Theory of self-focusing in photorefractive InP

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We present a theory of self-focusing and solitons in photorefractive InP, including the previously unexplained intensity resonance and the resonant enhancement of the space-charge field.

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Self-trapping of optical beams in photorefractive (PR) InP was observed in 1996.[1,2] Self-trapping of beams in such PR semiconductors offers attractive features: operation at communications wavelengths, fast response times (microseconds), and low power level (microwatts).[1,2] These properties suggest exciting applications, such as reconfigurable switches, interconnects, and self-induced waveguides. Unfortunately, PR semiconductors tend to have tiny electro-optic coefficients (1.5 pm/V), and thus a conventional screening soliton in such materials would require applied fields of 50 kV/cm and higher,[3] too large for most applications. Fortunately, Fe-doped InP crystals offer an exception to this rule. When the photoexcitation rate of holes is comparable with the thermal excitation rate of electrons, the space-charge field is resonantly enhanced by more than tenfold. That is, the internal photoinduced field exceeds 50 kV/cm when the applied field is 5 kV/cm. This enhancement, and other peculiar phenomena, appears only in the vicinity of a particular intensity of the beam.[1,2] These features cannot be explained by the theory of screening solitons,[3–5] as that theory does not show a resonance of any sort. Furthermore, despite the experimental observations,[1,2] thus far there is no proof that self-trapped beams exhibiting stationary propagation can form in this nonlinearity.

Experimentally, when a beam is launched into an InP:Fe crystal under various applied fields E_0 and intensity conditions, the main features observed are the following[1]:

1. For $E_0 < 0$ and $I_{\max} \sim I_{\mathrm{res}}$ (the resonance intensity), self-focusing occurs, and the peak of the intensity structure, $I_{\max}$, is shifted from the center (of the normally diffracted beam at $E_0 = 0$).
2. For $E_0 > 0$ and $I_{\max} < I_{\mathrm{res}}$, the beam self-focuses, but at the resonance itself ($I_{\max} \approx I_{\mathrm{res}}$) the beam breaks up.
3. Irrespective of the polarity of E_0, for high enough intensity, $I_{\max} \gg I_{\mathrm{res}}$, the beam goes through the crystal almost unaffected, that is, diffracting normally as if $E_0 = 0$.

The first feature highlights the unique property of this nonlinearity: the pronounced intensity resonance at which the space-charge field is enhanced. Slightly below resonance, self-focusing occurs at $E_0 < 0$. The first and second features imply that self-focusing occurs at both field polarities (at different intensity regimes), whereas in other PR media the sign of the nonlinearity is determined by the polarity of E_0 and does not depend on $I_{\max}$. The second and third features suggest that this nonlinearity not only saturates but also decreases with intensity as $I_{\max}$ exceeds I_{res}.

Here we present a theory describing resonant self-focusing effects in PR semiconductors. We explain the main features of the theory, extract new predictions, and show that, in at least one parameter regime, stationary self-trapped beams (spatial solitons) do exist in such media. The theory is based on a model[6,7] with two levels of dopants, one deep Fe-trap level and conduction of both electrons and holes. This model is successful in explaining two-wave mixing but it cannot explain self-focusing, because it relies on a periodic grating at low visibility. Even a superposition of gratings cannot be used to treat localized beams. Yet the assumptions about the rate equations hold for a localized beam and serve as a starting point. We start with the standard set of equations[6] in temporal steady state: the continuity equations for electrons and holes, the rate equation for the Fe traps, the transport equations, and Gauss's law:

$$e_n n_t - \gamma_n n p_t + \frac{1}{q}\frac{\mathrm{d}J_n}{\mathrm{d}x} = 0\,, \tag{1a}$$

$$e_p p_t - \gamma_p p n_t + \frac{1}{q}\frac{\mathrm{d}J_p}{\mathrm{d}x} = 0\,, \tag{1b}$$

$$-e_n n_t + \gamma_n n p_t + e_p p_t - \gamma_p p n_t = 0\,, \tag{1c}$$

$$J_n = q n \mu_n E + q D_n \frac{\mathrm{d}n}{\mathrm{d}x}\,, \tag{1d}$$

$$J_p = q p \mu_p E - q D_p \frac{\mathrm{d}p}{\mathrm{d}x}\,, \tag{1e}$$

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{q}{\epsilon}(Nd - Na + p - n - n_t)\,, \tag{1f}$$

where $e_n = e_n^{\text{th}} + \phi_n I(x)$, $e_p = e_p^{\text{th}} + \phi_p I(x)$, e_n^{th} and e_p^{th} are the thermal emission rates, ϕ_n and ϕ_p are the photoionization cross sections, I is the intensity, Nd and Na are the donor and acceptor densities, and n_t and p_t are the density of filled and ionized traps. We seek an expression for the space-charge field E as a function of I for a single beam whose width is much larger than the wavelength. In such cases, the diffusion terms are negligible in Eqs. (1d) and (1e).[3–5] The immediate implication is that the space-charge field cannot change its sign. It must be either always positive or always negative (depending on E_0) if the total current is to be kept constant in space. Furthermore, for typical InP parameters,[1,2,6] the photoexcitation of electrons is negligible. Thus, within a good approximation (see Appendix A), the electron density is proportional to the concentration of holes, i.e., $p(x) \propto n(x)$. This means that the electron and hole currents, not only their sum, are both constants. These approximations, as well as all other results here, are found to be accurate to a fraction of a percent. Using this relation in Eq. (1c) and substituting Eq. (1d) into (1c), we find that

$$E(x) = E_0 \, \frac{I_{\text{res}}(x) - I_b}{I_{\text{res}}(x) - I(x)}\,, \tag{2}$$

where I_b is the "dark irradiance" and $I_{\text{res}}(x) = e_n n_t(x)/[\phi_p p_t(x)]$ is the *local resonance intensity*. Here, $\phi_p p_t I$ is the rate of trap filling (by electrons) and $\phi_p p_t I_{\text{res}}$ is the rate of thermal ionization. Therefore $I_{\text{res}}(x) - I(x)$ is proportional to the net electron ionization rate of the traps. In steady state, this rate must be compensated by the net recombination rate, which is given by the numerator of Eq. (2) (times the concentration of electrons). Note that, through Gauss's law, $I_{\text{res}}(x)$ depends on $\mathrm{d}E/\mathrm{d}x$ (the density of free carriers in Gauss's law can be neglected). However, if $\mathrm{d}E/\mathrm{d}x$ is small enough, I_{res} is roughly constant, and therefore the space-charge field is

$$E(x) = E_0 \, \frac{I_{\text{res0}} - I_b}{I_{\text{res0}} - I(x)}\,, \tag{3}$$

where $I_{\text{res0}} = e_n n_{t0}/(\phi_p p_{t0})$ and $n_{t0} = Nd - Na$. Clearly, the space-charge field exhibits resonance effects when $I = I_{\text{res0}}$. Equation (3) is no longer valid at the resonance, yet numerical studies of Eq. (2) show that it is valid up to $I_{\max} = 0.8 I_{\text{res0}}$, where $I_{\max}$ is the peak intensity of the beam. Thus, the field expression Eq. (3) is valid in the regime of the first experimental point listed above.

Having found $E[I(x)]$ of Eq. (3), we obtain the refractive-index change, $\Delta n(x) \propto E(x)$, substitute it into the wave equation,[3,4] and seek stationary solutions (solitons) in the regime $I_{\max} < I_{\text{res}}$. Using methods similar to those reported in Refs. 3–5, we obtain the soliton wave functions and the existence curve. The wave function of a soliton at $I_{\max} = 0.8 I_{\text{res0}}$ is shown in Fig. 1(a) and compared with the soliton wave functions for Kerr and screening (saturable) nonlinearities. Figure 1(b) shows the existence curve of each nonlinearity. As shown in Fig. 1(a), the normalized width of the InP soliton is much narrower than in the Kerr and the saturable cases; i.e., solitons of the same width require a lower $\Delta n(x)$ (a factor of 2 at $I_{\max} = 0.8 I_{\text{res0}}$). This fact, and the resonant enhancement of $E(x)$, explain why PR solitons are observed in InP even though the electro-optic coefficient of InP is tiny.

As is well established, the PR screening nonlinearity saturates with increasing intensity (normalized to the background intensity).[3–5] Yet the resonant nonlinearity of solitons in InP does not saturate. Instead, $\Delta n(x)$ deforms: As $I_{\max}$ approaches I_{res} (or exceeds it), the induced $\Delta n(x)$ becomes asymmetric. This asymmetry arises from $\mathrm{d}E/\mathrm{d}x$ in Eq. (1f), which is no longer negligible. Figure 2 shows a numerical solution for $I_{\max} = 1.3 I_{\text{res0}}$. The local resonance intensity is modified by $\mathrm{d}E/\mathrm{d}x$ so that it is always larger than the optical intensity. Note that at extremum points (where $E' = 0$), Eq. (3) holds. This leads to an important conclusion: Since the numerator of Eq. (3) is positive, and since the sign of the field (neglecting diffusion) cannot be changed merely by an increase in the intensity, the denominator of Eq. (3) must be positive. This fact implies that the maximum value of the electric field occurs at a point for which $I(x) < I_{\text{res0}}$. When $I_{\max}$ is increased considerably above I_{res0}, the induced $\Delta n(x)$ shifts away from the beam until they no longer overlap, and $\Delta n(x)$ has a diminishing effect on the beam. This result explains why for $I_{\max} \gg I_{\text{res0}}$ the beam displays primarily linear diffraction. Experimentally,[1] at $I_{\max} \approx I_{\text{res0}}$ and slightly above this value, the induced $\Delta n(x)$ causes self-focusing. In those data, $I_{\text{res0}} \cong 38\ \text{mW/cm}^2$, and indeed at $I_{\max} \cong 35\ \text{mW/cm}^2$ self-focusing is apparent. However, at much higher intensities, self-focusing diminishes; this explains the third feature presented

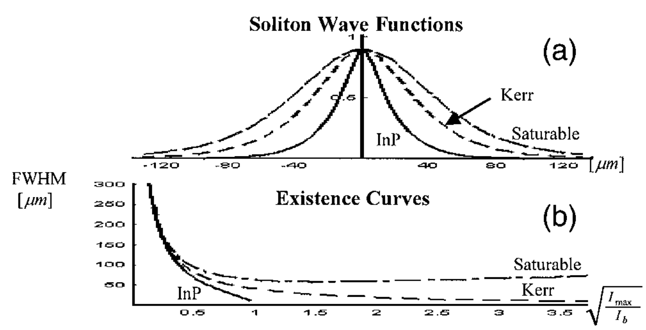

Fig. 1. (a) Soliton wave functions and (b) existence curves for Kerr ($\Delta n \propto I$), saturable [$\Delta n \propto -1/(1 + I)$], and InP [$\Delta n \propto 1/(1 - I)$] nonlinearities.

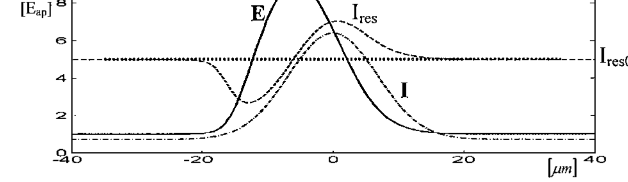

Fig. 2. Space-charge field $E(x)$ (solid curve) and local resonance intensity $I_{\text{res}}(x)$ (dashed curve) for a given intensity $I(x)$ (dashed–dotted curve). The peak intensity is $1.3 I_{\text{res0}}$. At the peak, $I < I_{\text{res}} = I_{\text{res0}}$.

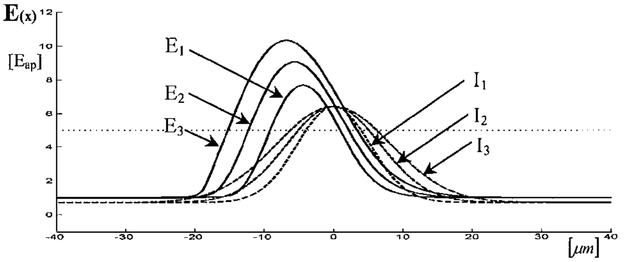

Fig. 3. Space-charge field $E(x)$ for three different beam widths. The peak intensity is the same for all three beams ($I_{\max} = 1.3I_{\mathrm{res}0}$).

above. Thus, the disappearance of self-focusing at intensities high above resonance results not from a decrease of Δn (as initially thought[1]) but because the induced waveguide is shifted to outside the beam.

The trends described above are valid irrespective of the polarity of E_0: Reversing E_0 should switch the nonlinearity from self-focusing to self-defocusing, as observed.[1] However, we note that we did not study the dynamic self-focusing process but rather developed a relation between $\Delta n(x)$ and $I(x)$ and then solved for solitons. Some of the results described in feature 2 (the beam breakup at resonance, which occurs only for $E_0 > 0$) cannot be explained on the basis of stationary propagation. Modeling beam breakup necessitates using a beam-propagation code and solving Eq. (2) at every step. But there is one simple dynamic case that can be readily explained. Inspection of Eq. (2) (and Gauss's law embedded in it) reveals that changing $x \to -x$ and $E \to -E$ (also $E_0 \to -E_0$) leaves the equation unchanged. This result means that, if $E(x)$ is a solution for $E_0 < 0$, then $-E(-x)$ is a solution for $E_0 > 0$ (for the same I). Therefore, if for $E_0 < 0$ and $I_{\max} \gg I_{\mathrm{res}0}$ the beam and $\Delta n(x)$ do not overlap, they do not overlap for $E_0 > 0$ either. This is why hardly any effect is observed at high enough intensities, at either E_0 polarity. Thus, the narrow beam mentioned in the second feature results from a shifted antiguide. Since the antiguide is dislocated with respect to the beam center [e.g., $E(x) \to -E(-x)$ in Fig. 2], most of the power in the beam is pushed to one side [where $\Delta n(x)$ is less negative]. This pushing to one side is why the beam appears self-focused, even though in this polarity the beam forms an antiguide.

Our numerical solution of Eq. (2) shows another effect that has yet to be observed experimentally. The enhancement of $E(x)$ above the resonance intensity depends on the beam width: Broader beams lead to larger field enhancement. Figure 3 shows how the electric field increases with beam width for a given $I_{\max}$. This increase is a consequence of the modification of the local resonance intensity by $\mathrm{d}E/\mathrm{d}x$, occurring only for $I_{\max} > I_{\mathrm{res}0}$. [Otherwise, at $I_{\max} < I_{\mathrm{res}0}$, Eq. (3) is valid; i.e., the resonant enhancement does not depend on the width of the beam.]

To conclude, we have formulated what we believe to be the first theory of self-focusing effects in PR media in which both (thermal) electrons and (photoexcited) holes contribute to the transport. We have shown that self-focusing is greatly enhanced by the proximity of an intensity resonance and derived an expression for the space-charge field in the subresonance regime. We have shown theoretically that spatial solitons form in this medium, interpreted the unique near-resonance features observed experimentally, and predicted the dependence of the field enhancement on the beam width. The theory relates to all PR materials supporting both types of charge carrier.

Appendix A

This Appendix justifies the $p(x) \propto n(x)$ approximation. We substitute Eqs. (1d), with the diffusion term neglected, and (1f) into Eq. (1a) and obtain an equation for $E(x)$ with x-dependent coefficients:

$$\frac{\mathrm{d}E}{\mathrm{d}x} + \frac{q\mu_n(\mathrm{d}n/\mathrm{d}x)}{q\mu_n n - (e_n + \gamma_n n)} E = \frac{q(\gamma_n n p_{t0} - e_n n_{t0})}{q\mu_n n - \epsilon(e_n + \gamma_n n)},$$

and a similar equation for the holes. These equations share the same solution [for $E(x)$]; therefore they must have the same coefficients. By comparing the electric field coefficients, we get

$$\frac{\mu_n(\mathrm{d}n/\mathrm{d}x)}{\mu_n n - (e_n + \gamma_n n)/\epsilon_0} = \frac{\mu_p(\mathrm{d}p/\mathrm{d}x)}{\mu_p p + (e_p + \gamma_p p)/\epsilon_0}.$$

For typical values[1,4] the second term in the denominators is negligible, yielding $p(x) \propto n(x)$. The proportionality factor is obtained by solution of Eqs. (1a) and (1b) far away from the beam, where all derivatives are zero.

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