality of the problem, the force between two solitons in such a medium is equivalent to the electrostatic force per unit length, between two oppositely charged wires. Thus, the attraction force between two solitons is $\bar{F} = -kp_1p_2/r(\hat{r})$, where p_i is the total power in the ith soliton, r is the distance between the solitons and $k = \tilde{\kappa}/2\pi n_0$. This equivalence to electrostatics reveals the nature of long-range interactions between two solitons, and fails only when the fields of the solitons are at very close proximity so they considerably overlap. The attraction between solitons that are far apart in a medium exhibiting the optical thermal nonlinearity is therefore proportional to the product of their powers, and inversely proportional to the distance between them. In the more general sense, solitons in non-local nonlinear media, whose nonlinearity is described by the archetypal Poisson equation, interact like charged particles, with a force $F \propto 1/r^{d-1}$, where d is the dimensionality of the problem $[(d+1)D$ solitons, $d = 1,2,3]$.

Equation (2) describes the transport process in the non-local nonlinear medium. To study the propagation of optical waves in this medium, equation (2) is augmented by the nonlinear wave equation. The quasi-monochromatic optical field has the form $E = A(x,y,z)e^{i(\omega t - kz)} + \mathrm{c.c.}$, with A being the slowly varying amplitude, $k = \omega n_0/c$, ω is the frequency, t is the time, n_0 is the unperturbed refracting index $(|\Delta n| \ll n_0)$, c is the vacuum light speed and $I = |A|^2$. The paraxial nonlinear wave equation describing the light in this medium is

$$(\partial_x^2 + \partial_y^2)A + 2ik\partial_z A + 2k^2(\Delta n/n_0)A = 0. \quad (3)$$

Past experiments have revealed that the medium is isotropic, hence the equation is scalar. To identify solitons, we seek solutions of the form $A(x,y,z) = u(x,y)e^{i\gamma z}$, where u is a complex function of x, y and γ is a real constant, for which A and ΔT are z-independent, hence so is $\Delta n(x,y)$. We find the solitons by solving equations (2) and (3) self-consistently, with optical boundary conditions such that A and its derivatives vanish at the boundaries[28]. The temperature boundary conditions and the parameters of the simulation are similar to the parameters of the experiments with their specific configuration. After identifying solitons and finding their wavefunction, we test their stability in the presence of noise with a standard beam propagation method[28]. After finding solitons, we study their interactions by numerically propagating two solitons in the experimental configurations, and solving equations (2) and (3) iteratively (as in ref. 28). Typical results of the simulations are presented in Figs 2 and 3.

It is now instructive to discuss the meaning and implications of the set of equations describing the nonlinear dynamics of light waves in thermal optically nonlinear media. Equation (1), which describes the transport of (light-induced) heat in our system, is linear with respect to the temperature. As such, it might be thought that the propagation of light waves in this medium is also linear. This view would be conceptually erroneous, because equations (2) and (3) together form a nonlinear set. In fact, equations (2) and (3) can be cast into a single, nonlinear, integro-differential equation that can support highly nonlinear entities as solitons.

In this vein, it is illuminating to emphasize the difference between interacting solitons in an optical thermal nonlinear medium, and a single light beam and an array of metallic wires that heat the sample at reconfigurable positions. The latter case is equivalent to a single particle in a fixed potential. The potential could be complicated, yet the problem is still a one-body problem, and cannot lead to complex dynamics. On the other hand, two soliton beams interacting nonlinearly form a two-body problem. Taking multiple solitons will make it a many-body problem, and its complexity is such that it can become chaotic, or evolve in patterns characteristic to complex nonlinear networks. The wiring technique demonstrated in Fig. 4a makes it possible for solitons to interact not only with their nearest neighbour, and as such it facilitates the construction of complex nonlinear networks of solitons.

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Competing financial interests

The authors declare that they have no competing financial interests.