

Observation of Vortex-Ring “Discrete” Solitons in 2D Photonic Lattices

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(Received 17 September 2003; published 25 March 2004)

We present the experimental observation of both on-site and off-site vortex-ring solitons of unity topological charge in a nonlinear photonic lattice, along with a theoretical study of their propagation dynamics and stability.

DOI: 10.1103/PhysRevLett.92.123904

PACS numbers: 42.65.–k

Coherent wave propagation in lattices is fundamental to various branches of physics, including lattice field theory [1], solid-state physics [2], photonics [3,4], and matter waves in optical traps [5]. The dynamics in these systems is defined by the phase relationship between different points on the wave front and by transport through tunneling or coupling between lattice sites. In nonlinear lattices, the additional feature of self-focusing or defocusing may balance the lattice diffraction, resulting in a discrete or lattice soliton. Until recently, experiments in nonlinear waveguide arrays had been limited to propagation in one dimension [6–8]. However, the development of a new technique [9] to optically induce nonlinear photonic lattices in photosensitive materials has allowed the observation of lattice solitons in both one [10] and two [11] dimensions. The induction technique has opened up new experimental possibilities to study soliton phenomena in periodic structures, and has generated numerous follow-up papers (e.g., [12,13]). In particular, the ability to create nonlinear waveguide arrays in 2D allows the study of lattice waves carrying angular momentum and the generation of vortex lattice solitons. Here, we report the first experimental observation of vortex-ring solitons in a nonlinear lattice [14]. We demonstrate both on-site and off-site vortex solitons in a square lattice, analyze these structures numerically, and address their stability properties in saturable nonlinear media. These vortex solitons are generic to nonlinear lattices in two dimensions and are among the building blocks for more extended and complicated wave structures. We anticipate that the dynamics observed here will appear in other systems, such as nonlinear fiber bundles [15], photonic crystal fibers [16], and Bose-Einstein condensates [5] in the near future.

While vorticity is fundamental to two-dimensional wave propagation [17], and vortex arrays are common to 2D systems [18], the study of vortex solitons in nonlinear lattices is relatively new. The optical case of discrete vortices was considered only recently, in Kerr nonlinear waveguide arrays, where on-site vortices (vortices whose singularity is located on a lattice site) [19] and off-site vortices (vortices whose singularity is located

between sites) [20] were studied. Both cases were found to be stable within a certain range of parameters. In this Letter, we present the experimental observation of both on-site and off-site vortex-ring lattice solitons of unity topological charge in a square lattice. Our experiments utilize the optical induction technique [9–11] in a photorefractive crystal, for which the nonlinearity is inherently saturable [21]. Hence, we also study numerically vortex lattice solitons in saturable nonlinear media, find their wave functions, and demonstrate their stability.

For the saturable nonlinearity, which applies to the photorefractive screening nonlinearity present in the experiment [9], the paraxial dynamics of the slowly varying amplitude ψ may be modeled as (in dimensionless units)

$$i \frac{\partial \Psi}{\partial z} + \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \right) - \left[\frac{1}{1 + V(x, y) + |\Psi|^2} \right] \Psi = 0, \quad (1)$$

where $V = V_0 \{ \cos[\pi(x+y)/d] + \cos[\pi(x-y)/d] \}^2$ is the square lattice induced by four interfering plane waves (as in [9,11]), where V_0 is peak intensity of the lattice and d is the lattice spacing. The lattice potential is written in this fashion because in the experiment, the lattice is optically induced in the photorefractive crystal by interfering beams of ordinary polarization, while the probe (soliton-forming) beam is extraordinarily polarized along the crystalline c axis [9–11]. (Note that to compare simulation with experiment, dimensional units can be obtained from $z \rightarrow z/k_0 \Delta n_0$ and $x \rightarrow x/\sqrt{2k_0^2 n_0 \Delta n_0}$, where $\Delta n_0 \simeq n_0^3 r_{33} V/2L$ is the index change caused by applying a voltage V , along the r_{33} direction, across a distance L .) The interference pattern of the (ordinarily polarized) array beams induces a 2D periodic index change (waveguide array) in the crystal with an index contrast proportional to the applied field. That is, *the probe experiences a periodic index change that becomes more pronounced as voltage is increased. For this reason, it is imperative that the experimental observation of the soliton be verified by lowering the probe intensity while keeping the voltage at the level at which the soliton forms.*

If, indeed, the elimination of diffraction arises from soliton effects, then lowering the probe intensity at this (high) voltage should lead to broadening of the output beam. This “test” can prove the inherent nonlinear (intensity-dependent) nature of the soliton and exclude the possibility that the reduced diffraction results from tighter (linear) waveguiding [11].

We find the wave functions of the unit-charge vortex lattice solitons by using the self-consistency method. This method is an iterative code, originally used to find soliton eigenfunctions in homogeneous media [22] and recently extended to find lattice solutions [23]. In this method, an initial intensity profile creates an index change (through the nonlinearity) acting as a defect in the lattice and localizes modes to its vicinity. In turn, the localized modal structure modifies the intensity profile, which again modifies index change (induced potential), and the process is iterated until a steady-state vortex mode is found. The profiles are then fed into Eq. (1), using a continuous beam propagation code (split-step Fourier scheme) [24] to model their linear and nonlinear propagation.

Typical beam propagation results showing on-site and off-site vortex solitons for $z = 800$ are given in Figs. 1(a)–1(d). Here, $d = 14$, $V_0 = 0.2$, and the vortices have a peak intensity of 0.04; the main four “lobes” all have the same peak intensity and, importantly, each lobe is $\pi/2$ out of phase with its neighbors. Note again that the singularity of the *on-site vortex* is centered on a lattice site [Figs. 1(a) and 1(b)], whereas the singularity of the *off-axis vortex* is centered between four lattice sites [Figs. 1(c) and 1(d)]. The soliton exhibits stationary propagation, and the shapes of the vortices remain unchanged; i.e., these are, indeed, vortex lattice solitons. For comparison, when the nonlinearity is set to zero, the vortices diffract by tunneling between lattice sites, as shown in Figs. 1(e) and 1(f) for the off-site vortex after $z = 800$. Note that the phase of the diffracting beam maintains its spiral structure throughout diffraction [Fig. 1(f)].

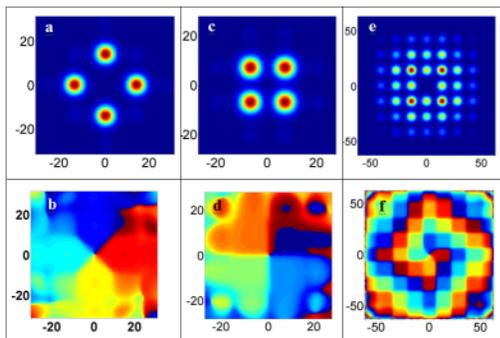


FIG. 1 (color online). Calculated intensity and phase of (a),(b) the on-site and (c),(d) the off-site vortex lattice solitons, along with the output diffraction pattern of (e),(f) the off-site vortex at $z = 800$.

Next, we investigate the stability properties of these discrete vortex states by simulating their propagation dynamics through the lattice in the presence of noise. Typical results are shown in Fig. 2. In the simulations, we add white noise in both amplitude and phase to a wide area encompassing the soliton. The noise is evenly distributed over the entire k space, and its power is at least 10% of the vortex power (in a section of k space 3 times larger than the soliton FWHM spectrum). Figure 2(a) shows stable propagation of the vortex soliton intensity for $z = 800$. We verify that the soliton is stable at least until $z = 6000$ (limited by our computation time). Figure 2(b) depicts the initial phase plane, with the coherent vortex structure clearly visible in a sea of noise, while Fig. 2(c) shows the phase after a propagation of $z = 800$. Note that higher frequency components of the noise have averaged out, leading to a smoother background than the input. We have also studied vortex lattice solitons in Kerr media, for the same conditions, and found that they are somewhat less robust than those of the saturable case, whose stability increases with saturation. In addition, we performed a linear stability analysis based on the discrete model for both cases, the details of which will be published elsewhere [25].

Note that the on-site vs off-site vortex solitons are the two-dimensional analogues of the Sievers-Takeno and Page modes, respectively, in a 1D array [26,27]. Intuition from the one-dimensional case suggests that the on-site vortex is energetically more favorable. In the 2D lattice, however, *both* solitons are possible, due to a combination of the supporting lattice potential and the

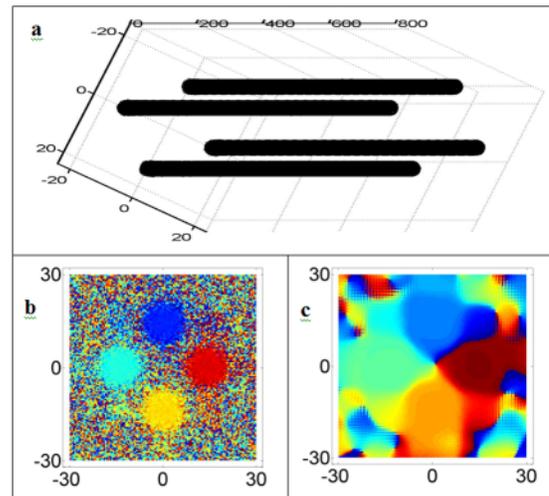


FIG. 2 (color online). Simulated propagation of the on-site vortex lattice soliton in the presence of noise. (a) The soliton intensity exhibits robust stationary propagation. (b) Phase at the input plane, clearly showing the presence of noise. (c) Phase after a simulated $z = 800$. Note that the high frequency components have averaged out, while the soliton maintains its vortex phase structure.

relative phase between sites. In this respect, the vortex soliton represents a pair of “twisted localized modes” [19,28], and the entire structure stays together under *focusing* nonlinearity. This is in distinct contrast to the homogeneous case, where bright rings are unstable [29] and only dark vortex solitons are possible with defocusing nonlinearity [30].

Experiments were performed using 488 nm light from an Ar⁺ laser and a 5 mm long SBN:75 photorefractive crystal. The experimental setup is sketched in Fig. 3(a). As in [11], two pairs of ordinarily polarized plane waves interfere to optically induce a square lattice inside the crystal. In this case, the lattice spacing was 14 μm , with each waveguide having a diameter of $\sim 7 \mu\text{m}$. The probe (soliton-forming) beam is extraordinarily polarized and is focused first onto a vortex mask (of unity topological charge) to create a vortex ring. This ring is then imaged with a $4f$ system, so that the vortex is mapped 1:1 onto the crystal input face with minimal distortion, and coupled into the waveguide array. Each array-forming plane wave has an intensity of $\sim 15 \text{ mW/cm}^2$, while the intensity ratio between the entire optical lattice and the probe beam is 7:1. The intensity pattern of the initial vortex ring, imaged at the input plane with the lattice, is shown for the on-site configuration in Fig. 3(b) and for the off-site configuration in Fig. 3(c).

Experimental results are shown in Figs. 4 and 5. The photorefractive screening nonlinearity is controlled by applying voltage against the crystalline c axis [21] and, in the proper parameter range, leads to localization of the probe beam and to the formation of lattice ring vortex solitons. At a low voltage ($\sim 100 \text{ V}$), the output diffraction of both the on-site and off-site vortices looks similar. Figures 4(a) and 4(b) show the diffraction pattern of an on-site vortex after 5 mm of propagation, showing that

both the hole and the width of the ring expand through the lattice. The ring expands to roughly 3 times its original size [Fig. 4(a)], while an interferogram, created by interfering the output pattern with a plane wave, clearly shows the $0 \rightarrow 2\pi$ spiral phase structure of the vortex [Fig. 4(b)]. *Note that the continuous symmetry of the homogeneous medium is broken by the lattice, so that angular momentum is not conserved in the periodic system. For this reason, phase information at the output is essential to show that the soliton maintains its vortex structure during propagation.* At around 700 V, the input vortices become vortex lattice solitons and maintain their structure (intensity and phase) while propagating. Figures 4(c) and 4(d) show the intensity and phase of an on-site soliton, while Figs. 4(e) and 4(f) show those features for an off-site soliton. Because phase information is very sensitive to noise in the system, only the relative phase of the relevant (soliton) lattice sites is shown for clarity.

As mentioned above, the relative strength of the waveguide array (index change experienced by the probe) is increased with increasing voltage bias [9]. Hence, to prove that the nondiffracting nature of the vortices indeed arises from the nonlinearity induced by the vortices themselves (and not from deepening the potential with voltage), it is necessary to reduce the probe intensity, while keeping the lattice parameters (i.e., voltage) fixed, and observe linear diffraction through the lattice. Experimentally, when the intensity of the probe beam is reduced by a factor of 12, at the same voltage as in Figs. 4(c)–4(f), the probe is too weak to form a lattice soliton; i.e., the vortices spread as they propagate through the lattice. This behavior is shown in Fig. 5(a) and 5(b) for the on-site and off-site vortex configurations, respectively. This is conclusive evidence of on-site and off-site vortex lattice solitons.

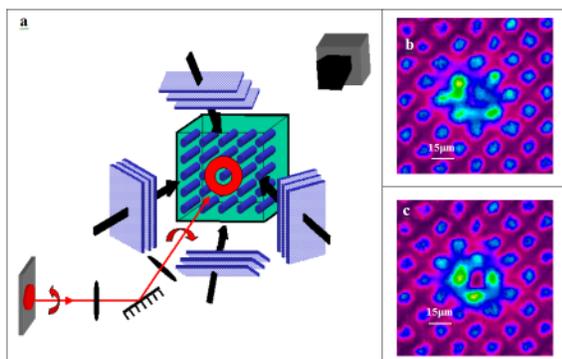


FIG. 3 (color online). Experimental scheme and vortex inputs with lattice. (a) A photonic lattice is optically induced in a photosensitive (photorefractive) crystal by interfering two pairs of plane waves, and a vortex probe is created by imaging a vortex mask with a $4f$ system. The output is then imaged into a charge coupled device camera. Experimental input of (b) on-site vortex and (c) off-site vortex, classified by the location of the phase singularity.

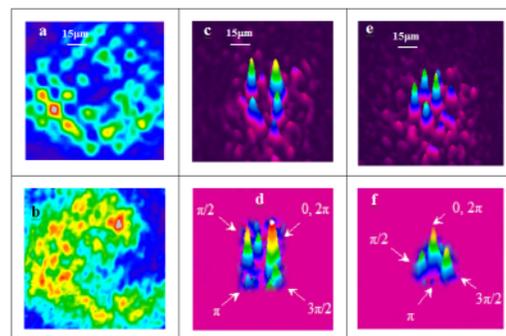


FIG. 4 (color online). Experimental results at the output face of crystal. (a),(b) Diffraction at 100 V: (a) intensity and (b) phase information formed by interference of output with a plane wave. (c) Typical on-site soliton at 700 V, and (d) relative phase from an interferogram. (e) Typical off-site soliton at 700 V, and (f) relative phase from an interferogram. Note that phase information is extremely sensitive to background noise, so only the interference patterns from the relevant lattice sites are shown.

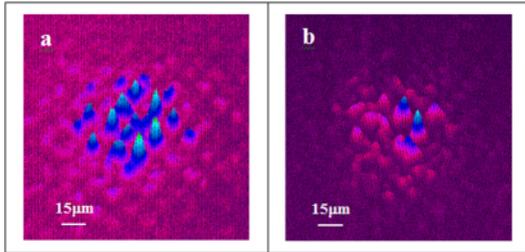


FIG. 5 (color online). Output images after reduction of the probe intensity by a factor of 12. (a) On-site vortex. (b) Off-site vortex. At this intensity, the probe can no longer support itself as a soliton and begins to diffract across the lattice by tunneling between sites.

The vortex solitons observed here are fundamental wave structures on 2D nonlinear lattices and can thus provide valuable insight into other systems in nature where similar dynamics can be potentially observed. For example, matter waves in optical traps [31] and charge density waves in planes of conductive polymers [2] can support solitons with topological charge. All-optical experiments also hold much potential for application, e.g., a 2D array of fiber lasers [15] can excite a vortex mode whose output can drive a phase-sensitive object.

In conclusion, we have experimentally demonstrated both the on-site and the off-site vortex solitons of unity topological charge in a nonlinear square lattice and have shown numerically that both configurations are stable in saturable nonlinear media. Extensions to vortex interactions, in this and other lattice geometries, will follow in future work.

This work was supported by the MURI program on solitons, the German-Israeli DIP project, and by the Ministry of Science, Israel. J.W.F. thanks the Lady Davis Foundation for generous support.

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