Spatial Thirring-type solitons via electromagnetically induced transparency

Inbal Friedler and Gershon Kurizki

Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Oren Cohen and Mordechai Segev

Physics Department, Technion—Israel Institute of Technology, Haifa 32000, Israel

Received August 3, 2005; revised manuscript received August 23, 2005; accepted August 25, 2005

We show that the giant Kerr nonlinearity in the regime of electromagnetically induced transparency in vapor can give rise to the formation of Thirring-type spatial solitons, which are supported solely by cross-phase modulation that couples the two copropagating light beams. © 2005 Optical Society of America

OCIS codes: 190.5530, 270.1670.

Coherence between two levels that is induced by a strong (drive) laser field can give rise to absorption cancellation on another transition in a Λ-shaped atomic level configuration for a weak probe field. Absorption cancellation occurs via destructive interference with the drive field. This phenomenon, known as electromagnetically induced transparency (EIT[1,2]), changes the probe-field dispersion, making its group velocity dependent on the drive field, so that by turning the drive field off one can slow a probe pulse down to a complete standstill.[3] Slow-light manifestations of EIT have attracted considerable attention, in view of their possible use for storing and regenerating quantum states of light in atomic quantum networks.[4] Another nonlinear manifestation of EIT is spatial solitons, which were predicted to form when diffraction is balanced by self-phase modulation[5] (SPM). Whereas the foregoing aspects of EIT pertain to a single probe beam, giantly enhanced Kerr nonlinear coupling of two probe beams is not less promising.[6,7] Its highlight is the dramatically enhanced phase shift (compared with similar shifts in other Kerr media), impressed by one ultraweak probe on another (cross-phase modulation, XPM) in the N-shaped atomic level configuration detailed below. This effect may bring about the deterministic entanglement of two single-photon pulses.[8,9]

Despite the extensive discussion of the giant XPM in EIT media and its recent experimental demonstration,[10] its analysis has been restricted mainly to one dimensional (1D) propagation without considering transverse (diffraction) effects of the cross-coupled beams. Here we study unexplored aspects of the giantly enhanced XPM between two beams subject to EIT: the formation of low-power spatial solitons that arise solely from the balance between diffraction and XPM with no contribution from SPM.

This kind of soliton generically conforms to the massive Thirring model.[11–13] In optics, Thirring-type (holographic) solitons were predicted to occur with the XPM arising from the grating induced by two mutually coherent fields,[14,15] having no SPM contribution. However, even though evidence for holographic focusing was reported,[16,17] optical Thirring-type solitons have thus far never been observed. This is because it is very difficult to find optical systems with large XPM but lacking SPM altogether. The system proposed in this Letter offers just that and thus supports the formation of such "exotic" optical Thirring-type solitons.

Consider a cold atomic medium containing two species of atoms, A with a Λ-shaped level configuration and B with an N-shaped level configuration (Fig. 1). All the atoms are optically pumped to the ground states $|b\rangle_{A,B}$. Atoms A and B resonantly interact with two running-wave fields driving the atomic transitions $|c\rangle_{A,B} \to |a\rangle_{A,B}$ with the Rabi frequencies $\Omega_d^{(A,B)}$, respectively. In the absence of level $|d\rangle_B$, this situation corresponds to the usual EIT for the fields $\mathcal{E}_{1,2}$ that are acting on the transitions $|b\rangle_{A,B} \to |a\rangle_{A,B}$: in the vicinity of a frequency corresponding to the two-photon Raman resonances $|b\rangle_{A,B} \to |c\rangle_{A,B}$, the medium becomes transparent for both weak fields.[1,2,18] This transparency is accompanied by a steep variation of the refractive index. The field $\mathcal{E}_1$ dispersively interacts with atoms B via the transition $|c\rangle_B \to |d\rangle_B$ with the detuning $\Delta = \omega_{dc}^{(B)} - \omega_1$. As a result, atoms of species B simultaneously provide EIT for the field $\mathcal{E}_2$ and its cross coupling with the field $\mathcal{E}_1$, known as XPM.[6,7,20] Note that the role of atoms A in Fig. 1 is only to provide EIT for the field $\mathcal{E}_1$. This is necessary to match

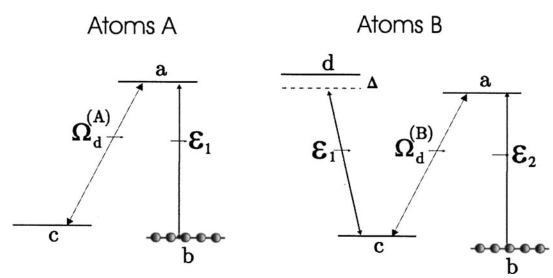

Fig. 1. Atomic level scheme involving two species of atoms A and B, both subject to EIT conditions. The fields $\mathcal{E}_1$, $\mathcal{E}_2$ interact via Kerr-nonlinear XPM. For the case in which $\mathcal{E}_{1,2}$ are cw fields, atoms A and $\Omega_d^{(A)}$ can be ignored, as they are unnecessary.

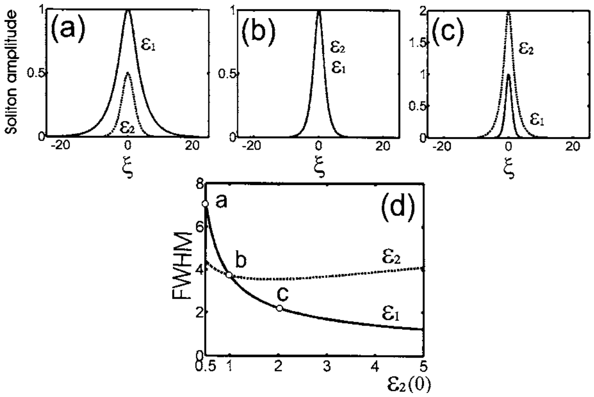

Fig. 2. Fundamental-mode Thirring-type solitons. (a)–(c) Soliton profiles for different ratios between the peak amplitudes. (d) Soliton existence curve: FWHM of each field versus the peak amplitude $\mathcal{E}_2(0)$ at a fixed peak amplitude $\mathcal{E}_1(0)=1$. Points a–c correspond to the solitons of (a)–(c), respectively.

the group velocities of the two copropagating weak pulses and thus maximize their interaction time.[20] In what follows we limit ourselves to continuous-wave (cw) fields, so that EIT conditions for field $\mathcal{E}_1$ are not required, i.e., atoms A, as well as the driving field $\Omega_d^{(A)}$, can be dropped.

Here we study optical beams with narrow transverse profiles, so that diffraction plays a significant role, as opposed to earlier treatments of XPM in EIT.[6–9,20–22] Another distinct feature of this problem is that SPM, caused by the coupling of the field $\mathcal{E}_2$ to the transition $c \rightarrow d$ in atoms B, or its equivalent for the field $\mathcal{E}_1$, is inversely proportional to the detuning of the field from that transition. Hence, assuming that the detuning of field $\mathcal{E}_2$ is much larger than the detuning of field $\mathcal{E}_1$ on the same transition, SPM can be neglected and only XPM survives. These two features are responsible for the ability to form a novel type of spatial soliton. The system is described by the following equations for the slowly varying envelopes, obtained perturbatively under the weak-field adiabatic approximation,[18,19] for cw fields and the standard paraxial conditions (i.e., optical beams much wider than the wavelength):

$$2ik_1\frac{\partial}{\partial z}E_1 + \nabla_\perp^2 E_1 = -k_1^2\eta|E_2|^2E_1, \tag{1a}$$

$$2ik_2\frac{\partial}{\partial z}E_2 + \nabla_\perp^2 E_2 = -k_2^2\eta|E_1|^2E_2, \tag{1b}$$

where $\eta=\mu_{dc}^{(B)2}\mu_{ab}^{(B)2}\rho_B/(\Delta|\Omega_d^{(B)}|^2\epsilon_0\hbar^3)$, ρ_B is the density of atoms B, $\mu_{dc}^{(B)}$ and $\mu_{ab}^{(B)}$ are the $c\rightarrow d$ and $a\rightarrow b$ transition dipoles, and $k_{1,2}$ are the wave vectors of the fields $E_{1,2}$, respectively.

In deriving the above equations we have made the experimentally realistic assumption that absorption of both weak fields is negligible over the propagation length.[6,9,20] Rewriting these equations in dimensionless form yields

$$\nabla_\perp'^2\mathcal{E}_1 + i\frac{\partial}{\partial\zeta}\mathcal{E}_1 + \sigma|\mathcal{E}_2|^2\mathcal{E}_1 = 0, \tag{2a}$$

$$a^2\nabla_\perp'^2\mathcal{E}_2 + ia\frac{\partial}{\partial\zeta}\mathcal{E}_2 + \sigma|\mathcal{E}_1|^2\mathcal{E}_2 = 0, \tag{2b}$$

where $a=k_1/k_2$ is the asymmetry parameter, $\nabla_\perp'^2=\partial^2/\partial\xi^2+\partial^2/\partial\varsigma^2$; $\xi=k_1x$; $\varsigma=k_1y$; $\zeta=(k_1/2)z$; and $\mathcal{E}_i=\sqrt{|\eta|}E_i$. The sign of the detuning Δ (appearing inside the constant η) is crucial, as it determines the sign of the nonlinearity: $\sigma=+1$ for positive (red) detuning (focusing) and $\sigma=-1$ for negative (blue) detuning (defocusing).

It is important to emphasize the basic difference between Thirring-type solitons and Manakov-like vector solitons.[23] The nonlinearity in Manakov-like systems depends on the sum of the intensities of the individual components; that is, SPM and XPM play a symmetric role. Consequently, the constituents of the Manakov-like solitons are bound states of the potential they jointly induce. For Thirring-type solitons, on the other hand, each component feels, and is guided by, a different potential: $\mathcal{E}_1$ feels the potential induced by $\mathcal{E}_2$ and vice versa. Note also that a must be differ-

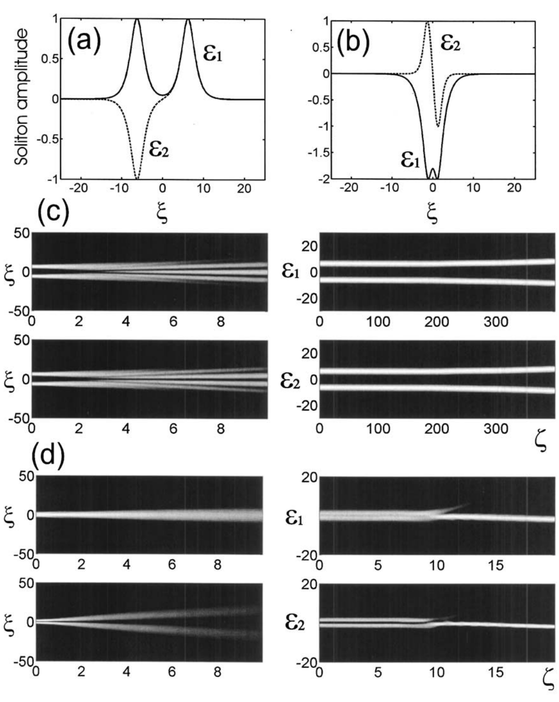

Fig. 3. (a) Composite Thirring-type soliton solutions with beam $\mathcal{E}_1$ in the fundamental (ground-state) mode and beam $\mathcal{E}_2$ in the dipole (first asymmetric) mode. The amplitude ratio is 1 (equal intensities). (b) Same as (a) for amplitude ratio 2. (c) Propagation of the solution shown in (a): beam $\mathcal{E}_1$ (upper) and beam $\mathcal{E}_2$ (lower) without (left) the nonlinearity for one diffraction length and with (right) the nonlinearity for $\sim$10 diffraction lengths. During nonlinear propagation the composite entity splits into two fundamental Thirring-type solitons diverging away from one another. (d) Propagation of the solution shown in (b) in the presence of initial noise. Both components fuse into a single fundamental Thirring-type soliton.

ent from 1: i.e., the fields $\mathcal{E}_1$ and $\mathcal{E}_2$, operating on different transitions, must have different wavelengths. If $a=1$ the SPM term cannot be neglected, and Eqs. (1) are no longer adequate.

The soliton solutions of Eqs. (2) have been found numerically through the self-consistency method and their propagation has been simulated by using the split-step Fourier propagation method. The asymmetry parameter a has been chosen to take the experimentally reasonable value of 1.0005 throughout our calculations. In what follows we discuss the main features of solitons thus obtained, referring to solutions that are bound in one or two transverse directions as 1D and 2D, respectively.

We first discuss the $\sigma=+1$ (focusing) case. We have studied various amplitude ratios of fields $\mathcal{E}_1$ and $\mathcal{E}_2$ in the presence of noise to find that the fundamental (ground-state mode) mode of the 1D system ($\nabla'^2_\perp=\partial^2/\partial\xi^2$) is stable. Figures 2(a)–2(c) present such fundamental Thirring-type solitons with different amplitude ratios. Figure 2(d) shows the existence curve versus the peak amplitude of the second component, $\mathcal{E}_2(0)$, when $\mathcal{E}_1(0)=1$. Note that the existence curve is governed only by the ratio between the peak amplitudes. Increasing the peak amplitudes of both components by a factor α results in new widths, FWHM_1/α and FWHM_2/α.

When seeking Thirring solitons in two transverse dimensions ($\nabla'^2_\perp=\partial^2/\partial\xi^2+\partial^2/\partial\varsigma^2$), we find that the 2D solitons suffer from a weak instability, similar to 2D Kerr solitons.

Next, we have searched for 1D composite (multimode) Thirring-type solitons for which each field is in a different mode.[24] Specifically, we have looked for solitons in which $\mathcal{E}_1$ and $\mathcal{E}_2$ are in the fundamental and second (dipole-type) modes, respectively, as was found for holographic solitons[25] and the Manakov-like system.[23] However, our system is not saturable (unlike the one in Refs. 14 and 25), and we find these solutions to be unstable. When the amplitude of the dipole mode is equal to or larger than that of the fundamental mode, the composite entity splits into two fundamental Thirring-type solitons diverging away from one another [Fig. 3(c)]. The splitting occurs irrespective of whether we add initial noise to the ideal solution. On the other hand, when the fundamental component is more intense than the dipole component, they fuse (within several diffraction lengths, depending on the noise) into a fundamental-mode Thirring-type soliton [Fig. 3(d)].

The case of defocusing nonlinearity [$\sigma=-1$ in Eq. (2)] might have been expected to yield dark 1D or 2D solitons. However, within our model with $a\neq 1$, we cannot find such solitons. This is related to the fact that a dark soliton (for any local nonlinearity) is the second bound state of the induced potential at cutoff energy.[26] A Thirring-type EIT dark soliton requires both components to be at the cutoff energy of each other's induced potential. This requirement prohibits the existence of a dark Thirring soliton unless $a=1$. Yet, as discussed above, the case $a=1$ does not represent our EIT system anymore.

To conclude, we have shown that the giant Kerr nonlinearity in the regime of EIT in vapor can lead to the formation of spatial Thirring-like vector solitons, supported solely by cross-phase modulation.

This work was supported by the Israeli Science Foundation through the Center of Excellence on Coherent Matter Wave Optics.

References

1. S. E. Harris, Phys. Today **50**, 36 (1997).
2. M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge U. Press, 1997), Chap. 7.
3. C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature **409**, 490 (2001).
4. D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
5. T. Hong, Phys. Rev. Lett. **90**, 183901 (2003).
6. H. Schmidt and A. Imamoğlu, Opt. Lett. **21**, 1936 (1996).
7. S. Harris and L. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
8. I. Friedler, G. Kurizki, and D. Petrosyan, Europhys. Lett. **68**, 625 (2004).
9. I. Friedler, G. Kurizki, and D. Petrosyan, Phys. Rev. A **71**, 023803 (2005).
10. H. Kang and Y. Zhu, Phys. Rev. Lett. **91**, 093601 (2003).
11. W. Thirring, Ann. Phys. (N.Y.) **3**, 91 (1958).
12. A. V. Mikhailov, JETP Lett. **23**, 320 (1976).
13. In the Thirring model dispersion and/or diffraction result only from linear coupling between two modes.
14. O. Cohen, T. Carmon, M. Segev, and S. Odoulov, Opt. Lett. **27**, 2031 (2002).
15. O. Cohen, R. Uzdin, T. Carmon, J. W. Fleischer, M. Segev, and S. Odoulov, Phys. Rev. Lett. **89**, 133901 (2002).
16. O. Cohen, S. Lan, T. Carmon, J. A. Giordmaine, and M. Segev, Opt. Lett. **27**, 2013 (2002).
17. M. Vaupel, C. Seror, and R. Dykstra, Opt. Lett. **22**, 1470 (1997).
18. M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
19. M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).
20. M. Lukin and A. Imamoğlu, Phys. Rev. Lett. **84**, 1419 (2000).
21. S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. **81**, 3611 (1998).
22. M. Yan, E. G. Rickey, and Y. Zhu, Phys. Rev. A **64**, 041801(R) (2001).
23. Z. Chen, M. Acks, E. Ostrovskaya, and Y. Kivshar, Opt. Lett. **25**, 417 (2000).
24. M. Mitchell, M. Segev, and D. N. Christodoulides, Phys. Rev. Lett. **80**, 4657 (1998).
25. J. R. Salgueiro, A. A. Sukhorukov, and Y. S. Kivshar, Opt. Lett. **28**, 1457 (2003).
26. A. W. Snyder, D. J. Mitchell, and B. Luther-Davies, J. Opt. Soc. Am. B **10**, 2341 (1993).