Self-trapping of bright rings

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We present experimental observations of self-trapped rings carrying zero topological charge, along with simulations that display the self-focusing dynamics of the rings and their stability features in materials with saturable nonlinearities.

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Spatial solitons are stable self-trapped wave packets propagating in nonlinear media for which diffraction is exactly balanced by the self-focusing effect of the nonlinearity.[1] Optical spatial solitons have been identified in many physical systems and can self-trap in one or two transverse dimensions.[2] Solitons of circular symmetry have been investigated extensively for both Kerr[3–9] and saturable[10–15] Thus far, two families of solitons with azimuthal (circular) symmetry have been identified. The first one corresponds to solitons with a maximum intensity in the center surrounded by bright rings,[3,10] and the second, to solitons with a dark spot in the center surrounded by bright rings.[11–15] In both cases these solitons have been shown to be unstable in Kerr media.[8,9] Depending on whether their power is above or below a critical point, Kerr solitons either focus to a zero radius (i.e., undergo catastrophic collapse) or diffract and broaden. Saturation suppresses this collapse, but the rings still suffer from azimuthal instabilities, which grow with propagation, and the ring disintegrates into multiple filaments.[10–15] In saturable nonlinear media, only the fundamental soliton, which is a bright spot with no surrounding rings, is free from azimuthal instabilities.[10] A bright ring, however, can self-trap in a stable fashion in a saturable medium (at least for hundreds of diffraction lengths) if it is a component of a composite soliton.[16] Another family of ring solitons comprises the necklace beams,[7] rings that become stable by having the intensity azimuthally modulated and resemble pearls in a necklace.

Here we study self-trapping of azimuthally uniform bright rings that carry zero topological charge in a saturable nonlinear medium. In particular, we present what is to our knowledge the first experimental study of the self-focusing dynamics of bright rings. We start with the normalized paraxial wave equation in cylindrical coordinates:

$$i\,\frac{\partial \Psi}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\,\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\,\frac{\partial^2 \Psi}{\partial \phi^2}\right) + \Delta n(|\Psi|^2)\Psi = 0\,, \quad (1)$$

where $\Delta n(|\Psi|^2)$ is the intensity-dependent refractive-index change. We seek self-trapped rings that are azimuthally uniform and look like one-dimensional (1D) solitons wrapped around their tails. Before we move on to saturable nonlinearities, it is instructive to review such rings for the Kerr case, where $\Delta n(|\Psi|^2) = |\Psi|^2$.[4–7] In Kerr media, a ring soliton that carries no topological charge can be approximated as $\Psi(r, z = 0) = a\,\mathrm{sech}[a(r - R)]$, where R and $1/a$ are the radius and the width of the ring, respectively. This approximation becomes an exact solution of Eq. (1) in the limiting case of $R \gg 1/a$, where the third term is negligible and the equation reduces to the nonlinear Schrödinger equation in one dimension.[4–7] In saturable media the solutions are not $\mathrm{sech}(r)$ and need to be calculated numerically, but the idea of taking the 1D soliton and wrapping it around into a ring still holds. Because these kinds of solution are not exactly solitons, they undergo evolution as they propagate. Numerical simulations in Kerr media have shown that these solutions shrink and eventually collapse to a point.[5] This is so because all the parts of the ring are in phase with one another, and therefore a net inward radial force is exerted on each point on the ring. A fast periodic azimuthal modulation, $\cos(\Omega\,\phi)$,[7]

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or a topological charge term $\exp(im\phi)$,[5] breaks the in-phase symmetry of the ring and can even reverse the shrinking to expansion.

We study numerically and experimentally the dynamics of optical rings carrying zero topological charge for a saturable nonlinearity of the form $\Delta n(|\Psi|^2) = -1/(|\Psi|^2 + 1)$, which applies to a homogeneously broadened electronic two-level system and to the photorefractive screening nonlinearity.[17] We first solve the paraxial wave equation in one dimension to find the 1D soliton. We then wrap the 1D soliton into a ring and propagate it numerically. In our first numerical experiment, the radius of the ring is 13, and the width is ≈ 2.5, and thus one diffraction length (the distance at which the ring width broadens by a factor of $\sqrt{2}$ in the absence of nonlinearity) is ≈ 3. Figure 1 shows the results of this simulation, displaying intensity cross sections at various propagation distances for the high (top row) and for the low (bottom row) intensity regimes, corresponding to deep and weak saturation, respectively. The ring undergoes considerable compression in both cases and forms a central peak surrounded by an annular ring. The rate of contraction is much slower than with the Kerr nonlinearity,[5] as expected, because instabilities grow more slowly in saturable nonlinear media. In both cases, after $z = 54$ the central beam undergoes cycles of compression and expansion, and, for as long as we are able to simulate the problem numerically, the cycles continue.

In our second numerical experiment, we increased the radius of the ring to 25. Now the attractive inward force was smaller and the shrinking tendency was greatly reduced, as shown in the top row of Fig. 2, representing the deep-saturation regime. The long-term evolution of the beam in the top row of Fig. 2 resembles the evolution of the beams of Fig. 1 but at a slower rate. In the weak-saturation regime (Fig. 2, bottom row), we observe the appearance of azimuthal instability along with the shrinking. At $z = 69$, the ring breaks into two-dimensional filaments, i.e., into an intermediate unstable state. On further propagation, the ring of filaments disintegrates, some of it coalesces into a central beam, and the rest decays into unbound radiation modes. This behavior is characteristic of rings with a large enough radius and at weak saturation. Interestingly, we observe that in the weak-saturation case the radius of the ring (Fig. 2, bottom) has not shrunk as much as with the same ring at the deep-saturation regime (Fig. 2, top). This is so because, owing to the azimuthal instability, different parts of the ring are no longer in phase but repel one another (similarly to what occurs in the research reported in Ref. 7), and the outward radial force slows down the shrinking tendency.

Our simulations indicate that the ring solitons (Figs. 1 and 2) do not undergo much change for as many as nine diffraction lengths. This means that self-trapped rings with little or no propagation evolution (contraction or instability) should be experimentally observable.

Our experiments were conducted in a photorefractive SBN:60 crystal in a standard setup in which an external electric field was applied along the c axis. The soliton beam was polarized along the c axis, and a uniform, orthogonally polarized background beam fixed the intensity ratio.[17,18] To generate the ring, we reflected the beam coming out of an Ar^+ laser off a fast rotating mirror and then demagnified and imaged it onto the input face of a photorefractive crystal. Because of the inclination of the mirror, the beam was constantly moving in a circular motion, which, when it was fast enough, gave the appearance of a ring. In our experiments the beam formed roughly 10 circles/s. The nonlinearity was of the photorefractive screening type, which occurs in temporal steady state in biased photorefractives. When photorefractive response times are set to be slow (with low enough intensity), the material effectively sees a ring. This means that the nonlinear change in the refractive index, after a temporal steady state is reached, responds to this rotating beam exactly as it responds to a stationary (in time) ring beam.

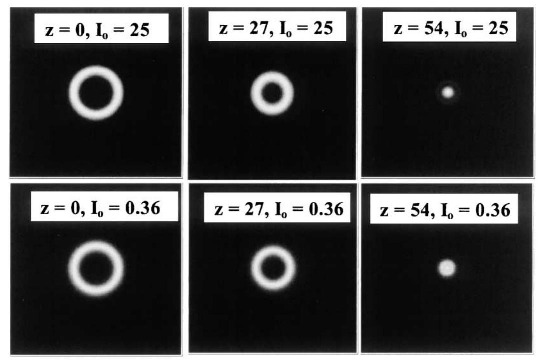

Fig. 1. Numerical simulations of ring solitons in a saturable nonlinear medium at high (top) and low (bottom) saturation. Intensity cross sections during propagation are shown. The ring undergoes compression in both cases. I_0 is the maximum intensity upon the ring. The ring radius, R, is 13. In dimensional units (representing typical parameters in photorefractives), these results correspond, for example, to $R = 43$ μm and a ring width of 8 μm for $\lambda = 0.488$ μm, $n = 2.37$, and $\Delta n_{\max} = 0.0002$. Propagation distance $z = 54$ corresponds to 18 mm.

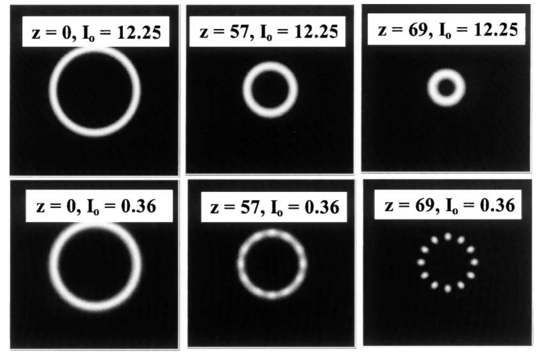

Fig. 2. Same as Fig. 1 but for $R = 25$. The high saturation (top) shrinks, and the low saturation (bottom) suffers azimuthal instability. In dimensional units, $R = 86$ μm and the ring width is 8 μm for $\lambda = 0.488$ μm, $n = 2.37$, and $\Delta n_{\max} = 0.0002$. Propagation distance $z = 69$ corresponds to 22.5 mm.

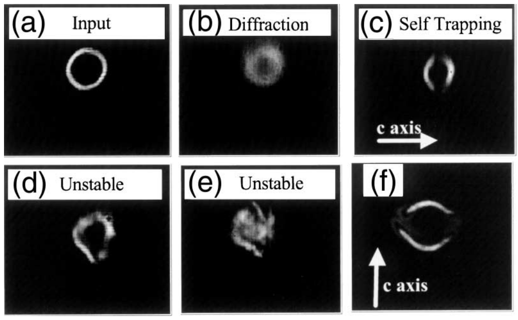

Fig. 3. Experimental results for a photorefractive SBN:60 crystal. (a) Input ring of 12-μm FWHM and a radius of 55 μm at an intensity ratio of 20. (b) Diffraction output. (c) Self-trapped output with nonlinearity on. (d), (e) Unstable soliton output for intensity ratios 0.33 and 0.10, respectively. (f) Same as (c) but with the c axis is rotated by 90°. The experiment is in the parameter regime of the simulation of Fig. 1.

In the first set of experiments, shown in Figs. 3(a)–3(c), we launched a ring at an intensity ratio of 20 (this is the maximum soliton intensity normalized to the background intensity) of 12-μm FWHM and a radius of 55 μm at the input [Fig. 3(a)]. When no external field was applied, the ring (linearly) diffracted after 5 mm [Fig. 3(b)]. With the application of 1.3 kV/cm the ring trapped to almost its original size [Fig. 3(c)]; yet instead of a trapped circular ring we observed two half-moons. This effect results from the anisotropy of the photorefractive nonlinearity. Most probably, circular rings cannot be trapped in photorefractive media; yet two narrow half-moons obviously can. We then reduced the intensity ratio and examined the dynamics in the weak-saturation regime. We observed that the ring became increasingly unstable. The output self-focused rings for intensity ratios of 0.33 and 0.10 are shown in Figs. 3(d) and 3(e), respectively. It is clear that the rings are much more unstable than expected from the simulations. Once again, the anisotropy of the underlying nonlinearity has rendered the rings more unstable than they should be. A nice way to highlight the anisotropy is to rotate the crystal such that the c axis is vertical and launch the ring under the conditions of Fig. 3(c). As shown in Fig. 3(f), the asymmetric focusing is indeed caused by the material anisotropy.

Before closing, we note that the azimuthal instability of ring solitons could be completely eliminated if the beam were made partially incoherent. As was shown in Ref. 19, the transverse instability of a 1D Kerr soliton is eliminated when the soliton is coherent in the trapped dimension and is sufficiently incoherent in the transverse dimension. In the same vein, we expect that, with a ring that is fully coherent in the radial direction (or with the ring width much narrower than the radial correlation distance) yet sufficiently incoherent in the azimuthal direction, all azimuthal instabilities should be eliminated. This will require launching a ring beam with an anisotropic coherence function.[20]

In conclusion, we have investigated numerically and experimentally ring solitons that carry zero topological charge in saturable nonlinear media. We found numerically that the rings eventually undergo collapse and that the effects of azimuthal instability become important only for large radii. Experimentally, we observed self-trapped semirings (two half-moons) in the high-saturation regime, but they are unstable for low saturation. This is to our knowledge the first experimental study of the self-trapping dynamics of bright rings in any nonlinear medium. It proves that, in principle, it should be possible to observe true self-trapped rings in isotropic saturable nonlinear media for considerable propagation distances. In fact, the only reason that we observed self-trapping of half-moons (semirings) is the anisotropy of the photorefractive nonlinearity used in our experiments.

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